

# Structural system reliability-based design optimization considering fatigue limit state

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**Abstract.** The fatigue-induced sequential failure of a structure having structural redundancy requires system-level analysis to account for stress redistribution. System reliability-based design optimization (SRBDO) for preventing fatigue-initiated structural failure is numerically costly owing to the inclusion of probabilistic constraints. This study incorporates the Branch-and-Bound method employing system reliability Bounds (termed the B<sup>3</sup> method), a failure-path structural system reliability analysis approach, with a metaheuristic optimization algorithm, namely grey wolf optimization (GWO), to obtain the optimal design of structures under fatigue-induced system failure. To further improve the efficiency of this new optimization framework, an additional bounding rule is proposed in the context of SRBDO against fatigue using the B<sup>3</sup> method. To demonstrate the proposed method, it is applied to complex problems, a multilayer Daniels system and a three-dimensional tripod jacket structure. The system failure probability of the optimal design is confirmed to be below the target threshold and verified using Monte Carlo simulation. At earlier stages of the optimization, a smaller number of limit-state function evaluation is required, which increases the efficiency. In addition, the proposed method can allocate limited materials throughout the structure optimally so that the optimally-designed structure has a relatively large number of failure paths with similar failure probability.

**Keywords:** failure sequence; fatigue; system reliability; system reliability-based design optimization

## 1. Introduction

Civil structures are often subjected to different oscillatory environmental and live loads such as wind on tall buildings, passing vehicles on bridges, or waves on offshore platforms. The failure of the Kemper Memorial Arena Roof was attributed to the repeated loading and unloading of wind-induced fatigue on the roof framing connection (Shepherd and Frost 1995). It was reported that overloading and fatigue contributed primarily to composite and steel bridge damages (Hobbacher *et al.* 2016, Moghadam *et al.* 2023, Moomen and Siddiqui 2022). An offshore platform named Alexander Kielland in the North Sea capsized during a storm owing to the fatigue-initiated failure of the supports (Moan 2005, Lee and Song 2011). These repeated loadings induce fatigue stress on the structure, which can eventually result in a catastrophic collapse. Therefore, the explicit consideration of fatigue during the design and maintenance of structural systems is necessary to avoid such disasters (Lee and Song 2011).

Suresh (1998) defines fatigue as the change in the material properties of metals caused by repeated stresses or strains, which causes cracking or failure. Most structures are highly redundant, and the fatigue-induced failure of a single member does not immediately cause the entire

structural system to collapse. However, the redistribution of forces will either overstress other members, resulting in the quasistatic failure of the entire structure, or accelerate the rate of fatigue of other members owing to increased stress (Martindale and Wirsching 1983). Hence, a system-level analysis considering stress redistribution is required. Owing to the inherent variability of both the external loads and the material properties of the structure, structural reliability theory should be applied. Structural system reliability (SSR) analysis, in addition to a failure-path searching scheme, was previously investigated to calculate system failure probability of structures against fatigue (Martindale and Wirsching 1983, Karamchandani *et al.* 1992, Lee and Song 2011, Shabakhty 2011, Gholizad *et al.* 2012, Lee and Kang 2014, Zuo *et al.* 2018, Zhao *et al.* 2020). The newly-developed Branch-and-Bound method employing system reliability Bounds (termed the B<sup>3</sup> method) can be used to calculate the system failure probability and identify the critical failure sequences of an offshore structure efficiently and accurately (Lee and Song 2011). The B<sup>3</sup> method, whose formulation is based on disjoint cut-set system events representing fatigue-induced failures and a new failure-path search scheme, can identify failure sequences in the decreasing order of their probabilities. The update rule for the lower and upper bounds of the system failure probability provides a reasonable termination criterion in the branch-and-bound search strategy. The B<sup>3</sup> method was further developed to perform a finite-element-based system reliability analysis of fatigue-induced failures on continuum structures (Lee and Song 2012).

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The detrimental consequences of failure against fatigue should be considered in the design process. Structural design often requires an optimization process that determines the best use of limited material resources considering constraints such as ultimate and serviceability limit states. The explicit consideration of the probability of failure (or conversely, reliability) as constraints in design optimization is often termed the reliability-based design optimization (RBDO). Extensive reviews regarding the different methods and approaches used in this field have been conducted (Youn and Choi 2004, Schuëller and Jensen 2008, Valdebenito and Schuëller 2010, Auoes and Chateauneuf 2009, Song *et al.* 2021). When a single-component event is considered in the optimization process, it is classified as component-RBDO (CRBDO) to differentiate it from system-RBDO (SRBDO) where a system event is considered using a logical (or Boolean) function of multiple component events (Nguyen *et al.* 2011). Several CRBDO studies considering fatigue in constraints have been conducted. Honarmandi *et al.* (2007) optimized the design of a cantilever beam under probabilistic fatigue constraints calculated using the first order second moment (FOSM) and first order reliability method (FORM). Ibrahim *et al.* (2014) developed an optimum safety factor methodology that considered fatigue damage for the optimal design of steel plates; it was discovered that reliability-based optimum solutions offered a more reliable design than deterministic design optimization. Hu *et al.* (2016) optimized the design of the blade of a wind turbine using a dynamic wind load uncertainty model to satisfy the target probability of failure against fatigue during its service life. Meanwhile, Yaich *et al.* (2018) developed a more reliable design using a robust hybrid method by optimizing a structure subjected to random vibrations with the fatigue limit state as the probabilistic constraint. The consideration of fatigue as a probabilistic constraint within a design optimization framework has been extensively investigated; however, the problems considered were those associated with CRBDO.

Song *et al.* (2021) reviewed the different techniques used in SRBDO. SRBDO can be categorized into five approaches: double-loop, single-loop, decoupled, simulation-based, surrogate-based, and heuristic approaches. The most straightforward technique in SRBDO is the double-loop where the inner loop optimization is performed to evaluate the probabilistic constraint and the outer loop optimization is performed to optimize the design variables (Auoes and Chateauneuf 2009, Song *et al.* 2021). To further improve computational efficiency, Liang *et al.* (2007) proposed a single-loop SRBDO where the key idea is to approximate the most probable point (MPP) of each constraint by solving the Karush-Kuhn-Tucker (KKT) conditions in each iteration of the search for the optimum design. A decoupled approach in which a sequence of cycle between deterministic optimization and reliability assessment was implemented for various optimization problems (Royset *et al.* 2001, Auoes and Chateauneuf 2008, Faes and Valdebenito 2020, Yuan *et al.* 2021). The SRBDO was essentially modified into an equivalent deterministic optimization with constraints linked to reliability analysis.

