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# The nano scale buckling properties of isolated protein microtubules based on modified strain gradient theory and a new single variable trigonometric beam theory

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**Abstract.** Microtubules (MTs) are the main part of the cytoskeleton in living eukaryotic cells. In this article, a mechanical model of MT buckling, considering the modified strain gradient theory, is analytically examined. The MT is assumed as a cylindrical beam and a new single variable trigonometric beam theory is developed in conjunction with a modified strain gradient model. The main benefit of the present formulation is shown in its new kinematic where we found only one unknown as the Euler-Bernoulli beam model, which is even less than the Timoshenko beam model. The governing equations are deduced by considering virtual work principle. The effectiveness of the present method is checked by comparing the obtained results with those reported by other higher shear deformation beam theory involving a higher number of unknowns. It is shown that microstructure-dependent response is more important when material length scale parameters are closer to the outer diameter of MTs. Also, it can be confirmed that influences of shear deformation become more considerable for smaller shear modulus and aspect ratios.

**Keywords:** protein microtubules; modified strain gradient theory; single variable beam theory; buckling

## 1. Introduction

Relating on the main structural elements of its cells, a living organism may be assigned to the prokaryotes or group of eukaryotic. Microtubule is the most stiffness component of eukaryotic cells that permits to play a considerable role in desired mechanical characteristics for preserving the shape of cell. One of the properties of MT that is important for their biological activities is their long shape and rigid that permits MTs to maintain other structures in a living cell. Consequently, the buckling investigation of MTs subjected to a mechanical force is very essential for the cytoskeleton located in living cells and during the mitosis' pro-phase stage in a living cell. Also, MTs are generally subjected to compressive forces leading to a buckling instability. Experimental investigations have been established in laboratories to study the elastic characteristics as well as mechanics behavior of MTs under external force (Venier *et al*. 1994, Kurachi *et al*. 1995, Fygenson *et al*. 1997, Odde *et al*. 1999). However, because of the experimental results dispersed over a large range, it has been seen that the coherence between the results is not significant.

Diminishing the structure's property dimensions to micron scale leads to a different mechanic response. Indeed, some experimental works have been shown that the scale effect plays a considerable role on mechanic behaviors of small-scale structures (Poole *et al*. 1996, Lam *et al*. 2003, McFarland and Colton 2005). The conventional elasticity theories contain no intrinsic size parameters and do not permit the predicting of the size dependent responses of nano- and micro-scaled structures. Consequently, various non-classical elasticity models have been introduced to examine the mechanic behaviors of structures in a reduced scale such as model of couple stress (Mindlin and Tiersten 1962, Koiter 1964, Toupin 1964, Akbaş 2017, Khabaz *et al*. 2020), micropolar model (Eringen 1967), nonlocal continuum model (Eringen 1972, 1983, Oveissi *et al*. 2016a, 2017, 2018, Foroutan *et al*. 2018, Mehar *et al*. 2018,

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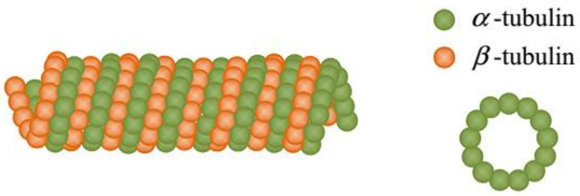



Fig. 1 Structure of a typical microtubule (Akgöz and Civalek 2014b)

Hashemian *et al*. 2019a, 2020, Pirmoradian *et al*. 2020a, b, c, Yousefzadeh *et al*. 2020) and models of strain gradient (Fleck and Hutchinson 1993, Vardoulakis and Sulem 1995, Aifantis 1999).

The modified theory of strain gradient (Lam *et al*. 2003) is one of the above-shown higher-order theories where is found that the density of strain energy contains second-order gradients of deformation in addition to the gradient of the first-order deformation. In the mathematical approaches of linear elastic isotropic material, three besides material length size coefficients are introduced by considering higher-order deformation gradients, additionally to two conventional ones. This widely employed model has been often proposed to examine mechanic behaviors of microbars (Akgöz and Civalek 2013, 2014a, Kahrobaiyan *et al*. 2011a, b, 2012, 2013, Narendar *et al*. 2012, Güven 2014), micro-beams (Akgöz and Civalek 2011a, 2012, Ansari *et al*. 2011, Wang *et al*. 2010, Asghari *et al*. 2012, Artan and Toksöz 2013, Ghayesh *et al*. 2013, Zhao *et al*. 2012, Lei *et al*. 2013, Tajalli *et al*. 2013) and nanostructures (Mirkalantari *et al*. 2017, Oveissi *et al*. 2016b, Hashemian *et al*. 2019b, Sourani *et al*. 2020, Enayat *et al*. 2020a, b, Merzouki *et al*. 2020).

