

Discussion on structural damage identification based on wavelet packet total energy of structural responses under random excitations

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Abstract. Structural damage identification based on wavelet packet energy has been applied due to its powerful time-frequency analysis capability. Current methods are mainly based on the wavelet packet total energy of structural acceleration or displacement responses, and these responses primarily reflect structural overall characteristics. Moreover, it is essential to investigate the influence of different excitations on identifying structural damage under random excitations. Therefore, three studies on structural damage identification based on wavelet packet total energy of structural responses under random excitations are discussed in this paper. First, structural damage identifications based on wavelet packet total energy of structural acceleration, displacement, and macro-strain/strain responses are discussed, respectively. Second, the effects of different excitations to intact and damaged structures on structural damage identification based on wavelet packet total energy of structural macro-strain/strain responses are also discussed. Finally, to eliminate the influence of random excitations, structural damage identifications based on transmissibility functions of wavelet packet total energy of structural macro-strain/strain responses are proposed. Through numerical studies of structural damage identification of both beam-type and truss structures, it is illustrated that the wavelet packet total energy change rate based on macro-strain/strain responses is more sensitive to local damage, different excitations significantly affect structural damage identification, and the influence of random excitations can be eliminated by the transmissibility functions of wavelet packet total energy of structural macro-strain/strain responses.

Keywords: random excitations; structural damage identification; structural strain responses; transmissibility functions; wavelet packet energy

1. Introduction

Due to the influence of various adverse factors such as environmental variations and external excitations, structures may experience different degrees of damage during their service period, and the accumulation of damage might subsequently endanger their safety. Consequently, structural damage identification methods using structural vibration responses have been extensively applied to ensure structural safety and durability (Gordan *et al.* 2022, Kiremidjian 2022). These methods are mainly divided into model-based and data-driven methods (Azhar 2024, Wang *et al.* 2023). The model-based methods rely on accurate finite element models, utilizing damage indexes or model updating techniques, such as optimization algorithms (Meysam and Bahar 2021) and Kalman filtering (Diaz *et al.* 2023) for damage identification. The data-driven methods employ signal processing techniques in the time domain, frequency domain, and time-frequency domain for extracting damage features from vibration responses, including autoregressive moving average (Achilli *et al.* 2021), power spectral analysis (Bayat *et al.* 2019) and wavelet packet energy

analysis (Pan *et al.* 2019), etc. Among them, wavelet packet energy analysis has a strong time-frequency analysis capability and is highly sensitive to structural damage. When local damages occur in a structure, dynamic responses at damage locations become more disordered, and changes in structural dynamic responses induce variation in their wavelet packet total energy (Pan *et al.* 2019). The wavelet packet total energy changes at damage locations are typically the most significant, which provides a basis for structural damage identification (Ouyang *et al.* 2024).

So far, structural damage identification methods have been developed based on wavelet packet energy of structural vibration responses. For example, Yu and Ma (2022) identified structural damage by analyzing wavelet packet energy ratio changes in key frequency bands of acceleration responses. Zhao *et al.* (2019) extracted wavelet packet energy from structural acceleration responses to define damage indexes, including standard deviation energy ratio change and energy ratio deviation, for detecting slight damage in a beam. Fang and Chen (2020) developed wavelet packet energy curvature from structural acceleration responses to locate steel beam damage. Wang and Shi (2018) used wavelet packet energy of structural acceleration responses to propose wavelet packet energy curvature difference for damage identification. These studies identify structural damage by analyzing changes in wavelet packet energy derived from acceleration responses. However, structural acceleration responses primarily reveal

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overall structural characteristics, while structural damage is often local.

Compared with structural acceleration responses, structural strain responses better reflect structural local characteristics and are more sensitive to local damage (Noori *et al.* 2018). Azim and Gül (2021) used structural strain responses based on principal component analysis to locate damage to railway truss bridges. Zhang *et al.* (2020) utilized structural strain responses to construct strain standard deviation for identifying damage in simply supported beams. Singh *et al.* (2018) identified frame structural damage by pseudo-inverse techniques using strain mode modes and frequencies. However, point strain sensors only collect structural local information. For beam-type structures, if sensors are installed far away from structural damage locations, damage information may not be fully captured. To solve this problem, Li and Wu (2007) proposed the concept of long-gauge fiber Bragg grating strain sensors, and structural health monitoring systems based on structural long-gauge strain (macro-strain) responses have been developed (Campanella *et al.* 2018). Saifeldien *et al.* (2024) proposed a method for detecting damage in reinforced concrete bridges using absolute long-gauge strain ratios derived from long-gauge strain responses. Noori *et al.* (2018) combined the wavelet packet energy rate index of long-gauge strain responses with the envelope area of strain-time curvature to propose a modified wavelet energy rate-based index for identifying damage in steel bridges. However, there is a lack of discussion on wavelet packet total energy damage identification based on structural acceleration, displacement, and strain/macro-strain responses, respectively.

In addition, structural responses are influenced by both external loads and damage, and external loads to intact and damaged structures are different for structures under random excitations. Therefore, it is necessary to investigate the effects of random excitations on structural damage identification based on wavelet packet total energy, which is based on the structural response of intact and damaged structures under different excitations. Furthermore, it is essential to eliminate the influence of different excitations and identify structural damage accurately based on the wavelet packet total energy of structural responses under random excitations. For this purpose, structural response transmissibility functions (TFs) have attracted much attention in structural damage identification. The damage identification methods based on structural response TFs are independent of excitations and damage-sensitive (Yan *et al.* 2019, Li *et al.* 2021). Wu *et al.* (2021) conducted frequency domain acceleration response TFs to detect and locate debonding in concrete pavement. Cheng and Cigada (2020) developed a damage index from intact and damaged frequency domain acceleration response TFs to identify damage in multi-degree-of-freedom systems and clamped-clamped beams. The authors (Liu *et al.* 2022) also used the Mahalanobis square distance of frequency domain strain response TFs before and after damage to locate structural damage under unknown seismic excitations. These studies

construct TFs from the ratio of two frequency domain responses, but their accuracy is highly sensitive to frequency band selection, with inappropriate choices potentially causing incorrect damage identification (Li *et al.* 2017). Moreover, since the Fourier transform is global, damage indexes based on frequency domain response TFs may not be sensitive to minor local damage (Li and Hao 2013). To avoid the problem of frequency band selection and enhance the ability of local feature extraction, Li *et al.* (2017) developed wavelet based TFs from displacement responses and constructed a damage index to detect local damage in a cantilever structure. To further improve the high-frequency resolution of signals, Wang *et al.* (2022) utilized wavelet packet total energy TFs from structural acceleration responses for fan blade fault detection. These studies identify local damage based on structural acceleration or displacement responses. The authors (Liu *et al.* 2023) used inverse Fourier transform to convert frequency domain TFs of structural strain responses into time domain signals and identified structural damage through energy changes of time domain signals. As the authors know, there is a lack of research on the influence of random excitations on structural damage identification, as well as the combination of wavelet packet total energy of structural macro-strain/strain responses with TFs to identify structural damage under random excitations.