These approaches can be classified based on the optimization algorithm procedure. Simulation-based algorithms were implemented because of the complications of SSR analysis (Taflanadis and Beck 2009, Jia and Taflanidis 2013, Suksuwan and Spence 2018, 2019). Furthermore, techniques for approximating limit-state functions using surrogate models have been tested and proven to be effective (Dubourg *et al.* 2011, Moustapha *et al.* 2016, Ghiasi and Ghasemi 2018, Yoon *et al.* 2020). The motivation of this approach is to replace the numerically expensive evaluation of functions (i.e., structural analysis) with surrogate models. The kriging metamodeling technique, as a substitute for the performance function, and subset simulation were integrated into gradient-based optimizers to solve RBDO problems in structural mechanics (Dubourg *et al.* 2011). An RBDO algorithm was developed using a Gaussian mixture model with Monte Carlo simulation (MCS) for its reliability analysis, and the sensitivity was derived based on the Gaussian process (GP) predictions (Li and Wang 2019). Kim and Song (2021) approximated the quantiles of a performance function using adaptive GP to verify design samples while conforming to target probability constraints to solve RBDO problems. Metaheuristic optimization algorithms have garnered attention owing to their applicability to a wide range of problems. Inspired by nature, optimal designs were developed without calculating the sensitivity of constraints with respect to the design variables. Several researchers have successfully integrated metaheuristic algorithms into the RBDO framework (Yang and Hsieh 2011, Abbasnia *et al.* 2014, Chen *et al.* 2014, Hamzehkolaei *et al.* 2015, Chakri *et al.* 2017, Liao and Biton 2019, Zaeimi and Ghoddosian 2019). Genetic algorithm (GA) was used to optimize the weight of a truss structure subjected to probabilistic constraints (Liu *et al.* 2014). An improved differential evolution algorithm was used for the RBDO of nonlinear inelastic truss structures (Truong and Kim 2018). Particle swarm optimization and symbiotic organism search as the optimizer and method of moments with the equivalent Pearson's distribution system for the calculation of reliability were used to solve RBDO problems involving linear, highly nonlinear, and implicit probabilistic constraints with normal and non-normal variables (Liao and Biton 2019). The radial basis function neural network was used to approximate the structural response, and the MCS was performed to determine the system failure probability. A new searching scheme for the dominant failure mode based on differential evolution together with extended sequential compound method was used in an SRBDO algorithm (Xing *et al.* 2021). Meanwhile, Thampan *et al.* (2001) investigated the configuration and areas of truss members considering system reliability constraints based on strength and serviceability limit states. A modified branch-and-bound algorithm was employed to calculate the system failure probability, whereas the GA was used as the optimization algorithm for the design variables. Additional bounding conditions were introduced to limit the tracing of the failure paths. Meng *et al.* (2020) performed a review and comparison of different RBDO's using metaheuristic algorithms. Recent metaheuristic algorithms can

successfully solve five complicated engineering problems with multimodal functions and mixed design variable types, thereby enabling the investigation of global convergence, robustness, accuracy, and computational speed. The multiverse optimizer, grey wolf optimizer (GWO), and artificial bee colony achieved superior robustness and successfully yielded the best optimal solution.

However, most of the previous studies pertaining to SRBDO consider strength and serviceability limit states either in series or parallel, in which the associated system failure event is defined comprehensively. Several issues occur when addressing sequential failures based on the fatigue limit state. For complex structures with a high level of structural redundancy, the system failure event is unknown a priori. A failure path approach is required to determine local or global failure sequences. Another challenge is that the sensitivity of the system failure probability with respect to the design parameter is difficult to calculate because of the unknown system failure event. A sensitivity analysis is essential for efficient gradient-based SRBDO algorithms. Moreover, the failure-path SSR approach is computationally expensive when several structural members are present.

In this study, a new SRBDO method is developed considering fatigue-induced sequential failure that addresses the challenges mentioned above. The coupling of fatigue failure and other failure modes is beyond the scope of this study. Application examples are selected wherein fatigue limit states govern. The advantages of the B<sup>3</sup> method as an SSR analysis tool that can identify critical failure sequences efficiently are integrated into an SRBDO framework to obtain the optimal design of redundant structures considering fatigue limit states. Because the sensitivity of the failure probability cannot be calculated explicitly, a metaheuristic optimization algorithm was used in the proposed method. The superior performance of the GWO in RBDO problems (Meng *et al.* 2020) was exploited in the proposed framework. In this study, the B<sup>3</sup> method is implemented in a system reliability-based design optimization algorithm using the GWO-based metaheuristic algorithm. To further improve the efficiency of the proposed algorithm, an additional bounding rule is proposed in the context of SRBDO against fatigue using the B<sup>3</sup> method.

## 2. Proposed SRBDO method

### 2.1 B<sup>3</sup>-Method

The B<sup>3</sup> method (Lee and Song 2011, 2012) has a structured way of finding failure sequences in order to calculate the overall system failure probability. Three processes are defined in this search scheme: system failure evaluation, branching, and bounding. System failure evaluation is the process of determining the system failure probability of a sequence and identifying whether the sequence will produce an overall system failure. Branching is the process of finding a particular permutation of structural members that constitutes a failure sequence. Lastly, bounding is the process of updating the lower and upper bounds of the system failure probability. There are

two goals in B<sup>3</sup> method: finding for failure sequences and narrowing the gap between upper and lower bounds of the system failure probability. To find a failure sequence, the searching scheme starts with the evaluation of failure probability for each member when no prior member has failed yet. In system failure evaluation, a failure sequence is checked whether it produces overall system failure (such as the loss of stability and large displacements) of the structure. After the system failure probabilities are calculated, the bounding process commences. When a system failure is observed, the lower bound,  $\alpha_L$ , is increased. However, if there are non-failure observed, the upper bound,  $\alpha_U$ , is decreased. The branching process starts at the non-failure node that has the highest probability. Another structural analysis is then executed to consider stress redistribution due to failed members. The entire procedure is repeated until the gap between the lower bound and upper bounds becomes small enough.