In the literature review, it is noted that the diameter and length of MTs are found in the order of micrometers and nanometers, respectively. Consequently, the mathematical modeling of MTs using non-classical elasticity models has been become more popular recently. Fu and Zhang (2010) and Gao and Lei (2009) studied the buckling behavior and the persistence length of MTs by employing the modified theory of couple stress and the model of nonlocal elasticity, respectively. The nonlocal model of the beam of Timoshenko is employed by Heireche *et al*. (2010) to examine the dynamic response of MTs embedded in viscoelastic cytoplasm. The buckling responses of MTs have been investigated by Gao and An (2010) by considering the nonlocal theory of anisotropic shell. Shen (2010a, b, c) presented a nonlocal shear theory of deformable shell for nonlinear and linear analysis of microtubules. Furthermore, bending and vibration investigation of MTs was studied based on Euler-Bernoulli beam and theories of the non-classical continuum (Civalek *et al*. 2010, Akgöz and Civalek 2011b). Recently, many shear deformation theories are developed (Hadji *et al*. 2019, Zouatnia and Hadji 2019a, b, Batou *et al*. 2019, Nebab *et al*. 2019, 2020, Sahouane *et al*. 2019, Safa *et al*. 2019, Gafour *et al*. 2020, Daikh *et al*. 2020, Rachedi *et al*. 2020, Merzoug *et al*. 2020, Tayeb *et al*. 2020) and the need for a simple and effective one is always desired.

In this research, buckling behavior of MTs is investigated. For this purpose, a single variable shear deformation beam model is developed by including the modified model of strain gradient elasticity. This model is constructed from Euler-Bernoulli Beam Theory (EBT) by considering the sinusoidal function in terms of thickness coordinate to include shear deformation effect and it involves only one governing equation. The proposed theory will be employed by the large community of biophysicist researching in the field of mechanical filament.

## 2. The modified theory of strain gradient elasticity

The Modified Strain Gradient Elasticity Theory (MSGT) initiated by Lam *et al*. (2003) is one of the most popular higher-order (non-classical) continuum theories. Contrary to the classical continuum theories, this one takes into account a higher order strain gradient in addition to the classical strain tensor in mathematical approaches like dilation gradient vector, "deviatoric stretch gradient" and symmetrical "rotation gradient tensor". The energy of strain $U$ for the MSGT can be given by utilizing infinitesimal deformations (Lam *et al*. 2003) as

$$U = \frac{1}{2}\int_0^L \int_A \left(\sigma_{ij}\varepsilon_{ij} + p_i\gamma_i + \tau_{ijk}^{(1)}\eta_{ijk}^{(1)} + m_{ij}^s\chi_{ij}^s\right) dA\ dx \quad (1)$$

in which $\varepsilon_{ij}$, $\gamma_i$, $\eta_{ijk}^{(1)}$ and $\chi_{ij}^s$ present the components of the "strain tensor" $[\varepsilon]$, the "dilatation gradient vector" $\gamma$, the "deviatoric stretch gradient tensor" $[\eta]^{(1)}$ and the "symmetric rotation gradient tensor" $\chi^s$, respectively and are given by

$$\varepsilon_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \quad (2)$$

$$\gamma_i = \frac{\partial \varepsilon_{mm}}{\partial x_i} \quad (3)$$

$$\eta_{ijk}^{(1)} = \frac{1}{3}\left(\frac{\partial \varepsilon_{jk}}{\partial x_i} + \frac{\partial \varepsilon_{ki}}{\partial x_j} + \frac{\partial \varepsilon_{ij}}{\partial x_k}\right) - \frac{1}{15}\left[\delta_{ij}\left(\frac{\partial \varepsilon_{mm}}{\partial x_k} + 2\frac{\partial \varepsilon_{mk}}{\partial x_m}\right) + \delta_{jk}\left(\frac{\partial \varepsilon_{mm}}{\partial x_i} + 2\frac{\partial \varepsilon_{mi}}{\partial x_m}\right) + \delta_{ki}\left(\frac{\partial \varepsilon_{mm}}{\partial x_j} + 2\frac{\partial \varepsilon_{mj}}{\partial x_m}\right)\right] \quad (4)$$