Based on these literature reviews, three studies of structural damage identification based on wavelet packet total energy of structural responses under random excitations are discussed in this paper. First, structural damage identifications based on structural acceleration, displacement, and macro-strain/strain wavelet packet total energy are discussed, respectively. Second, the effects of random excitations on structural damage identification, namely, different excitations for intact and damaged structures are also discussed based on wavelet packet total energy of structural macro-strain/strain responses. Third, to eliminate the interference from random excitations, structural damage identifications based on TFs of wavelet packet total energy of structural macro-strain/strain responses are proposed. Some numerical examples of structural damage identification of both beam-type and truss structures based on structural responses wavelet packet total energy are used in these three studies.

The rest of the paper is organized as follows: Section 2 briefly illustrates structural damage index based on structural responses wavelet packet total energy. In Section 3, the damage identification performance of wavelet packet total energy using structural acceleration, displacement, and macro-strain/strain responses are discussed respectively, for both beam-type and truss structures. In section 4, the influence of different excitations on damage identification of beam-type and truss structures is also discussed. In Section 5, damage identification based on wavelet packet total energy transmissibility functions of structural macro-strain/strain responses is proposed, and its effectiveness is verified by both beam-type and truss structures. The conclusions of this paper are drawn in Section 6.

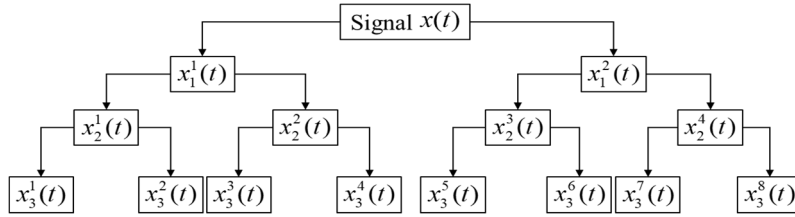


Fig. 1 Schematic diagram of three-layer wavelet packet decomposition

2. Damage index based on structural responses wavelet packet total energy

Wavelet packet analysis can not only decompose the low-frequency component of signals in detail but also analyse the high-frequency part of signals deeply. In addition, it can flexibly select the most suitable frequency range according to the specific characteristics of signals, thus significantly improving signal resolution ability. Take three-layer wavelet packet decomposition as an example, its wavelet packet decomposition tree can be shown in Fig. 1.

A signal $x(t)$ is decomposed by an i -layer wavelet packet, it can be expressed as

$$x(t) = \sum_j^{2^i} x_i^j(t) \quad (1)$$

where i is the number of decomposition layers; j stands for the number of components; $x_i^j(t)$ represents the wavelet packet component of the signal $x(t)$ after wavelet packet decomposition

$$x_i^j(t) = \sum_{k=-\infty}^{+\infty} c_{i,k}^j \psi_{i,k}^j(t) \quad (2)$$

where $\psi_{i,k}^j(t)$ and $c_{i,k}^j$ are wavelet packet function and wavelet packet coefficient respectively; k represents the position transform parameter of the wavelet packet function.

The energy of each frequency band reveals the signal distribution across different frequency ranges. The component energy E_i^j in the j -th frequency band of the i -layer wavelet packet decomposition can be shown as

$$E_i^j = \int_{-\infty}^{\infty} x_i^j(t)^2 dt \quad (3)$$

Then, the wavelet packet's total energy E can be shown as follows

$$E = \sum_j^{2^i} E_i^j \quad (4)$$

If a structure is damaged, its dynamic characteristics will inevitably change, leading to corresponding variations in the wavelet packet total energy of its dynamic responses. More severe damage induces greater alterations in structural responses and greater variations in wavelet packet total

energy between intact and damaged structures. Therefore, based on the wavelet packet total energy of structural acceleration responses, Ouyang *et al.* (2024) defined the change rate of wavelet packet total energy of the m -th measuring location as a Damage Index (DI_m) for structural damage identification, as shown in follows

$$DI_m = \frac{E_m^d - E_m^u}{E_m^u} \quad (5)$$

where E_m^u and E_m^d represent the wavelet packet total energy of the m -th measuring location response in intact and damaged structures, respectively. The DI_m is calculated for each measuring location, and the vector $DI = [DI_1, DI_m, \dots, DI_n]$ of a structure under all n measuring locations can be obtained. Structural responses of damaged locations are more disordered, and their wavelet packet total energies are increased, that is, the E_m^d at damaged locations increase compared with E_m^u at intact locations, and their changes are the largest. Thus, structural damage can be located according to the DI values.

It is worth noting that when structural responses are decomposed using wavelet packet analysis, an appropriate wavelet packet function and decomposition layer should be selected. Daubechies (DB) family of wavelet packet functions are widely used in wavelet packet analysis due to their desirable regularity and high-order vanishing moments, which have significant advantages in structural damage identification (Silik *et al.* 2022). In this paper, the Db8, Db20, and Db45 wavelet packet functions are employed to decompose structural responses, and various decomposition layers are discussed for each wavelet packet function.

3. Discussions on wavelet packet total energy of structural acceleration, displacement, and macro-strain/strain responses

Existing damage identification methods primarily rely on wavelet packet total energy derived from structural acceleration or displacement responses, which are less sensitive to local damage than macro-strain/strain responses. Moreover, there is a lack of comprehensive discussions on using wavelet packet total energy for damage identification based on acceleration, displacement, and strain responses. To address this gap, the damage identifications based on wavelet packet total energy are discussed for structural acceleration, displacement, and macro-strain responses in beam-type and truss structures.

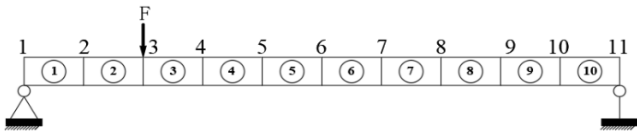


Fig. 2 Simply supported beam model

3.1 Discussion on beam-type structures

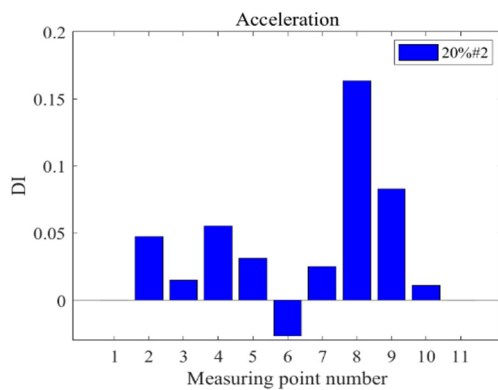
A numerical model of a beam-type structure (simply supported beam) under single-point white noise excitation is first studied. The simply supported beam model is depicted in Fig. 2, which consists of 10 elements with a total length of 5 m. The cross-section height and width are 0.015 m and 0.05 m, respectively. The mass density and elastic modulus are 7800 kg/m³ and 200 GPa respectively, and the damping ratios of the first two modes are 2%. Vertical random white noise excitation is applied at node 3. Acceleration and displacement measuring points are arranged at nodes 1-11, while macro-strain responses on the upper surface of elements 1-10 are observed. The sampling frequency and sampling time are 100 Hz and 15 s, respectively.