In the context of fatigue limit-state, the failure event is defined when the time to attain the critical crack length,  $a_{ci}$ , exceeds the predefined inspection time,  $T_s$ . A detailed derivation of the fatigue limit-states in the context of B<sup>3</sup> method is given in Lee and Song (2011), and it is briefly reviewed here. For the  $i$ th member of a structure, the time to attain the critical crack length,  $T_i$ , is given by

$$T_i = \frac{1}{C v_0 (S_i^0)^m} \int_{a_i^0}^{a_{ci}} \frac{1}{[Y(a)\sqrt{\pi a}]^m} da \quad (1)$$

where  $S_i^0$  is the far-field stress range,  $Y(a)$  is the geometry function with the crack length  $a$ ,  $c$  and  $m$  are the material properties in the Paris equation (Paris and Erdogan 1963, Lim *et al.* 2019),  $v_0$  is the frequency of application of external loadings,  $a_{ci}$  is the critical cracked length of the  $i$ th member, and  $a_i^0$  is the initial cracked length of the  $i$ th member. To ascertain disjoint events in a failure sequence, the events that the member fails before the failures of any other remaining members under the given loading condition are additionally included in the system event definition. Thus, the component limit states are defined as

$$g_i(\mathbf{X}) = \begin{cases} T_i - T_s \leq 0 \\ T_i - T_l \leq 0 \text{ for } l \neq i \end{cases} \quad (2)$$

where  $\mathbf{X}$  is the vector of random variables. When a member is included in a failure sequence, the stress redistribution should be considered when calculating for  $T_i^{1,\dots,i-1}$ . Mathematical induction was used by Lee and Song (2011) to derive the expression

$$T_i^{1,\dots,i-1} = \frac{1}{C v_0 (S_i^{1,\dots,i-1})^m} \int_{a_i^0}^{a_{ci}} \frac{1}{[Y(a)\sqrt{\pi a}]^m} da - \sum_{k=1}^{i-1} \left( \frac{S_i^{1,\dots,k-1}}{S_i^{1,\dots,i-1}} \right)^m T_k^{1,\dots,k-1} \quad (3)$$

Due to the aforementioned formulation, each failure sequence identified during the B<sup>3</sup> analysis are mutually exclusive. The probability of system failure is then defined as a series of parallel subsystems. Each failure sequence

identified are disjoint to each other; thus, the system failure probability  $P(E_{sys})$  is the sum of each probability  $P(C_i)$  of identified failure sequence  $C_i$ .

$$P(E_{sys}) = P\left(\bigcup_{i=1}^{N_{fs}} C_i\right) = \sum_{i=1}^{N_{fs}} P(C_i) \quad (4)$$

where  $N_{fs}$  is the number of failure sequences. Finding all failure sequences is a computationally expensive and time-consuming task; thus, only dominant failure sequences are used to identify a narrow gap between the upper and bounds where  $P(E_{sys})$  exist (i.e.,  $\alpha_L \leq P(E_{sys}) \leq \alpha_U$ ).

## 2.2 GWO algorithm

The grey wolf's predation process of encircling, hunting, searching and attacking the prey is translated to mathematical models to solve optimization problems. Grey wolves follow a caste system divided into – alpha, beta, delta and omega wolves (Mirjalili *et al.* 2014). The position of  $i$ th wolf on a pack of  $N$  wolves represents a solution of the form  $\mathbf{X}_i = (x_1, x_2, \dots, x_k)_i$  for  $i = 1, 2, \dots, N$  in a  $k$ -dimensional search space. The wolves' hierarchy is mathematically modeled by assuming the best solution as the alpha ( $\mathbf{X}_\alpha$ ), second and third best solution as the beta ( $\mathbf{X}_\beta$ ) and delta ( $\mathbf{X}_\delta$ ), respectively. All the remaining candidate solutions are the omega wolves. At the end of the iteration, the alpha's wolf position corresponds to the location of the prey or correspondingly the optimal design.

The encircling process during the cooperative hunting of the wolf pack around the prey in terms of a  $k \times 1$  distance vector,  $\mathbf{D}$ , is mathematically modeled as

$$\mathbf{D} = |\mathbf{C} \cdot \mathbf{X}_p^{(t)} - \mathbf{X}_p| \quad (5)$$

$$\mathbf{X}_p^{(t+1)} = \mathbf{X}_p^{(t)} - \mathbf{A} \cdot \mathbf{D} \quad (6)$$

where  $\mathbf{X}_p$  is the position of the prey,  $\mathbf{X}_p^{(t)}$  and  $\mathbf{X}_p^{(t+1)}$  are the positions of the wolf at the iteration  $t$  and  $t + 1$ , respectively.  $\mathbf{A}$  and  $\mathbf{C}$  are the coefficient  $k \times 1$  vectors whose values affect the locations of the omegas around the best wolf. These parameters are computed using the following equations

$$\mathbf{A} = 2a \cdot \mathbf{r}_1 - a \quad (7)$$

$$\mathbf{C} = 2 \cdot \mathbf{r}_2 \quad (8)$$

The parameter  $a$  is decreased from 2 to 0 linearly over the entire iterations. Coefficient vectors  $\mathbf{r}_1$  and  $\mathbf{r}_2$  with size  $k$  are randomly selected in the interval  $[0, 1]$ .

In the hunting process, the alpha wolf leads followed by its subordinate's beta and delta, while the rest of the wolves, omegas, chases the prey with directions and supervision from the superior wolves. This is formulated by saving the three best solutions obtained so far and drive other wolves to adjust their positions according to the locations of the alpha, beta, and delta solutions found thus far in Eq. (11).