$$\theta_i = \frac{1}{2} e_{ijk} \frac{\partial u_k}{x_j} \quad (5)$$

$$\chi_{ij}^s = \frac{1}{2}\left(\frac{\partial \theta_i}{\partial x_j} + \frac{\partial \theta_j}{\partial x_i}\right) \quad (6)$$

where $u_i$ indicates the components of the "displacement vector" $u$ and $\theta_i$ presents the components of the "rotation vector" $\theta$, also $\delta$ and $e_{ijk}$ are the "Kronecker delta" and symbols of permutation, respectively. In addition, the components of the "Cauchy stress tensor" $[\sigma]$ and "higher-order stress tensors" $p$, $\tau^{(1)}$ and $m^s$ (conjugated with $\varepsilon_{ij}$, $\gamma_i$, $\eta_{ijk}^{(1)}$ and $\chi_{ij}^s$ respectively) are given by Lam *et al*. (2003)

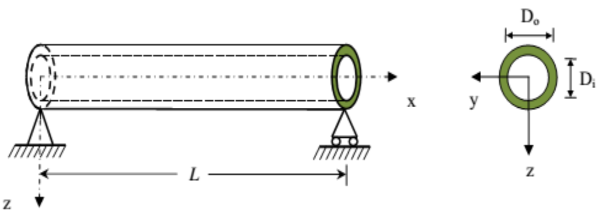



Fig. 2 Continuum modeling of an isolated microtubule as a simply supported cylindrical hollow beam

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{mm} + 2G\varepsilon_{ij} \tag{7}$$

$$p_i = 2Gl_0^2\gamma_i \tag{8}$$

$$\tau_{ijk}^{(1)} = 2Gl_1^2\eta_{ijk}^{(1)} \tag{9}$$

$$m_{ij}^s = 2Gl_2^2\chi_{ij}^s \tag{10}$$

where $l_0$, $l_1$, $l_2$ are additional material "length scale parameters" related to "dilatation gradients", "deviatoric stretch gradients" and "rotation gradients", respectively. In addition, $\lambda$ and $G$ are the Lamé constants given by

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \text{ and } G = \frac{E}{2(1+\nu)} \tag{11}$$

## 3. Formulation of single variable trigonometric beam theory

Microtubules are made up of both $\alpha$-tubulin and $\beta$-tubulin (see Fig. 1). MTs can be assumed as a hollow beam in the mathematical modeling. Modeling of an isolated MT like a simply supported cylindrical hollow element of beam is indicated in Fig. 2 in which $D_i$, $D_o$ and $L$ present the inner, outer diameters and the length of the MT, respectively.

The kinematics of the present theory can be defined as

$$\begin{aligned} u(x,z) &= -z\frac{\partial w}{\partial x} - \beta\, f(z)\frac{\partial^3 w}{\partial x^3} \\ v(x,z) &= 0,\ w(x,z) = w(x) \end{aligned} \tag{12}$$

where $w$ is the $z$-component of the vector of displacement. $f(z)$ is a shape function given by

$$f(z) = \frac{h}{\pi}sin\left(\frac{\pi z}{h}\right) \tag{13}$$

where $h$ indicates the height of the beam ($D_0$). Substituting Eq. (12) in Eq. (2) provides non-zero deformation components as

$$\left.\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} - f(z)\beta\frac{\partial^4 w}{\partial x^4} \\ \varepsilon_{xz} &= \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = -\frac{1}{2}f'(z)\beta\frac{\partial^3 w}{\partial x^3} \end{aligned}\right\} \tag{14}$$

where

$$f'(z) = \frac{df}{dz} = cos\left(\frac{\pi z}{h}\right) \tag{15}$$

Using Eq. (14) in Eqs. (3)-(5), the non-zero components of higher-order deformation gradients are obtained as