3.1.1 Damage identification based on wavelet packet total energy of structural acceleration responses

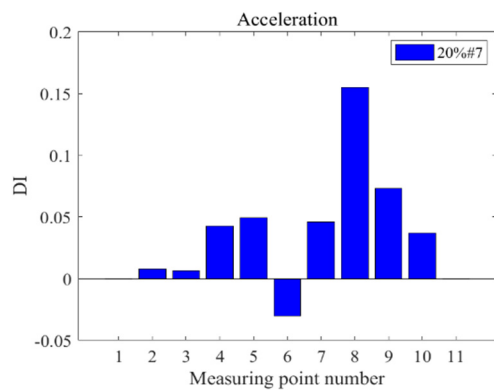
Structural damage is simulated by reducing the elastic modulus of specific elements, and two damage cases are considered. Damage case 1 is 20% damage to element 2, and damage case 2 represents 20% damage to element 7. The same white noise excitation is applied both before and after structural damage. Initially, random load sample 1 is applied to node 3. To mitigate the influence of load randomness on damage identification, another random load sample 2 is applied to node 3 under the same two damage cases. For intact and damaged cases, acceleration responses at nodes 1-11 are observed to estimate their wavelet packet energies. Subsequently, the DI values are derived from these wavelet packet energies to identify structural damage.

It is found that the same damage identification results can be obtained using the wave packet functions of the Db8, Db20, and Db45 under different decomposition layers. Subsequently, the 3-layer wavelet packet decomposition of the Db45 is taken as an example, and the corresponding DI values are utilized to identify structural damage. The damage identification results based on acceleration responses under two random load samples are presented in Figs. 3 and 4, respectively.

Figs. 3 and 4 display that even when random loads applied before and after damage are identical, the DI values

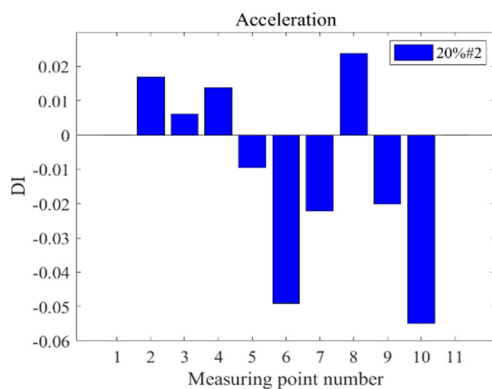


(a) Damage case 1

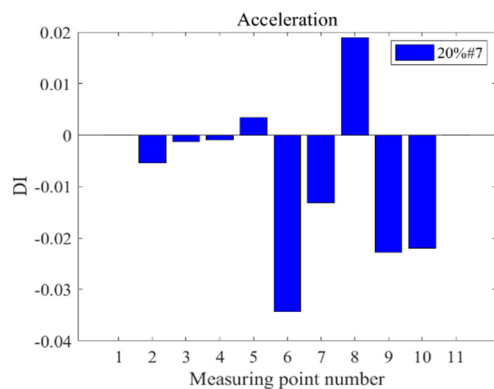


(b) Damage case 2

Fig. 3 Histogram based on wavelet packet total energy of structural acceleration responses under random load sample 1



(a) Damage case 1



(b) Damage case 2

Fig. 4 Histogram based on wavelet packet total energy of structural acceleration responses under random load sample 2

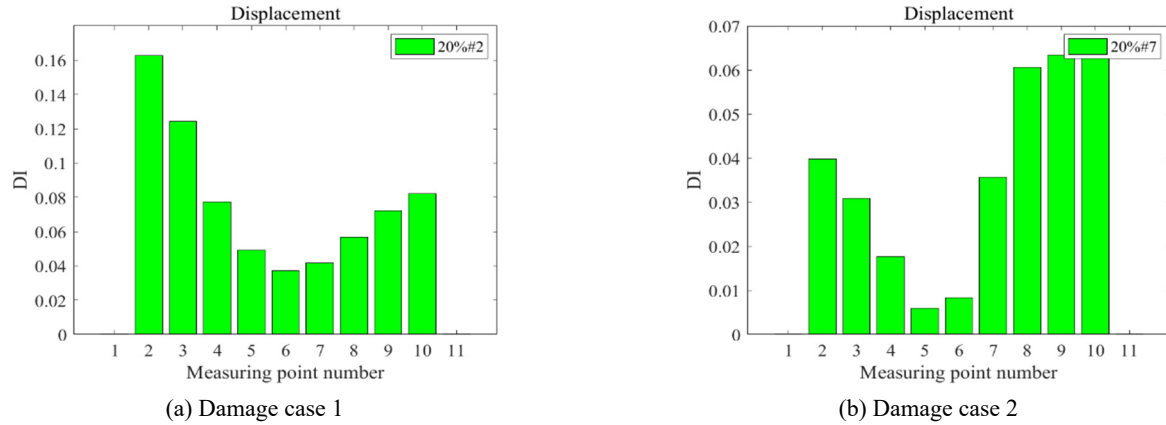


Fig. 5 Histogram based on wavelet packet total energy of structural displacement responses under random load sample 1

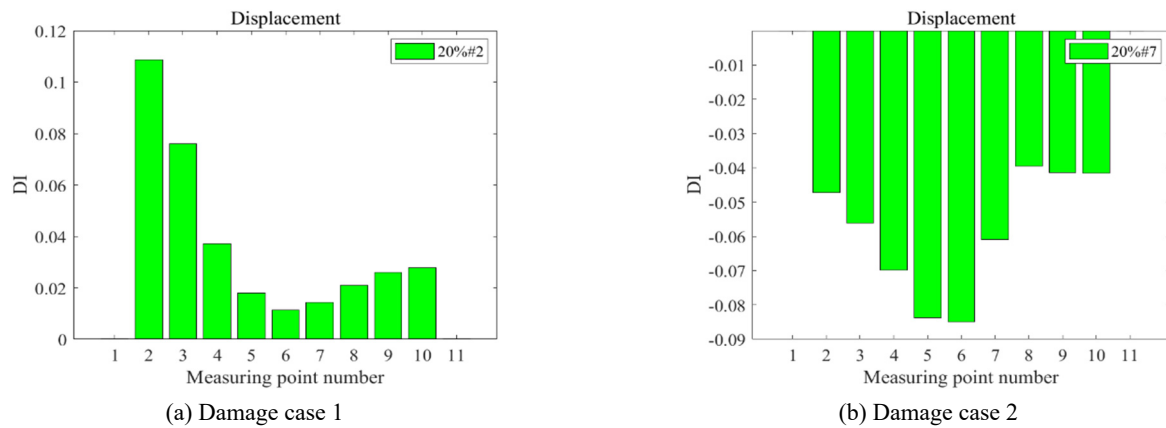


Fig. 6 Histogram based on wavelet packet total energy of structural displacement responses under random load sample 2

estimated from structural acceleration responses do not reliably indicate damage locations under all damage cases. This is because acceleration responses contain more macro-structural information, and accelerations of undamaged elements will also be affected by structural damage, resulting in misjudgment of damage localization results.