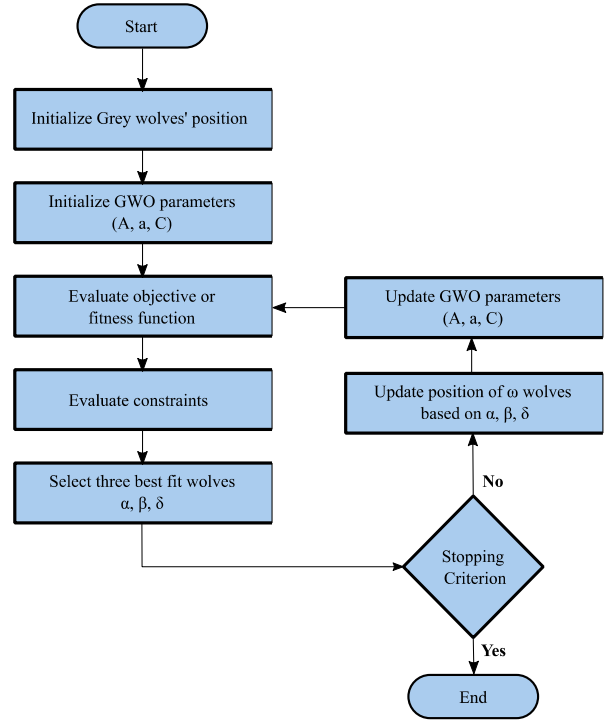


Fig. 1 Grey wolf optimization flow chart

$$\mathbf{D}_\alpha = |\mathbf{C}_1 \cdot \mathbf{X}_\alpha - \mathbf{X}_i^{(t)}|, \quad \mathbf{D}_\beta = |\mathbf{C}_2 \cdot \mathbf{X}_\beta - \mathbf{X}_i^{(t)}|, \quad (9)$$

$$\mathbf{D}_\delta = |\mathbf{C}_3 \cdot \mathbf{X}_\delta - \mathbf{X}_i^{(t)}|$$

$$\mathbf{X}_1 = \mathbf{X}_\alpha - \mathbf{A}_1 \cdot \mathbf{D}_\alpha, \quad \mathbf{X}_2 = \mathbf{X}_\beta - \mathbf{A}_2 \cdot \mathbf{D}_\beta, \quad (10)$$

$$\mathbf{X}_3 = \mathbf{X}_\delta - \mathbf{A}_3 \cdot \mathbf{D}_\delta$$

$$\mathbf{X}_i^{(t+1)} = \frac{\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3}{3} \quad (11)$$

The algorithm starts by initializing the position of the pack of wolves and parameters  $A$ ,  $a$  and  $C$ . The objective or fitness function and constraints are evaluated for each wolf. The best three solution are then selected and named the alpha, beta and delta. The rest of wolves' position are updated using Eqs. (9)-(11). The whole process is repeated until stopping criteria is achieved such as the max number of iterations. The GWO algorithm is summarized in a flow chart as shown in Fig. 1.

## 2.3 Proposed SRBDO method

In this section, the proposed method for performing SRBDO on structures considering the fatigue limit state is discussed. The  $B^3$  method is integrated with the GWO algorithm to determine the optimal design that satisfies probabilistic constraints. The optimal structural design problems with a single objective function can be formulated by specifying (a) the reliability of the structure as the objective function, or (b) the reliability as the constraint (Royset *et al.* 2001). In this study, SRBDO is formulated to minimize the cost of the design subject to probability constraints. The cost of a structure is assumed to be proportional to its weight. The relevant mathematical

formulation is as follows

$$\begin{aligned} & \text{Min } f(\mathbf{d}) \\ & \text{subject to} \\ & P(E_{sys}) = P\left(\bigcup_{i=1}^{N_{fs}} C_i\right) \leq P_{sys}^t \quad (12) \\ & \mathbf{d}^l \leq \mathbf{d} \leq \mathbf{d}^u \end{aligned}$$

where  $f(\cdot)$  is the objective function,  $P(E_{sys})$  is the system failure probability,  $P_{sys}^t$  is the target/threshold system failure probability,  $\mathbf{d}$  is the vector of the design variables, and  $\mathbf{d}^l$  and  $\mathbf{d}^u$  are the lower and upper limits of the design variables, respectively. The current study focuses on a new methodology that accounts for fatigue-induced sequential failure in an SRBDO framework. However, other failure mechanisms (e.g., sidesway due to formation of plastic mechanism) or other failure modes (e.g., yielding, buckling, etc.) can be accounted for by including it as an additional constraint in Eq. (12). Previous studies have already considered system failures based on plastic mechanism and elasto-plastic behavior (Kim *et al.* 2013).

The B<sup>3</sup> method can be used to determine the different failure sequences  $C_i$  and bound the value of  $P(E_{sys})$  with the upper bound  $\alpha_U$  and lower bound  $\alpha_L$ . Because the probabilistic constraint in Eq. (12) considers only one value to be compared with  $P_{sys}^t$ , the assumption for the  $P(E_{sys})$  is as follows

$$P(E_{sys}) \approx \alpha_U \quad (13)$$

This assumption can be realistic because the gap between  $\alpha_U$  and  $\alpha_L$  is small. GWO is used to obtain the optimal design that satisfies the given constraints. The objective function is formulated such that the weights of the infeasible designs are penalized. It is defined as

$$f(\mathbf{d}) = \sum_{i=1}^n A_i(\mathbf{d})L_i\rho_i + \sum_{j=1}^m r_j \cdot \max(0, G_j)^2 \quad (14)$$

where  $A_i$  is the area,  $L_i$  is the length,  $\rho_i$  is the density for  $i = 1, 2, \dots, n$ , and  $n$  is number of structural members,  $G_j$  is the inequality constraints (i.e.,  $P(E_{sys}) - P_{sys}^t \leq 0$ ),  $r_j$  is the penalty factor for  $j = 1, 2, \dots, m$ , and  $m$  is number of inequality constraints.