$$\gamma_x = \frac{\partial \varepsilon_x}{\partial x} = -z\frac{\partial^3 w}{\partial x^3} - f(z)\beta\frac{\partial^5 w}{\partial x^5}, \tag{16a}$$

$$\gamma_z = \frac{\partial \varepsilon_x}{\partial z} = -\frac{\partial^2 w}{\partial x^2} - f'(z)\beta\frac{\partial^4 w}{\partial x^4} \tag{16b}$$

and

$$\eta_{111}^{(1)} = \frac{1}{5}\left[2\left(-z\frac{\partial^3 w}{\partial x^3} - f(z)\beta\frac{\partial^5 w}{\partial x^5}\right) - \frac{\pi^2}{h^2}f(z)\beta\frac{\partial^3 w}{\partial x^3}\right] \tag{17a}$$

$$\eta_{113}^{(1)} = \eta_{131}^{(1)} = \eta_{311}^{(1)} = -\frac{4}{15}\left(\frac{\partial^2 w}{\partial x^2} + 2f'(z)\beta\frac{\partial^4 w}{\partial x^4}\right) \tag{17b}$$

$$\begin{aligned} &\eta_{122}^{(1)} = \eta_{212}^{(1)} = \eta_{221}^{(1)} \\ &= -\frac{1}{5}\left[-z\frac{\partial^3 w}{\partial x^3} - f(z)\beta\frac{\partial^5 w}{\partial x^5} + \frac{\pi^2}{3h^2}f(z)\beta\frac{\partial^3 w}{\partial x^3}\right] \end{aligned} \tag{17c}$$

$$\begin{aligned} &\eta_{133}^{(1)} = \eta_{313}^{(1)} = \eta_{331}^{(1)} \\ &= -\frac{1}{5}\left(-z\frac{\partial^3 w}{\partial x^3} - f(z)\beta\frac{\partial^5 w}{\partial x^5} - \frac{4}{3}\frac{\pi^2}{h^2}f(z)\beta\frac{\partial^3 w}{\partial x^3}\right) \end{aligned} \tag{17d}$$

$$\eta_{223}^{(1)} = \eta_{232}^{(1)} = \eta_{322}^{(1)} = \frac{1}{15}\left(\frac{\partial^2 w}{\partial x^2} + 2f'(z)\beta\frac{\partial^4 w}{\partial x^4}\right) \tag{17e}$$

$$\eta_{333}^{(1)} = \frac{1}{5}\left(\frac{\partial^2 w}{\partial x^2} + 2f'(z)\beta\frac{\partial^4 w}{\partial x^4}\right) \tag{17f}$$

$$\chi_{xy}^s = \frac{1}{2}\left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\right) = -\frac{1}{2}\left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\beta\, f'(z)\frac{\partial^4 w}{\partial x^4}\right] \tag{18a}$$

$$\chi_{yz}^s = \frac{1}{2}\left(\frac{\partial \theta_y}{\partial z} + \frac{\partial \theta_z}{\partial y}\right) = \frac{\pi^2}{4h^2}\beta\, f(z)\frac{\partial^3 w}{\partial x^3} \tag{18b}$$

Employing Eq. (14) in Eq. (7), the non-zero components of classical stress tensor $\sigma$ can be written as

$$\sigma_x = E\eta\left(-z\frac{\partial^2 w}{\partial x^2} - f(z)\beta\frac{\partial^4 w}{\partial x^4}\right) \tag{19a}$$

$$\tau_{xz} = -Gf'(z)\beta\frac{\partial^3 w}{\partial x^3} \tag{19b}$$

$$\sigma_y = \sigma_z = \lambda\left(-z\frac{\partial^2 w}{\partial x^2} - f(z)\beta\frac{\partial^4 w}{\partial x^4}\right) \tag{19c}$$

where

$$\eta = \frac{(1-\nu)}{(1+\nu)(1-2\nu)} \quad \text{and} \quad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \tag{20}$$