3.1.2 Damage identification based on wavelet packet total energy of structural displacement responses

Additionally, structural displacement responses are adopted to evaluate the effectiveness of structural damage identification based on the wavelet packet total energy. Damage cases and random load samples remain consistent with those in Section 3.1.1. Displacement responses at nodes 1-11 are observed, and the corresponding DI values are estimated to identify structural damage. Since consistent damage identification results are obtained by the Db8, Db20, and Db45 wavelet packet functions under different decomposition layers, only the results of 3-layer wavelet packet decomposition based on the Db45 are presented here. The damage identification results based on displacement responses under random load samples are displayed in Figs. 5 and 6, respectively.

It can be seen from Figs. 5 and 6 that the DI values using structural displacement responses cannot accurately locate damage locations under all damage cases. This is

because displacement responses also encompass more global structural information, displacement responses of undamaged elements are susceptible to local damage.

3.1.3 Damage identification based on wavelet packet total energy of structural macro-strain responses

Structural strain responses are more sensitive to damage, and measurements obtained using long-gauge strain (macro-strain) sensors can address the limitations of point strain sensors, which may lack sensitivity to local damage when positioned at certain distances from the damaged areas. Therefore, the macro-strain responses on the upper surface of elements are further examined for comparative study. Macro-strain is defined as the average strain within a specified length of a structure. For the m -th element, the relationship between the macro-strain response and the rotational displacements at both nodes of the element is given in Eq. (6)

$$\bar{\varepsilon}_m = \frac{h_m}{L_m} [\theta_l - \theta_r] \quad (6)$$

where L_m denotes the length of the m -th element; h_m is the distance from the sensor to the neutral axis of the m -th element; θ_l and θ_r represent the rotation displacement at the left and right nodes of the m -th element, respectively.

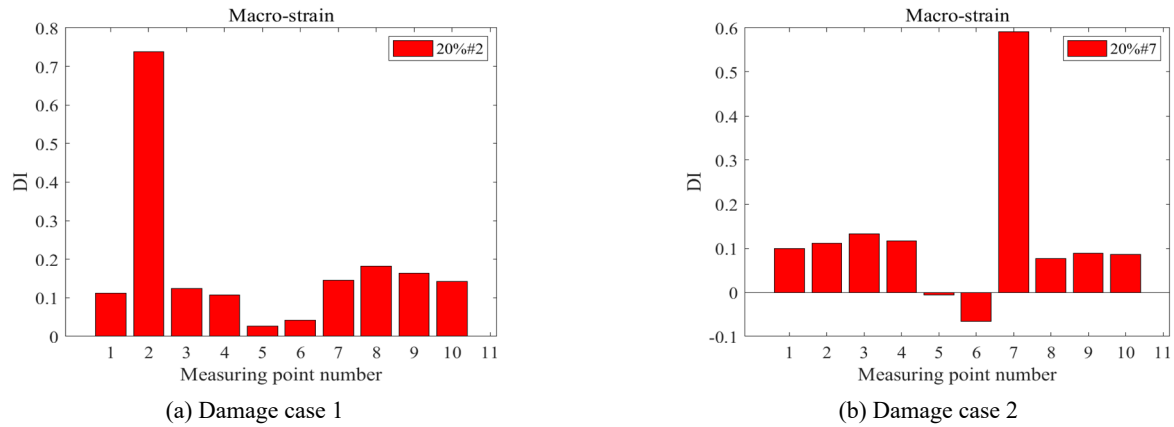


Fig. 7 Histogram based on wavelet packet total energy of structural macro-strain responses under random load sample 1

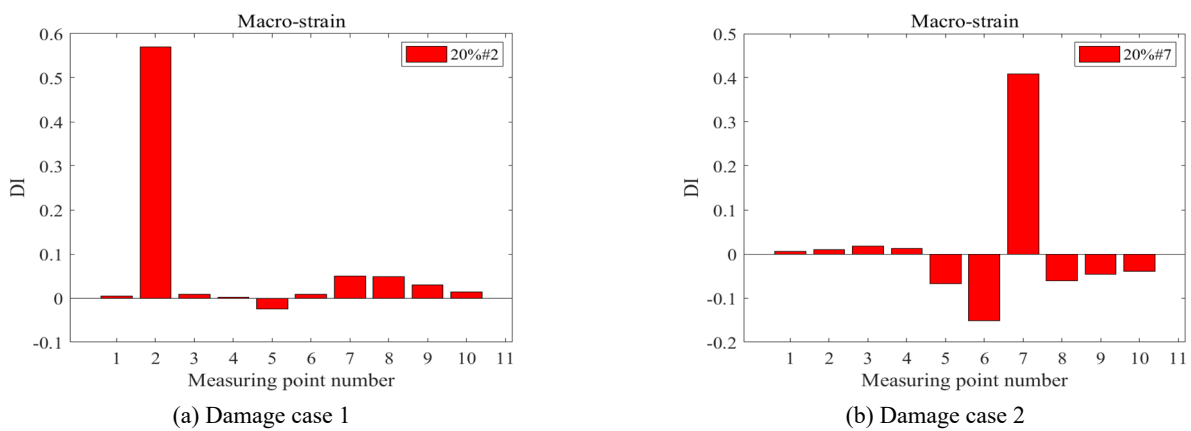


Fig. 8 Histogram based on wavelet packet total energy of structural macro-strain responses under random load sample 2

The simply supported beam model from Section 3.1.1 is employed here, with the same damage cases and random load samples, to compare structural damage identification using the DI values derived from structural acceleration, displacement, and macro-strain responses. The wavelet packet functions of the Db8, Db20, and Db45 are also used to perform various decomposition layers on macro-strain responses, and the corresponding DI values are obtained, which can get the same damage identification results. Similarly, the 3-layer decomposition with the Db45 is presented as an example. The damage identification results based on macro-strain responses under random load samples are shown in Figs. 7 and 8, respectively.

Figs. 7 and 8 illustrate that when excitation remains consistent before and after damage, the DI values calculated by structural macro-strain responses can accurately locate damage and reduce the misjudgement of other undamaged

elements under all damage cases and random load samples.

3.2 Discussion on beam-type structures

To further evaluate the damage identification performance of the wavelet packet total energy change rate in different structural types, a truss structure is analysed. The truss structure is shown in Fig. 9, which is composed of 15 rods. There are 7 unconstrained nodes and 2 bearing nodes with a total of 14 degrees of freedom (DOFs). The bar element length is 0.5 m, and the section height and width are 0.03 m and 0.12 m. The elastic modulus is 200 GPa. The lumped mass and linear stiffness are $m_i = 10 \text{ kg}$ ($i = 1, 2, \dots, 7$) and $k_j = 1.08 \times 10^5 \text{ N/m}$ ($j = 1, 2, \dots, 15$), respectively. The structural damping is modeled as Rayleigh damping with the first two modal damping ratios of 3%, and Rayleigh damping coefficients are $\alpha = 0.6965$ and $\beta = 0.0011$.