The proposed framework of the SRBDO algorithm uses both the upper and lower bound probabilities obtained by the B<sup>3</sup> method. The upper bound  $\alpha_U$  is set as the system failure probability of the structure, as in Eq. (13), and the lower bound as an additional stopping criterion in the branching process of the B<sup>3</sup> method. The specific formulation of the B<sup>3</sup> method, which yields a monotonically-increasing lower bound, is exploited in the proposed framework; it terminates the branching process when the threshold system failure probability is exceeded.

$$\text{Stopping Criterion: } \left| \frac{\alpha_L}{\alpha_U} \right| > \epsilon \text{ AND } \alpha_L > P_{sys}^t \quad (15)$$

At the earlier stage of the optimization process, the stopping criterion of the B<sup>3</sup> method used in this study may yield designs that have a wider gap between the failure

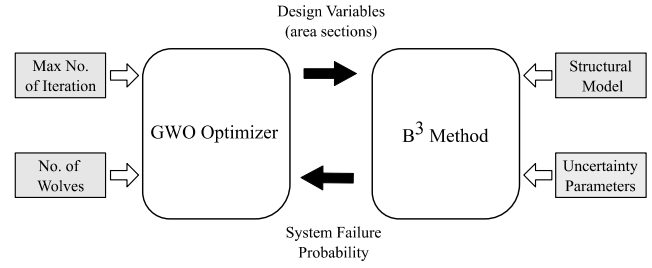


Fig. 2 Proposed SRBDO method

bounds. This is acceptable in the exploration stage, when the design space is searched globally for a promising location.

The population of searching agents will likely be obtained in promising areas with  $\alpha_L < P_{sys}^t$  and  $\alpha_U < P_{sys}^t$  because of the penalizing function in Eq. (14) for infeasible designs. As the optimization process progresses to the exploitation stage when the promising areas are locally searched, a narrower gap between the failure bounds is desirable.

The number of wolves and the maximum number of iterations as the stopping criterion of the GWO is first defined into the optimizer. The structural model (i.e., finite element model) and the uncertainty parameters (i.e., distribution type and parameters) are defined into the B<sup>3</sup> framework. In every iteration, each of the wolf's position corresponds to the design variables and is considered as a candidate solution. In each design, the B<sup>3</sup> method is used to evaluate the system failure probability. After determining the system failure probability, this information is fed back to the GWO optimizer for the calculation of the objective function or weight of the design considering a penalizing factor for infeasible designs in Eq. (14). This loop is illustrated in Fig. 2 and repeated until an optimal design is found.

### 3. Numerical examples

#### 3.1 Application example I: Multilayer Daniel's system

The SRBDO algorithm is illustrated using a hypothetical structural system problem adapted from Lee and Song (2011). The structure comprises three layers of elements with six brittle bars, as shown in Fig. 3. It is assumed that each layer has the same area and a length of 1 m. The design variables considered in this example are the area for each layer,  $\mathbf{d} = [A_1, A_2, A_3]$  with lower and upper limits of 0.01 and 0.05 m<sup>2</sup>, respectively. The SRBDO problem is formulated as

$$\begin{aligned} \text{Min } f(\mathbf{d}) &= (A_1 + 2A_2 + 3A_3)L\rho \\ &+ \sum_{j=1}^m r \cdot \max(0, G_j)^2 \quad (16) \end{aligned}$$

subject to

$$G_1: P(E_{sys}) \leq 5 \times 10^{-3}$$

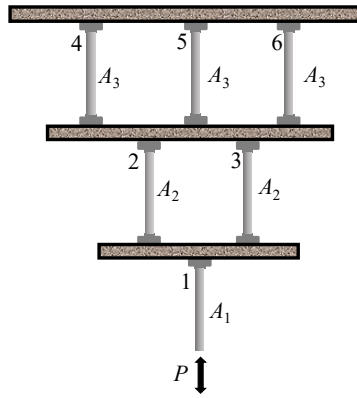


Fig. 3 Multilayer Daniel's system

$$G_2: \begin{cases} P(C_1) \leq 0.5 \times 10^{-3} \\ 0.01 \text{ m}^2 \leq \mathbf{d} \leq 0.05 \text{ m}^2 \end{cases} \quad (16)$$

The random variables considered are:  $C$  (the material property parameter in the Paris equation), the initial crack length  $a_{i0}$ , and the external load  $P$ , which are assumed to follow lognormal distributions with means  $1.36 \times 10^{-13}$  mm/cycle/(MPa mm) $^m$ , 0.11 mm, and 1,200 kN, respectively. For the sake of simplicity, all random variables are assumed to be statistically independent, with coefficient of variation (COV) of 0.1. The other parameters are as follows: loading frequency = 500,000 cycles per year, inspection cycle  $T_s = 4$  years,  $m = 3$ ,  $Y(a) = 3$ , and critical length  $a_{ci} = A_i/1$  m for  $i = 1, 2, 3$ .

The 1,200 kN force is assumed to be equally distributed to the bars per layer. When a bar element fails (the member attains its critical length before the inspection cycle), the force is redistributed equally to the remaining bars on that specific layer.

The overall system failure of the structure occurs when all bar elements fail in one of the three layers. Two cases are considered in this example. In Case 1, the overall system failure probability of  $P(E_{sys})$  is assumed to be less than the threshold failure probability of  $5 \times 10^{-3}$ . In Case 2, additional constraint is imposed, i.e., the dominant failure sequence  $P(C_1)$  must be less than  $5 \times 10^{-4}$ .

The optimization results are presented in Table 1. The optimal weights for Cases 1 and 2 converged to 710.39 and

719.68 kg, respectively. The lower and upper bounds of the system failure probability of Case 1, as determined by the proposed method, were  $4.719 \times 10^{-3}$  and  $4.957 \times 10^{-3}$ , respectively. This system failure probability was comparable to the result of the crude MCS of  $4.824 \times 10^{-3}$  with 1,000,000 samples. Meanwhile, the lower and upper bound system failure probabilities of Case 2 were  $3.678 \times 10^{-3}$  and  $3.866 \times 10^{-3}$ , respectively, which were comparable to the MCS result of  $3.79 \times 10^{-3}$ . The  $P(C_1)$  for Case 2 was  $4.67 \times 10^{-4}$ , which is similar to the threshold of  $5 \times 10^{-4}$ .