Also, substituting Eqs. (16)-(18) in Eqs. (8)-(10) gives the non-zero components of higher order stresses as

$$p_x = 2Gl_0^2\gamma_x = 2Gl_0^2\left(-z\frac{\partial^3 w}{\partial x^3} - f(z)\beta\frac{\partial^5 w}{\partial x^5}\right) \quad (21a)$$

$$p_z = 2Gl_0^2\gamma_z = -2Gl_0^2\left(\frac{\partial^2 w}{\partial x^2} + f'(z)\beta\frac{\partial^4 w}{\partial x^4}\right) \quad (21b)$$

$$\tau_{111}^{(1)} = 2Gl_1^2\eta_{111}^{(1)} = \frac{2}{5}Gl_1^2\left[2\left(-z\frac{\partial^3 w}{\partial x^3} - f(z)\beta\frac{\partial^5 w}{\partial x^5}\right) - \frac{\pi^2}{h^2}f(z)\beta\frac{\partial^3 w}{\partial x^3}\right] \quad (22a)$$

$$\tau_{113}^{(1)} = \tau_{131}^{(1)} = \tau_{311}^{(1)} = -\frac{8}{15}Gl_1^2\left(\frac{\partial^2 w}{\partial x^2} + 2f'(z)\beta\frac{\partial^4 w}{\partial x^4}\right) \quad (22b)$$

$$\tau_{122}^{(1)} = \tau_{212}^{(1)} = \tau_{221}^{(1)} = -\frac{2}{5}Gl_1^2\left[-z\frac{\partial^3 w}{\partial x^3} - f(z)\beta\frac{\partial^5 w}{\partial x^5} + \frac{\pi^2}{3h^2}f(z)\beta\frac{\partial^3 w}{\partial x^3}\right] \quad (22c)$$

$$\tau_{133}^{(1)} = \tau_{313}^{(1)} = \tau_{331}^{(1)} = -\frac{2}{5}Gl_1^2\left(-z\frac{\partial^3 w}{\partial x^3} - f(z)\beta\frac{\partial^5 w}{\partial x^5} - \frac{4}{3}\frac{\pi^2}{h^2}f(z)\beta\frac{\partial^3 w}{\partial x^3}\right) \quad (22d)$$

$$\tau_{223}^{(1)} = \tau_{232}^{(1)} = \tau_{322}^{(1)} = \frac{2}{15}Gl_1^2\left(\frac{\partial^2 w}{\partial x^2} + 2f'(z)\beta\frac{\partial^4 w}{\partial x^4}\right) \quad (22e)$$

$$\tau_{333}^{(1)} = \frac{2}{5}Gl_1^2\left(\frac{\partial^2 w}{\partial x^2} + 2f'(z)\beta\frac{\partial^4 w}{\partial x^4}\right) \quad (22f)$$

$$m_{xy}^s = 2Gl_2^2\chi_{xy}^s = -Gl_2^2\left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\beta\, f'(z)\frac{\partial^4 w}{\partial x^4}\right] \quad (23a)$$

$$m_{yz}^s = 2Gl_2^2\chi_{yz}^s = \frac{Gl_2^2\pi^2}{2h^2}\beta\, f(z)\frac{\partial^3 w}{\partial x^3} \quad (23b)$$

The first variation of the strain energy for the MTs can be expressed employing the above classical and non-classical (higher-order) stress and strain components into Eq. (1) (by neglecting the Poisson effect) as

$$\begin{aligned}\delta U &= \int_0^L\int_A\left(\sigma_{ij}\delta\varepsilon_{ij} + p_{ij}\delta\gamma_{ij} + \tau_{ijk}^{(1)}\delta\eta_{ijk}^{(1)} + m_{ij}^s\delta\chi_{ij}^s\right)dA\,dx \\ &= \int_0^L\left(-\frac{6BI}{\pi^2}\beta^2\frac{\partial^{10}w}{\partial x^{10}} + \left(S_4\beta^2 - 4BA\beta\frac{h^2}{\pi^3}\right)\frac{\partial^8 w}{\partial x^8} + \right. \\ &\quad\left. + (2S_3\beta - S_1\beta^2 - BI)\frac{\partial^6 w}{\partial x^6} + S_2\frac{\partial^4 w}{\partial x^4}\right)\delta w\Bigg]dx\end{aligned} \quad (24)$$

where

$$\begin{aligned}B &= 2G\left(l_0^2 + \frac{2}{5}l_1^2\right) \\ S_1 &= GA\left(\frac{1}{2} + \frac{\pi^2}{h^2}\left(\frac{4}{15}l_1^2 + \frac{1}{8}l_2^2\right)\right) \\ S_2 &= EI + GA\left(2l_0^2 + \frac{8}{15}l_1^2 + l_2^2\right) \\ S_3 &= \frac{24EI}{\pi^3} + \frac{GA}{\pi}\left(4l_0^2 + \frac{4}{3}l_1^2 + l_2^2\right) \\ S_4 &= \frac{6EI}{\pi^2} + GA\left(l_0^2 + \frac{2}{3}l_1^2 + \frac{1}{8}l_2^2\right)\end{aligned} \quad (25)$$

where $A$ and $I$ are the "cross section" of area and the "second moment" of area, respectively.