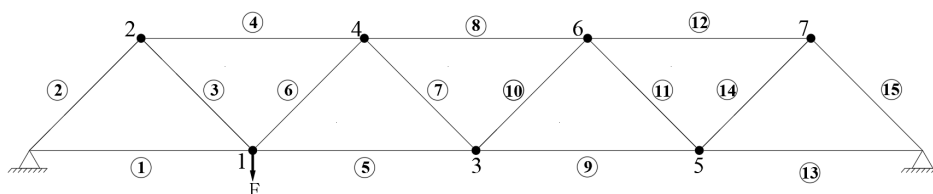


Fig. 9 Schematic diagram of the truss structure

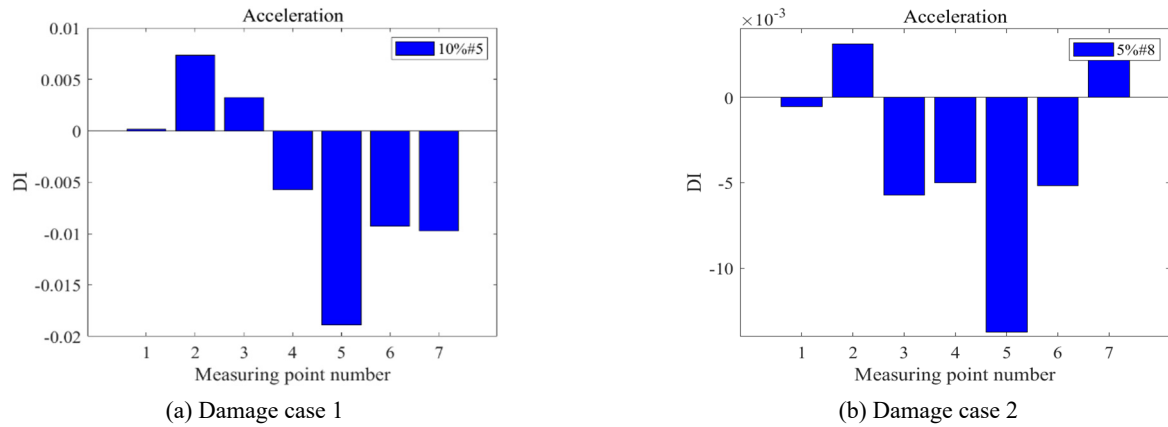


Fig. 10 Histogram based on wavelet packet total energy of structural acceleration responses

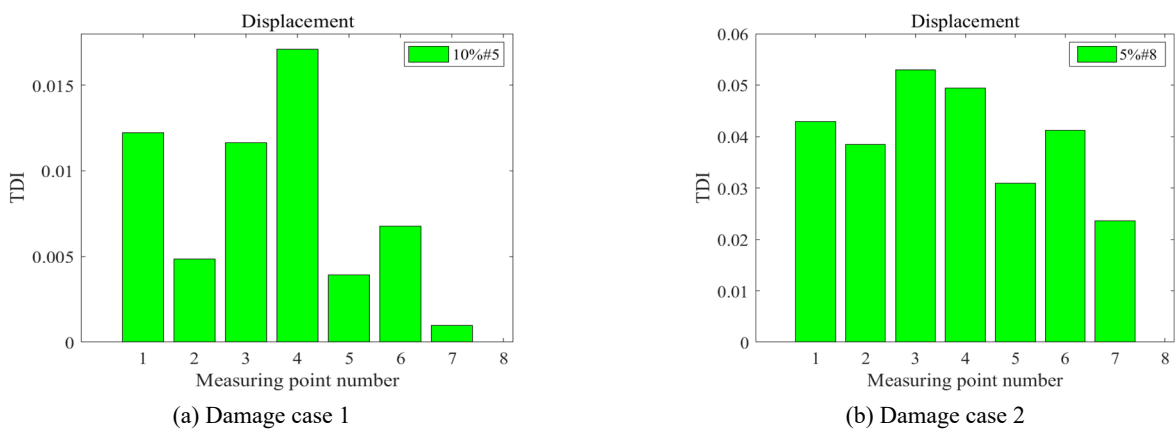


Fig. 11 Histogram based on wavelet packet total energy of structural displacement responses

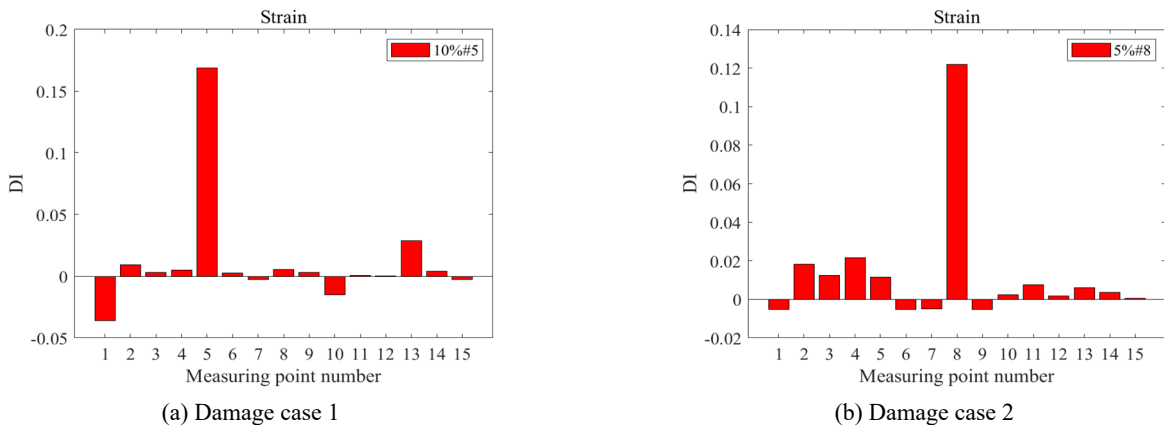


Fig. 12 Histogram based on wavelet packet total energy of structural strain responses

Random white noise excitation is applied to the 1st node of the truss structure, as shown in Fig. 9. Structural strain responses of elements 1-15 as well as structural acceleration and displacement responses at nodes 1-7, are observed. The sampling frequency and sampling time are 200 Hz and 10 s, respectively. Damage cases are introduced by reducing the elastic modulus of specific elements. Damage case 1 is 10% damage to element 5 (acceleration and displacement measurements at nodes 1 and 3), and damage case 2 is 5% damage to element 12 (acceleration and displacement

measurements at nodes 6 and 7, respectively). The same random load sample is used before and after structural damage, and structural acceleration, displacement, as well as strain responses are measured in both damaged and intact states. The DI values for damage identification are estimated using the Db8, Db20, and Db45 wavelet packet functions across different decomposition levels, and consistent damage identification results are obtained. Accordingly, the 3-layer decomposition of the Db45 is taken as an example to present the damage identification

results. The damage identification results based on structural acceleration, displacement, and strain responses are demonstrated in Figs. 10-12.