From the presented results, it was confirmed that the proposed SRBDO algorithm could determine an optimal design that satisfied the given constraints successfully. As a result, the materials were concentrated into member 1, which was forced to have the largest optimal area among the members. Because the failure of member 1 constituted an overall system failure of the structure, a large percentage of the total weight was provided to this member. Additionally, the dominant failure sequence determined for both cases was the failure of member 1; hence, its contribution to the total overall system failure probability was higher than those of the other failure modes. The proposed method allocated more materials to members that contributed more significantly to the overall system failure event, which was member 1 in this case. In fact, the proposed method prioritizes certain members (by assigning larger areas).

### 3.2 Application example II: Three-dimensional tripod jacket structure

To demonstrate the applicability of the proposed method to more complex and realistic structural problems, a three-dimensional tripod structure (Lee and Song 2011, Karamchandani *et al.* 1992) was analyzed against fatigue-induced sequential failures. In this problem, repeated pounding of waves may result in cracks and failure owing to fatigue stress. A structural analysis was performed to determine the stresses of undamaged and damaged structures based on the assumption that the material behaves linearly until it experiences crack failure.

The details of the geometry of the structure can be found from previous literature (Lee and Song 2011) and is illustrated in Fig. 4. The truss structure comprised 66 members. Subsequently, the truss was subcategorized into

Table 1 Optimization results from proposed SRBDO method for multilayer Daniel's system

	Case 1	Case 2
Optimal design, m <sup>2</sup>	[0.0297, 0.0151, 0.0102]	[0.0305, 0.0154, 0.0101]
Total weight, kg	710.39	719.68
Upper bound (Proposed method)	$4.958 \times 10^{-3}$	$3.866 \times 10^{-3}$
Lower bound (Proposed method)	$4.720 \times 10^{-3}$	$3.678 \times 10^{-3}$
Dominant sequence, $C_1$ (Proposed method)	1	1
Probability of dominant sequence, $P(C_1)$ (Proposed method)	$1.5 \times 10^{-3}$	$4.94 \times 10^{-4}$
System failure probability (MCS)	$4.824 \times 10^{-3}$	$3.789 \times 10^{-3}$
Dominant sequence, $C_1$ (MCS)	1	1
Probability of dominant Sequence, $P(C_1)$ (MCS)	$1.453 \times 10^{-3}$	$4.67 \times 10^{-4}$

Table 2 Cross section database (EN 10220 2002)

No.	Section
1	ø406.4×12.5
2	ø457×12.5
3	ø406.4×16
4	ø508×12.5
5	ø457×16
⋮	⋮
159	ø1,422×50
160	ø1,524×50

Table 3 Statistical parameters of tripod jacket structure

Random variable	Mean	COV	Distribution type	No. of random variables
$C$	$1.202 \times 10^{-13}$	0.533	Lognormal	66
$a_{oi}$	0.11	1	Exponential	66
$I$	1	0.1	Lognormal	1

three main groups (vertical, horizontal, and diagonal) based on the orientation of the member (Fig. 4). Each main group was further classified into five subgroups based on the story level. The area of the members in a subgroup is represented by 15 design variables, i.e.,  $\mathbf{d} = [V_1, V_2, V_3, V_4, V_5, H_1, H_2, H_3, H_4, H_5, D_1, D_2, D_3, D_4, D_5]$ . The design section for each member was selected from 160 commercially available tubular sections. Each tubular section in Table 2 was assigned a unique index to be used for calling its properties (diameter, area, and thickness). The deterministic and random variables are listed in Table 3.

The uncertainties in the initial crack length, Paris equation parameter  $C$  of each member and the load scale factor  $I$  were considered. A total of 133 random variables were considered in this example. The explicit formulation of the SRBDO problem is as follows

$$\text{Min } f(\mathbf{d}) = \sum_{i=1}^{66} A_i L_i \rho + \sum_{j=1}^m r \cdot \max(0, G_j)^2 \quad (17)$$

subject to

$$G_1: P(E_{sys}) \leq 2 \times 10^{-3}$$

$$\mathbf{d}_{\text{lower}} \leq \mathbf{d} \leq \mathbf{d}_{\text{upper}}$$

The B<sup>3</sup> method was used to determine the system failure probability of a specified design  $\mathbf{d}$ . Failure at the member level is defined as when a section reaches the critical length before the inspection cycle. In the searching scheme of the B<sup>3</sup> method, this damaged member is removed in another iteration of the structural analysis where stress redistribution is considered. The failure at the system level is adapted from (Lee and Song 2011). A system failure event is observed when at least one of the following criteria occurs:

1. Global statistical determinacy or instability condition:  $3 \times (\text{number of nodes}) - (\text{number of members}) - (\text{number of reaction degrees of freedom}) > 0$
2. Local instability condition: less than three members are connected to a non-supporting node
3. Condition of global stiffness matrix: the condition number of the global stiffness matrix is exceedingly large
4. Unreasonably large displacement occurs.

The optimal truss design for the tripod jacket structure is presented in Table 4. The total weight of the structure is 131,747.7 kg. The convergence plot in Fig. 5 shows that the proposed SRBDO framework can identify the optimal

Table 4 List of sections for the optimal design

Group	Outer diameter, mm	Thickness, mm	Area, mm <sup>2</sup>
$V_1$	1,321	31	40,895.40
$V_2$	1,118	25	28,387.04
$V_3$	914	31	28,312.20
$V_4$	864	35	30,161.23
$V_5$	965	21	19,907.06
$H_1$	610	19	11,612.88
$H_2$	406	21	8,381.919
$H_3$	406	19	7,741.92
$H_4$	457	21	9,429.66
$H_5$	508	21	10,477.40
$D_1$	660	13	8,387.08
$D_2$	711	22	15,806.42
$D_3$	610	29	17,419.32
$D_4$	660	21	13,620.62
$D_5$	660	31	20,447.70