The variation of "potential energy" of the applied loads can be expressed as

$$\delta V = \int_A -P\frac{d^2 w}{dx^2}\delta w dA \quad (26)$$

where $P$ is applied axial compressive load.

The governing equations are derived from the principle of minimum total potential energy. It states that

$$\delta U + \delta V = 0 \quad (27)$$

and substituting Eqs. (24) and (26) into Eq. (27) and setting the coefficients of $\delta\, u_0$, $\delta\, w_0$ equal to zero, the governing equations of MTs can be obtained as

$$\begin{aligned}\delta w: &-\frac{6BI}{\pi^2}\beta^2\frac{\partial^{10}w}{\partial x^{10}} + \left(S_4\beta^2 - 4BA\beta\frac{h^2}{\pi^3}\right)\frac{\partial^8 w}{\partial x^8} \\ &+ (2S_3\beta - S_1\beta^2 - BI)\frac{\partial^6 w}{\partial x^6} + S_2\frac{\partial^4 w}{\partial x^4} - P\frac{d^2 w}{dx^2} = 0\end{aligned} \quad (28)$$

and

$$\beta = \frac{S_3\pi^3 + 2\alpha^2 BAh^2}{\pi(S_1\pi^2 + S_4\alpha^2\pi^2 + 6BI\alpha^4)} \quad (29)$$

## 4. Analytical solutions

In this section, Navier solution method is employed to derive analytical solutions for buckling problem of the simply supported MTs. The solution is considered to be of the form

$$w = \sum_{m=1}^{\infty} W_m\, sin(\alpha x) \quad (30)$$

where $\alpha = m\pi/a$ and $W_m$ are arbitrary parameters to be determined.

Substituting Eq. (30) into Eq. (28), the following equation is obtained.

$$\alpha^2\begin{bmatrix}\frac{6BI}{\pi^2}\beta^2\alpha^8 + \left(S_4\beta^2 - 4BA\beta\frac{h^2}{\pi^3}\right)\alpha^6 \\ +(S_1\beta^2 + BI - 2S_3\beta)\alpha^4 + S_2\alpha^2 - P\end{bmatrix}W_m = 0 \quad (31)$$

## 5. Results and discussion

In order to highlight the influence of material length scale parameters and shear deformation, some numerical

Table 1 Critical buckling loads (nN) of the simply supported microtubule ($l = D_o$)

| $r$ | Beam theory | $L = 50D_o$ | | | $L = 500D_o$ | | |
|---|---|---|---|---|---|---|---|
| | | CT | MCST | MSGT | CT | MCST | MSGT |
| Isotropic | EBT | 0.1054 | 0.5824 | 1.7913 | 0.0011 | 0.0058 | 0.0179 |
| | Ref[(a)] | 0.1053 | 0.5822 | 1.7893 | 0.0011 | 0.0058 | 0.0179 |
| | Present | 0.1053 | 0.5822 | 1.7893 | 0.0011 | 0.0058 | 0.0179 |
| $10^{-4}$ | EBT | 0.1054 | 0.1055 | 0.1059 | 0.0011 | 0.0011 | 0.0011 |
| | Ref[(a)] | 0.0220 | 0.0493 | 0.0726 | 0.0010 | 0.0010 | 0.0011 |
| | Present | 0.0220 | 0.0493 | 0.0726 | 0.0010 | 0.0010 | 0.0011 |
| $10^{-5}$ | EBT | 0.1054 | 0.1054 | 0.1055 | 0.0011 | 0.0011 | 0.0011 |
| | Ref[(a)] | 0.0040 | 0.0097 | 0.0199 | 0.0008 | 0.0009 | 0.0010 |
| | Present | 0.0040 | 0.0097 | 0.0199 | 0.0008 | 0.0009 | 0.0010 |
| $10^{-6}$ | EBT | 0.1054 | 0.1054 | 0.1054 | 0.0011 | 0.0011 | 0.0011 |
| | Ref[(a)] | 0.0018 | 0.0024 | 0.0037 | 0.0002 | 0.0005 | 0.0007 |
| | Present | 0.0018 | 0.0024 | 0.0037 | 0.0002 | 0.0005 | 0.0007 |