Figs. 10-12 demonstrate that structural damages are not located using the DI values based on acceleration or displacement responses under consistent excitation before and after damage. In contrast, the DI values based on structural strain responses can accurately identify damage locations and reduce misjudgements.

3.3 Summary of discussion results

As shown in Sections 3.1 and 3.2, the discussion results for simply supported beam and truss structures demonstrate that, even with consistent excitations before and after damage and without considering the influence of noise, structural damage cannot be accurately identified by the wavelet packet total energy change rate based on acceleration and displacement responses. In contrast, damage under all damage cases and random load samples can be accurately identified using the wavelet packet total energy change rate of macro-strain/strain responses, and the misjudgment of undamaged elements can be minimized. Therefore, compared with the damage identification using acceleration and displacement responses, the damage identification using macro-strain/strain responses is accurate.

4. The influence of different excitations on damage identification based on wavelet packet total energy of structural micro-strain/strain responses

In practical engineering, excitations are often random, particularly in the form of random white noise, which induces significant randomness in structural dynamic responses. Additionally, there is a lack of exploration into the impact of random excitations on structural damage identification. Therefore, the influence of different excitations on intact and damaged structures is investigated in this section.

4.1 Damage identification of beam-type structures

Based on the wavelet packet total energy change rate of structural macro-strain responses of beam-type structures, random load cases 1 and 2 are excited to further consider the randomness of excitations. The damage case 2 in Section 3.1, i.e., 20% damage to element 7 remains considered, but different random load samples are used before and after structural damage. Fig. 13 shows the damage identification results based on structural macro-strain responses under 3-layer wavelet packet decomposition by the Db45 in damage case 2.

Fig. 13 illustrates that even with the DI values of

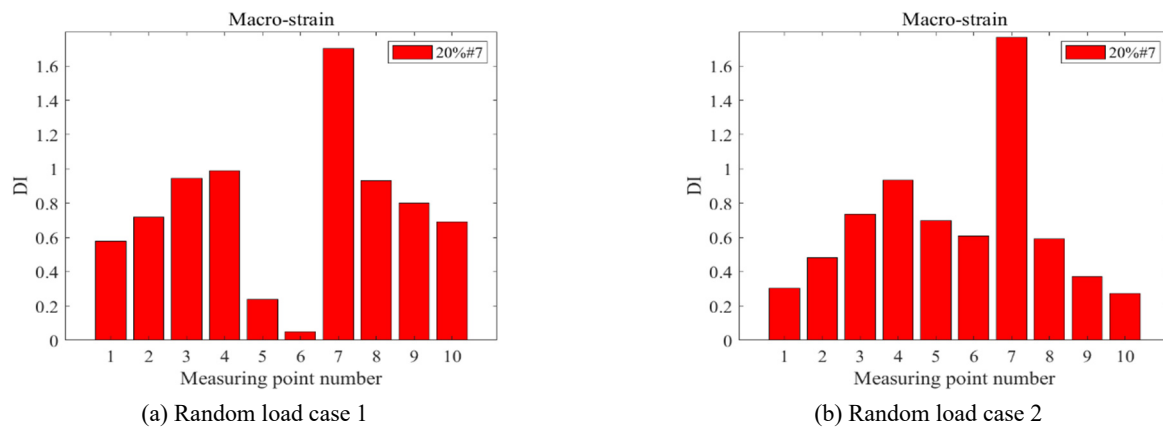


Fig. 13 Histogram based on wavelet packet total energy of structural macro-strain responses (damage case 2)

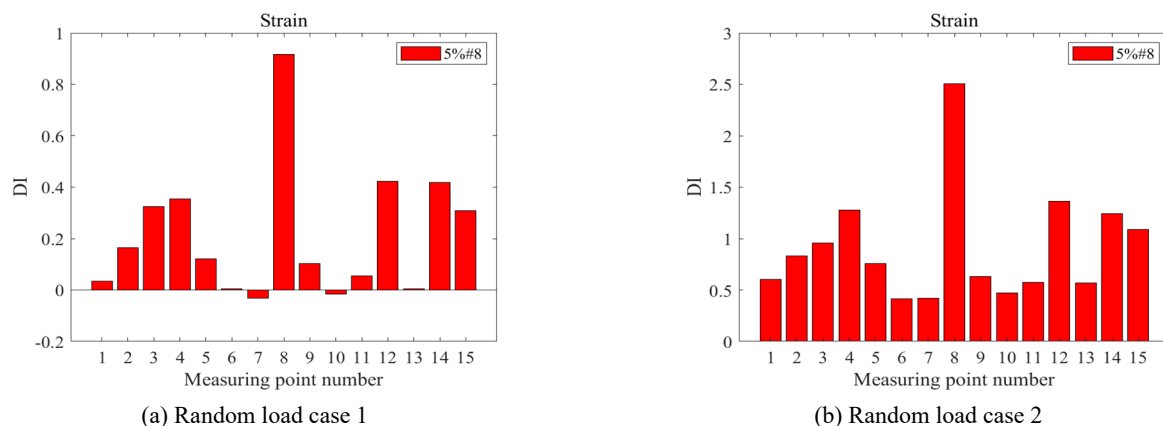


Fig. 14 Histogram based on wavelet packet total energy of structural strain responses (damage case 2)

structural macro-strain responses, large DI values are obtained for undamaged elements due to inconsistencies in random load samples between intact and damaged structures. Consequently, it is difficult to discern whether structures occur single damage or multiple damages.

4.2 Damage identification of truss structures

In addition, random load cases 1 and 2 are considered for the truss structure to investigate the influence of different excitations on intact and damaged structures. The damage case 2 in Section 3.2 is still adopted, that is, 5% damage to element 8 is considered, but different random load samples are used before and after damage. Fig. 14 shows the damage identification results based on structural strain responses under 3-layer wavelet packet decomposition by the Db45 in damage case 2.

Similarly, it is found from Fig. 14 that when random load samples before and after damage in the truss structure are inconsistent, structural damage cannot be accurately located by the DI values of structural strain responses.

The discussion results in Sections 4.1 and 4.2 indicate that damage localizations are significantly affected by variations in random loads before and after damage, even when measurement noises are not considered. The wavelet packet total energy of structural responses cannot be directly utilized for damage identification under random excitations. Therefore, it is necessary to develop a novel damage index from structural dynamic responses that is insensitive to excitations but only related to structural states, and then use it to identify structural damage.

5. The proposed structural damage identification based on wavelet packet total energy transmissibility functions of structural micro-strain/strain responses under random excitations

Under single-point excitation, structural responses transmissibility functions (TFs) are the ratio of responses at two measuring locations, which relies solely on structural responses and avoids the measurement of excitations. Hence, structural responses TFs are not affected by

excitation, and depend only on the characteristics of structures, making them sensitive to structural damage (Li *et al.* 2015). To address the challenge of damage identification under different excitations before and after structural damage, a damage identification method based on wavelet packet total energy TFs is proposed.