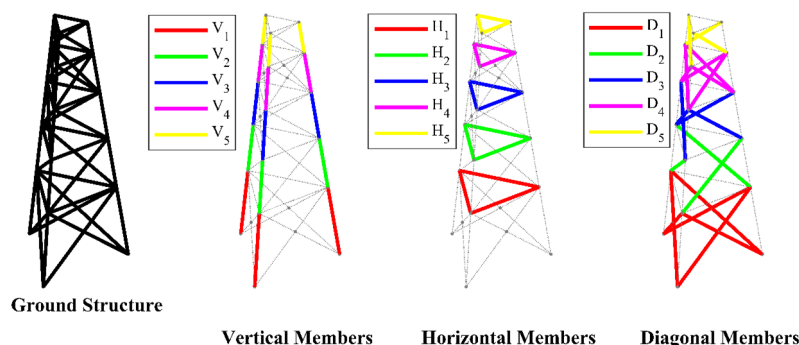


Fig. 4 Offshore tripod structure groupings of structural members

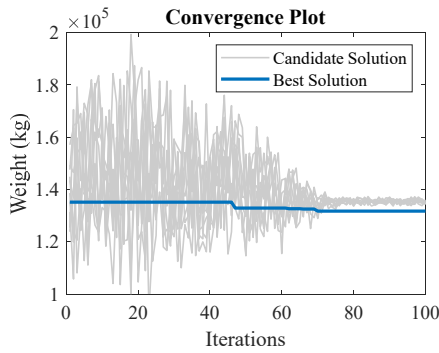


Fig. 5 Convergence plot of offshore tripod structure

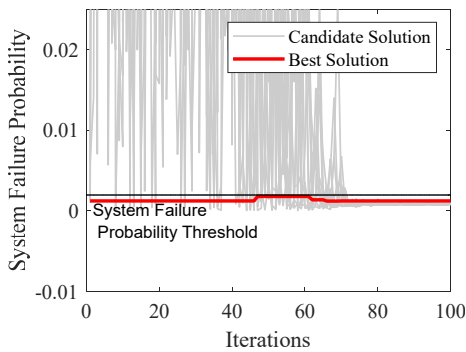


Fig. 6 System failure probability per iteration

design (i.e., the best solution) among the candidate solutions (represented by the other wolves' position). As shown in Fig. 5, the scattered weights in the early stage of the optimization process represent the exploration characteristics of GWO. The member sections per iteration vary significantly, which corresponds to a significant difference in weight among the candidate solutions. The design space is searched for a promising location (i.e., different combinations of member sections) where an optimum can be obtained. In the latter iteration, smaller variations in the areas of the members were observed in each new iteration. At approximately the 70th iteration, the scatter in the weights reduced to a smaller range, which commenced the exploitation stage of the optimization process.

Fig. 6 shows the system failure probability of the best solution below the system failure threshold of  $2 \times 10^{-3}$ . In the optimal design, the system failure probabilities of the upper and lower bounds were  $1.0 \times 10^{-3}$  and  $1.5 \times 10^{-3}$ , respectively. The results were comparable to those of the MCS with 10,000,000 samples, i.e.,  $1.2 \times 10^{-3}$ . As shown in the figure, at the earlier stages, the system failure probability was significantly greater than the target system failure probability and converged gradually immediately below the target probability. Higher values of system failure probability were observed at the exploration stage when smaller areas were assigned to members (i.e., vertical

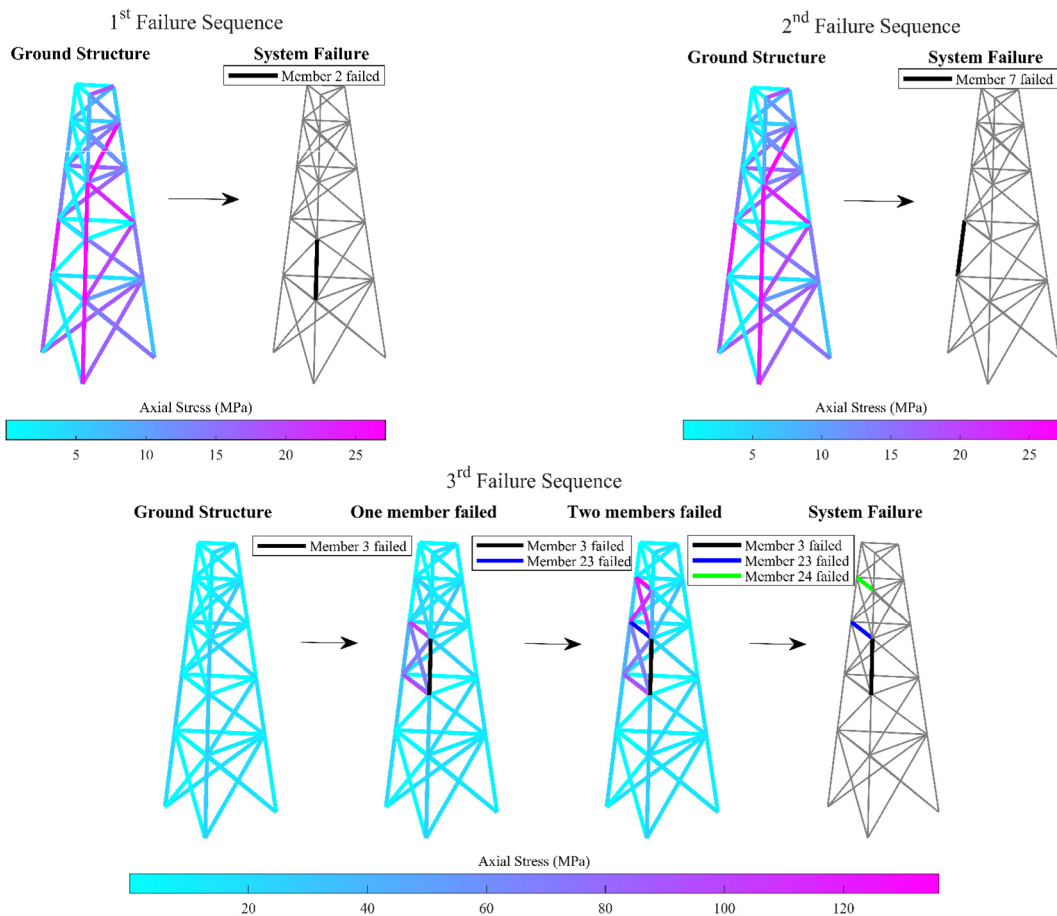


Fig. 7 Critical failure sequences and stress redistribution

Table 5 List of critical failure sequences

Failure sequence	Probability by proposed method ( $\times 10^{-4}$ )	Probability by MCS ( $\times 10^{-4}$ )
2	4.83	4.68
7	0.79	0.74
3→23→24	0.74	0.66
3→22	0.72	0.74
40→19	0.58	0.57
37→19	0.58	0.53
3→23→22	0.57	0.60
37→18	0.36	0.37
40→18	0.36	0.35
37→2	0.22	0.23

members, members connected to supports) that promote or accelerate the overall system failure.