[(a)] Taken from Akgöz and Civalek (2014b)

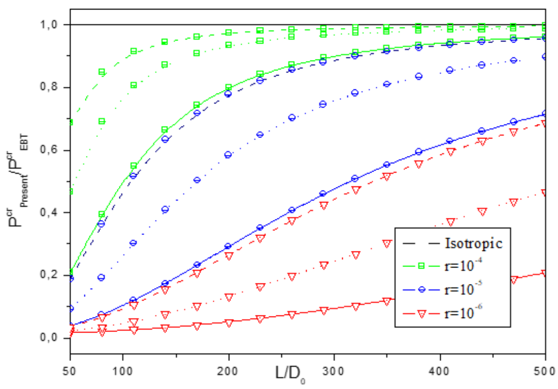
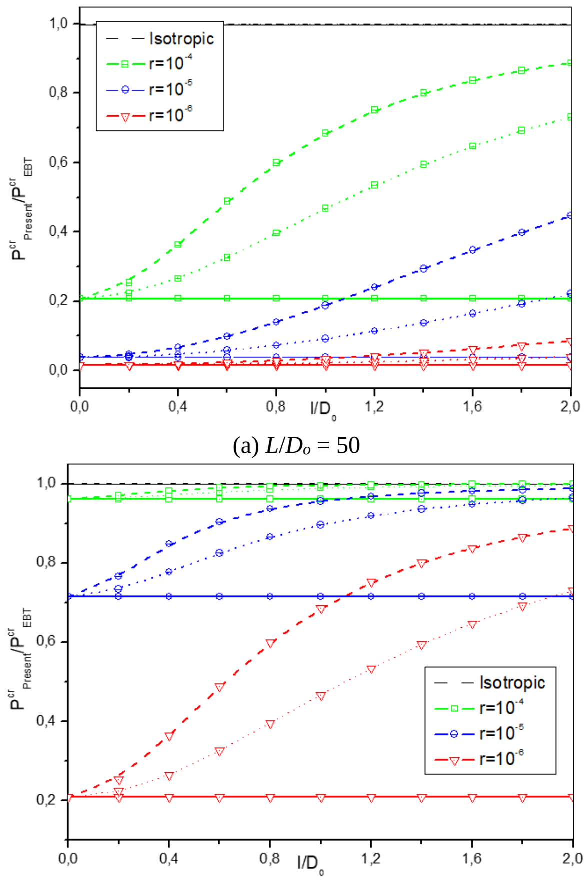



Fig. 3 Variation of critical buckling ratio for various geometric ratio and different shear modulus ratio




(a) $L/D_o = 50$

(b) $L/D_o = 500$

Fig. 4 Influences of length scale parameters on critical buckling load ratio for different shear modulus ratio (a) $L/D_o = 50$; (b) $L/D_o = 500$

examples are considered on buckling responses microtubules.

For illustrative purposes, a simply supported MT is taken as an example with the following material and geometric properties (Heireche *et al.* 2010): Young's modulus $E$ = 1 Gpa, Poisson ratio $\nu$ = 0.3, shear modulus ratio $r = G/E$ varying between $10^{-6}$ and $10^{-4}$, inner radius $r_i$= 7.5 nm, outer radius $r_o$= 12.5 nm and the length of MT $L$ varying between 1.25 μm and 12.5 μm. All material length scale parameters are considered to be equal to each other as $l_0 = l_1 = l_2$. If two parameters of length scale ($l_0$ and $l_1$) or all of them are zero, the proposed model will be transformed Modified Couple Stress (MCST) and Classical Models (CT), respectively. The results obtained by the proposed single variable sinusoidal beam model are compared with those obtained by Akgöz and Civalek (2014b) and EBT. In addition, it should be noted that the solid lines, the dotted lines, and the dashed lines of the figures indicate the results for CT, MCST and MSGT, respectively.