The wavelet packet total energy TFs are constructed based on structural micro-strain/strain responses between measuring location m and reference location r , namely $\frac{E_m}{E_r}$. Subsequently, the Transmissibility Damage Index (TDI) is constructed by utilizing the difference in the wavelet packet total energy TFs between intact and damaged structures to identify structural damage. It can be presented as follows

$$TDI_m = \frac{E_m^d}{E_r^d} - \frac{E_m^u}{E_r^u} \quad (7)$$

where $\frac{E_m^u}{E_r^u}$ and $\frac{E_m^d}{E_r^d}$ represent the wavelet packet total energy TFs of structural strain responses in intact and damaged structures, respectively. The TDI value is estimated for each measuring location and the TDI vector $TDI = [TDI_1, TDI_m, \dots, TDI_n]$ of a structure under n measuring locations can be obtained. Therefore, in the case of different excitations before and after damage, TDI_m at damage location is larger than those at the adjacent locations on both sides, so structural damage can be located. The damage identification method that combines wavelet packet total energy with structural response TFs can eliminate the influence of different excitations effectively and locate structural damage.

5.1 Damage identification of beam-type structures

Based on the discussion results in Sections 3-4, the structural damage identification based on structural macro-strain responses wavelet packet total energy TFs under different excitations before and after structural damage is further studied by the simply supported beam. The same damage case and random load cases (random load cases 1 and 2) as in Section 4.1 are adopted to verify the effectiveness of structural macro-strain wavelet packet total energy TFs in eliminating the influence of random loads. First, the macro-strain responses of the simply supported

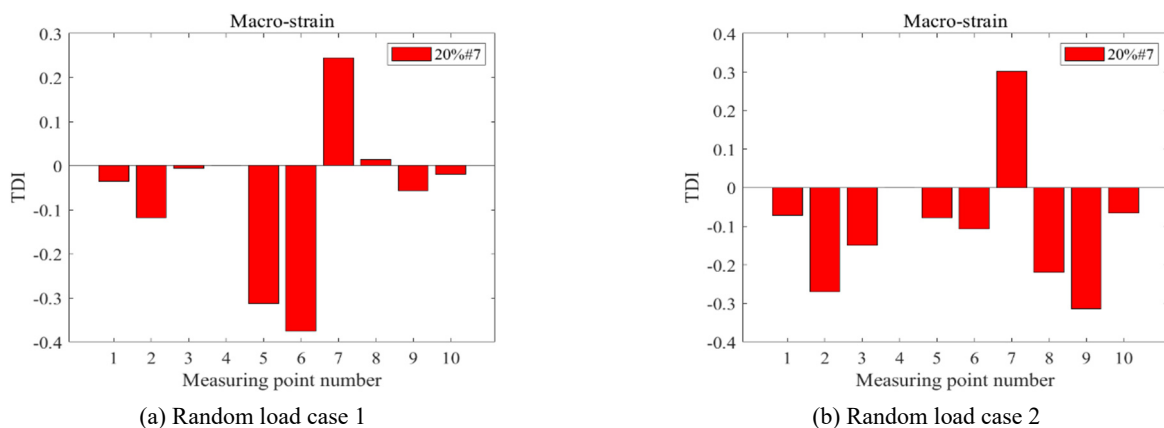


Fig. 15 Histogram based on change of macro-strain wavelet packet total energy TF (damage case 2)

Table 1 Damage cases of the simply supported beam

Damage cases	Damage elements	Damage extent
Case 1	2#	20%
Case 2	6#	10%
Case 3	2#, 6#	20%, 15%
Case 4	5#, 7#	10%, 10%

beam in both intact and damaged states are obtained under two random load cases. Then, the Db45 wavelet packet function is used to decompose the structural macro-strain responses in a 3-layer, and the wavelet packet total energies are obtained. The energy of the 4-th element is selected as the reference location, and the change in wavelet packet total energy TFs of each element is estimated to identify structural damage. Fig. 15 shows the results of structural damage identification based on the TDI values of structural macro-strain responses under damage case 2.

Compared with Fig. 13, Fig. 15 demonstrates superior damage localization performance even when the random loads before and after structural damage are inconsistent. The misjudgement of undamaged elements can be eliminated, and damage can be accurately located through the variation in the wavelet packet total energy TFs of structural macro-strain responses.

Furthermore, to verify the damage identification performance of the TDI values based on structural macro-

strain responses to identify other single-damage and multi-damage cases, four damage cases as shown in Table 1 are added under random load cases. In addition, Gaussian white noise is considered in the structural macro-strain responses corresponding to the above four damage cases. The noise level is 10% in root-mean-square (RMS) values as follows

$$\bar{\epsilon}_m^* = \bar{\epsilon}_m + \eta \times rms(\bar{\epsilon}_m) \times randn \quad (8)$$

where $\bar{\epsilon}_m$ and $\bar{\epsilon}_m^*$ are structural macro-strain/strain response of the m -th element before and after noise pollution, η is the noise level, and $randn$ is a random vector with standard normal distribution.

The structural macro-strain responses with noise are analysed by wavelet packet, and the wavelet packet total energy at the 4-th element is selected as the reference location to calculate the TDI values. The damage identification results under 10% noise are shown in Fig. 16.

As can be seen from Fig. 16, when 10% noise is considered in the observed structural macro-strain responses, there is a small amount of singularity for the TDI values at undamaged elements, indicating that noise has a certain interference effect. However, damage locations can still be identified from the peak value of the TDI values in all damage cases affected by 10% noise. Consequently, even with differing excitations before and after structural damage, the proposed TDI values still show good noise robustness for identifying structural damage.