The  $B^3$  analysis found 35 critical failure sequences until its termination, and the ten most critical failure sequences are listed in Table 5. It can be observed in the table that the failure probabilities determined by the  $B^3$  method are close to those calculated by MCS. The stress profile and redistribution of forces for the first three failure sequences are illustrated in Fig. 7.

In the first and second critical failure sequence, the failure of one member triggers unreasonable large displacements

which constitutes an overall structural system failure. In the third failure sequence, the stress redistribution leading to large displacements when three sequential failures occur triggered the structural system failure.

The proposed SRBDO framework terminates the branching process of the  $B^3$  method when the failure sequence exceeds the target system failure probability. It is evident from Fig. 8 that the total number of branches (for all wolves/candidate solutions) is lower in the earlier iteration and increases in the latter iterations. For example, if one of the vertical members attached to the support has a probability of failure greater than the target system failure probability, then no further branching will be performed because the design is already in the infeasible space, and further searching for other failure modes will increase the system failure probability of the structure. The monotonic increase in the lower bound of the  $B^3$  method owing to its disjoint formulation is applied in this proposed framework efficiently to reduce the computational cost of the optimization process. This is evident in Fig. 8, wherein the number of limit-state evaluations is lower at the earlier stages when the design exhibits a relatively high probability of system failure. For a feasible design, more branches are created, and more failure modes are determined; hence, the number of limit state evaluations is increased.

Fig. 9 shows that the failure sequences determined at the earlier stages were fewer than those determined based on the optimal design. More failure sequences implies that the material is distributed well within the structure, and that more members contribute to the overall system failure probability.

Fig. 10 shows the process by which the proposed

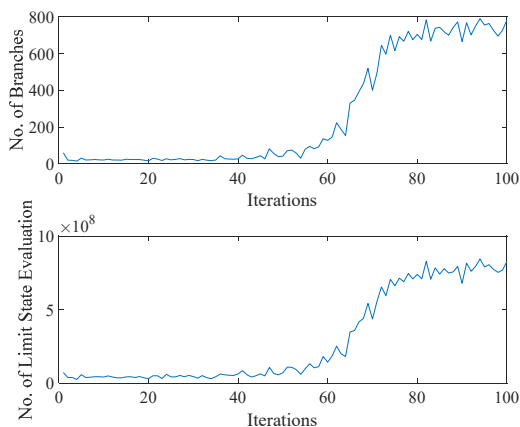


Fig. 8 Total number of branches and total number of limit state evaluation

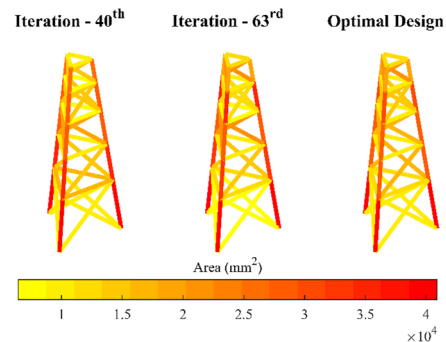


Fig. 10 Distribution of area sections

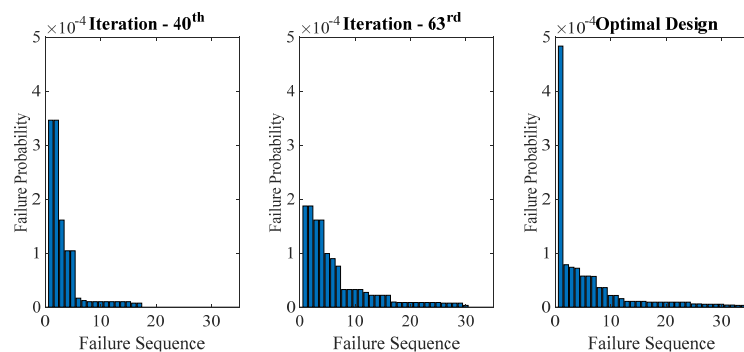


Fig. 9 Probability of failure sequences at 40th, 63rd and optimal design

Fig. 10 also shows the gradual increase in the size of sections per level (from bottom to top) in the optimized design for the vertical members. By contrast, the 63<sup>rd</sup> iteration design exhibits larger sections on the fourth level compared with that on the third level. This indicates that the proposed framework can yield the optimal design by allocating the appropriate materials to regions in the structure that contribute to the overall system failure.

#### 4. Conclusions

A new SRBDO method was developed to determine an optimal design that considers fatigue-induced sequential failure. The B<sup>3</sup> method was applied to estimate the overall system failure probability, in addition to GWO to identify the least weight of a structure that complies with the set target system failure probability. The proposed method can determine the optimal design efficiently by considering the lower bound as an additional stopping criterion in the branching process of the B<sup>3</sup> method. The branching process ceases when the lower bound exceeds the target system failure probability. This strategy significantly decreases the number of limit state evaluations in the exploration stage of the optimization, thereby enhancing the computational efficiency of the proposed algorithm. To demonstrate the proposed SRBDO method, it was applied to the examples of a multilayer Daniel's system and a three-dimensional offshore structure with 66 members. It was discovered that the convergence plots converged to an optimal design after several iterations. The overall system failure probability calculated using the B<sup>3</sup> method was verified via a crude MCS. This new approach can allocate the appropriate materials throughout the structure, thereby increasing the number of failure sequences in the optimal design. Furthermore, it explicitly incorporates system reliability into design optimization while considering the fatigue limit states, thereby resulting in cost-efficient and safer structures. The proposed method focuses on fatigue sequential failure. Different failure modes aside from fatigue can govern the system failure of a structure. Thus, future research could focus on the joint consideration of fatigue and other failure modes within one SRBDO framework. The most likely failure mode of the optimal design can then be assessed by its calculated probability value.

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