Comparison of critical buckling loads with different shear modulus ratio and geometric ratio corresponding to various beam theories and models are provided in Table 1.

It is observed that the results computed by the present single variable sinusoidal theory are in excellent agreement with those predicted by Akgöz and Civalek (2014b).

However, it should be noted that the proposed theory uses only one variable whereas the theory of Akgöz and Civalek (2014b) uses three variables. It can be observed that the critical buckling loads computed by the proposed theory and CT are lower than those evaluated by EBT and MSGT and also these situations are more frequent for smaller shear modulus ratios and geometric ratios as $r = 10^{-6}$ and $L/D_o = 50$. It can be seen that the results computed by EBT and the proposed theory are almost equal for isotropic case and $L/D_o = 50$. On the other hand, the differences between the results corresponding to EBT and the proposed theory are more obvious for the lower geometric ratios ($L/D_o = 50$), but they diminish for the larger ones, like $L/D_o = 500$. Also, it can be concluded that shear deformation influence becomes more considerable for small MTs.

Variations of critical buckling ratios versus various geometric ratio and different shear modulus ratio are plotted in Fig. 3. It is seen from this figure that the critical buckling ratios are much lower than one for smaller geometric ratios. However, these ratios tend to be closer to one by increasing geometric ratio and shear modulus ratio, particularly for MSGT. It can also be emphasized that these ratios corresponding to isotropic case are almost equal to one. It can be deduced from the results that difference between elastic and shear modulus, resulting from composite structure and anisotropic molecular architecture of MTs, plays a considerable role on buckling properties of MTs.

Influences of material length scale coefficient-to-outer diameter ratio on critical buckling load ratios of the simply supported MTs corresponding to various shear modulus ratios for $L/D_o = 50$ and $L/D_o = 500$ are presented in Fig. 4. It is observed that an increase in material length scale coefficient-to-outer diameter ratio leads to a decrement on impacts of shear deformation while the shear deformation impacts become more prominent by reducing shear modulus ratio. Also, it is clear that these ratios are closer to one for MSGT as a function of the increase in the material length scale parameter-to-outer diameter ratio. In addition, it can be seen that the critical buckling ratios for $L/D_o = 50$ are always farther than those for $L/D_o = 500$.

## 6. Conclusions

In this article, a microstructure-dependent “shear deformation beam model” is presented for buckling analysis of microtubules on the basis of “modified strain gradient elasticity theory”. The present single variable sinusoidal beam model captures impacts of shear deformation with no need for “shear correction factors” and this employing only one variable as the EBT. The equations of motion are obtained by utilizing the principle of minimum total potential energy. The buckling response of simply supported isolated microtubules is examined. Analytical solution for critical buckling load is obtained with the help of Navier solution method. Impacts of “shear deformation”, “material length scale parameter”, “geometric ratio” and “shear modulus ratio” on critical buckling load of MTs are examined in detail. The results are compared with those obtained by EBT and Akgöz and Civalek (2014a) in conjunctions with CT and MCST. It is observed that microstructure-dependent behavior is more important when material length scale parameters are close to the outer diameter of MTs. It can be remarked that the beam models based on higher order elasticity models and simple beam model are stiffer than those based on classical theory and shear deformation beam models. Also, it can be confirmed that the impacts of shear deformation become larger because of the composite structure and anisotropic molecular architecture of MTs, particularly for smaller geometric ratios. An improvement of present approach will be taken into account in the future work to consider other shear deformation models and other types of materials (Panda and Singh 2013, Akil 2014, Panda and Katariya 2015, Katariya and Panda 2016, Kar *et al*. 2016, 2017, Timesli *et al*. 2017, Saffari *et al*. 2017, Katariya *et al*. 2017a, b, Bensattalah *et al*. 2018, 2019, 2020, Eltaher *et al*. 2018, Fakhar and Kolahchi 2018, Faleh *et al*. 2018, Panjehpour *et al*. 2018, Akbaş 2018, Selmi and Bisharat 2018, Lata and Singh 2019, Mehar *et al*. 2019, Hussain and Naeem 2019, Mehar and Panda 2019, Dash *et al*. 2018, 2019, Avcar 2019, Suleiman *et al*. 2019, Hadipeykani *et al*. 2020, Kiani *et al*. 2020, Boulal *et al*. 2020).

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