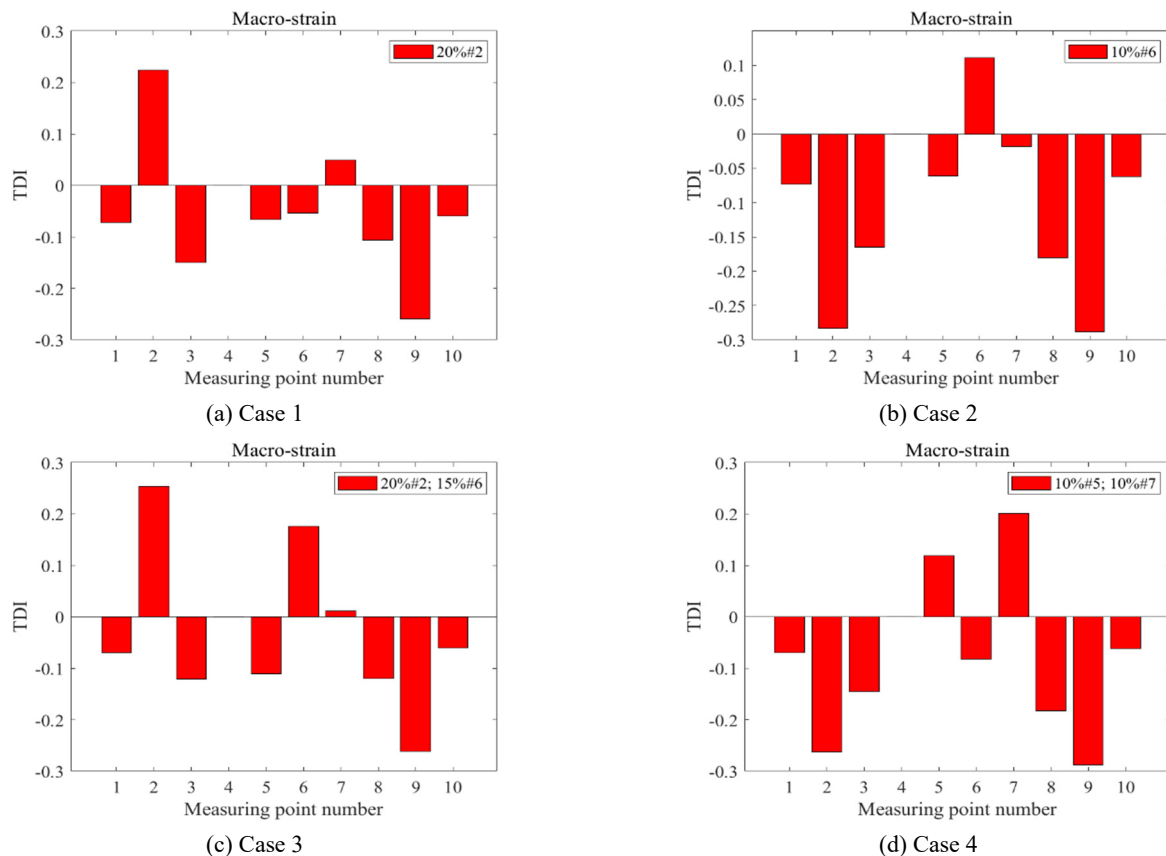


Fig. 16 Damage identification results for various damage cases

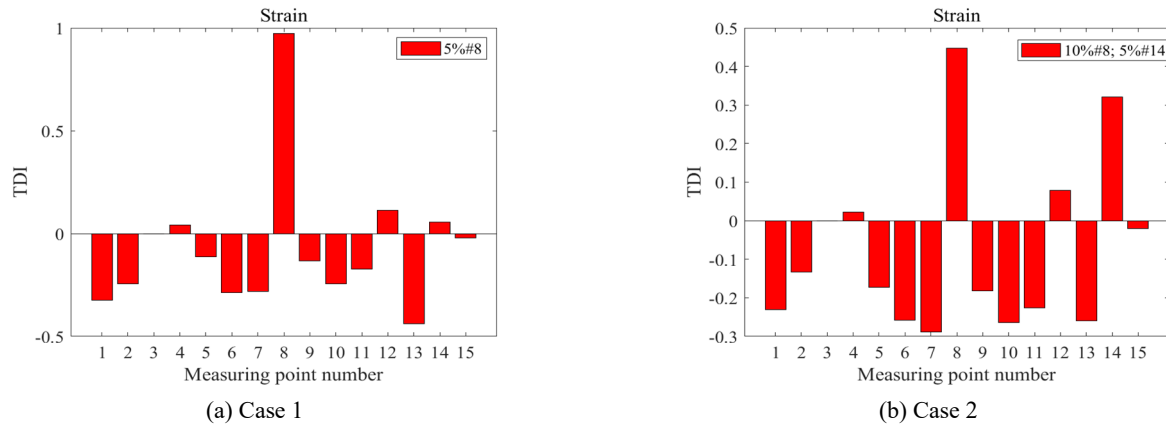


Fig. 17 Histogram based on change of strain wavelet packet total energy TF for various damage cases

5.2 Damage identification of truss structures

Based on Section 4.2, the same damage case and random load cases (random load cases 1 and 2) are applied to evaluate the effectiveness of the structural strain wavelet packet total energy TFs in eliminating the influence of random loads in truss structures. The results indicate that damage locations can be accurately located and misjudgement of undamaged elements can be effectively eliminated under two random load cases. Due to limited space, the corresponding results are not presented here.

Then, two damage cases are added to verify the damage identification performance of the TDI values based on structural strain responses when random excitation variations before and after structural damage. Case 1 is 5% damage to the element 8; Case 2 is 10% and 5% damage to the elements 8 and 14 respectively. Different random load samples before and after damage are used to verify the damage identification performance of this method on single damage and multiple damages. The 3-layer wavelet packet decomposition of the Db45 is performed on the measured strain responses of damaged and intact cases. The wavelet packet total energy of the 3-rd element is selected as the reference location, and then the TDI values are calculated to identify damage. In addition, to verify the robustness of the proposed method, 10% Gaussian white noise is added to the measured strain responses to consider the influence of measurement errors. The damage identification results are shown in Fig. 17.

From Fig. 17, it can be seen that damage is identified at element 8 in Case 1, and damages are identified at elements 8 and 14 in Case 2. Thus, even under the influence of 10% noise, the TDI values of structural strain responses can still locate single and multiple damages, and the identification results are the same as those of damage cases. In summary, the TDI values based on structural macro-strain/strain responses can eliminate the influence of different excitations, be robust to noise, and identify structural damage accurately.

6. Conclusions

In this paper, discussions on structural damage identification based on wavelet packet total energy of structural responses under random excitations are conducted. Some numerical investigations of beam-type and truss structures are used in these discussions and the following conclusions are obtained:

It is shown that structural damage identification based on wavelet packet total energy of structural macro-strain for beam-type structures and strain responses for truss structures can accurately locate structural damage, which reduces the misjudgments of undamaged elements compared with using structural acceleration or displacement responses.

Different excitations to intact and damaged structures seriously affect structural damage identification based on wavelet packet total energy even using structural macro-strain/strain responses. Therefore, care should be taken for structural damage identification based on the wavelet packet total energy of structural responses under random excitations.

A method of combining structural response transmissibility functions and wavelet packet total energy of structural macro-strain/strain responses is proposed to eliminate the influence of different excitations to intact and damaged structures for accurate identification of structural damage under random excitations. The identification results of both beam-type and truss structures show that single-damage and multi-damage can be accurately located even with 10% measurement noises.

The contributions of this paper are as follows: (1) a comprehensive discussion on damage identification based on various structural responses, emphasizing the superior sensitivity of macro-strain/strain responses to local damage; (2) an investigation of the significant influence of random excitations on structural damage identification; and (3) A novel method based on transmissibility functions of wavelet packet total energy of structural macro-strain/strain responses is proposed to eliminate the influence of random excitations, and then extract damage-sensitive features for structural damage identification.

In this paper, structural damage identification under a single random excitation is considered. To adapt to multiple

random excitations, structural damage identification based on the structural response transmissibility matrix is investigated by the authors.

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