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Feasibility study on an acceleration signal-based translational and rotational mode shape estimation approach utilizing the linear transformation matrix

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Abstract. In modal analysis, the mode shape reflects the vibration characteristics of the structure, and thus it is widely performed for finite element model updating and structural health monitoring. Generally, the acceleration-based mode shape is suitable to express the characteristics of structures for the translational vibration; however, it is difficult to represent the rotational mode at boundary conditions. A tilt sensor and gyroscope capable of measuring rotational mode are used to analyze the overall behavior of the structure, but extracting its mode shape is the major challenge under the small vibration always. Herein, we conducted a feasibility study on a multi-mode shape estimating approach utilizing a single physical quantity signal. The basic concept of the proposed method is to receive multi-metric dynamic responses from two sensors and obtain mode shapes through bridge loading test with relatively large deformation. In addition, the linear transformation matrix for estimating two mode shapes is derived, and the mode shape based on the gyro sensor data is obtained by acceleration response using ambient vibration. Because the structure's behavior with respect to translational and rotational mode can be confirmed, the proposed method can obtain the total response of the structure considering boundary conditions. To verify the feasibility of the proposed method, we pre-measured dynamic data acquired from five accelerometers and five gyro sensors in a lab-scale test considering bridge structures, and obtained a linear transformation matrix for estimating the multi-mode shapes. In addition, the mode shapes for two physical quantities could be extracted by using only the acceleration data. Finally, the mode shapes estimated by the proposed method were compared with the mode shapes obtained from the two sensors. This study confirmed the applicability of the multi-mode shape estimation approach for accurate damage assessment using multi-dimensional mode shapes of bridge structures, and can be used to evaluate the behavior of structures under ambient vibration.

Keywords: angular velocity; linear transformation matrix; modal analysis

1. Introduction

System identification is the process of identifying the mathematical model of a system using the input and output data of an actual physical system. Several system identification techniques have been used to estimate the physical properties of various engineering disciplines. Commonly, the input sources of the actual system are the applied forces, and the outputs are the structural dynamic responses (i.e., displacements, velocities, and accelerations). Therefore, studies have been conducted to derive the equations of motion for structural dynamics and structural health monitoring. Generally, methods for identifying the motion equation are based on data obtained through numerical analysis and forced vibration experiments.

However, for large structures, such as bridges, a forced vibration test using known input sources for system identification is almost impossible. This is because artificial shaking or impact excitation are limited when used to excite an entire structure. To solve this problem, output-only system identification approaches have been studied over the past few decades. Felber (Felber 1994) proposed a practical peak-picking method that uses a reference sensor and all the outputs. This approach identifies the natural frequencies and corresponding mode shapes by examining the peaks of the spectra, which can be calculated by converting the time-domain responses into frequency-domain responses using a discrete Fourier transform technique. An autoregressive moving average vector (ARMAV) model has also been used for output-only system identification (Bodeux and Golinval 2001). The AR part of the output sources was related to the MA part of the stationary Gaussian white noise input sources. Therefore, it is possible to extract the modal parameters from the output signal without requiring excitation measurements. The prediction error approach (Ljung 1987) can identify unknown system matrices based on the outputs. The proposed method estimates the parameters of the dynamic model based on recorded observations, rather than on statistical approaches. Generally, output-only system identification uses modal analysis approaches for observations acquired through long-term experiments. Several studies have been conducted to determine the dynamic properties of bridges. However,

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these approaches are disadvantageous because they are often inaccurate and contain nonlinearities.

Farrar and James (Farrar and James Iii 1997) proved that the cross-correlation between two responses has properties identical to the system's Markov parameter. The development of such a natural excitation technique (NExT) was a significant step for output-only system identification (Caicedo *et al*. 2004, Chiang and Lin 2010, Chang and Pakzad 2013). Specifically, once the correlation functions are calculated from the responses, a variety of output-only system identification approaches can be applied.

In this study, we propose a multiscale modal analysis approach based on a linear transformation matrix. To extract structural modal parameters, such as natural frequencies and mode shapes, an eigensystem realization algorithm (ERA) based on NExT, which is an output-only system identification approach, was considered. The basic concept of the proposed approach is to obtain another type of mode shape using one type of response measured by the accelerometers. To this end, a linear transformation matrix should be calculated from a pre-measurement test to relate the mode shapes estimated from the accelerometer responses to the other mode shapes estimated from the gyroscope responses. Next, another type of mode shape related to the sensor can be estimated without installation using the responses of the accelerometers and linear transformation matrix. Thus, once the linear transformation matrix is calculated from the pre-measurement for the two different types of sensors, the two types of mode shapes related to the sensors can be obtained simultaneously using only the dynamic responses of the accelerometers.

The final goal of this study was to apply the proposed method to a multiscale sensing-based damage assessment of a bridge-like structure capable of one-dimensional modeling (Kaloop *et al*. 2020, Liu *et al*. 2020, Ali *et al*. 2019). In a previous study (Sung *et al*. 2013), it was confirmed that multiscale sensing-based damage assessment is advantageous for multiple damage diagnoses when accelerometers and gyroscopes are used simultaneously. However, gyroscopes have a low signal-to-noise ratio compared to accelerometers (Sung *et al*. 2013). Therefore, angular velocities can be measured in the event of a large bridge deformation under strong winds. Thus, they are unsuitable for continuous structural monitoring based on gyroscopes. However, once the linear transformation matrix is established from the measured angular velocities and accelerations under the event based on the proposed method, the acceleration- and angular-velocity-based mode shapes can be simultaneously estimated using only the signals measured by the accelerometers. Therefore, multiscale-sensing-based damage diagnosis can be performed using only acceleration signals. Moreover, multiscale sensing-based continuous bridge-health monitoring is possible.

In this study, the experimental validation is performed to confirm the applicability of the proposed method under the condition that the accelerometers and gyroscopes are simultaneously installed on the one-dimensional lab-scale bridge model, which was used in the previous research (Sung *et al*. 2013).

2. Theory

2.1 System identification for dynamic responses as measured by accellerometers

The discrete time-state space model for a dynamical system can be expressed as

$$\mathbf{z}(\mathrm{k}+1)=\mathbf{A}\mathbf{z}(\mathrm{k})+\mathbf{B}\mathbf{u}(\mathrm{t}) \tag{1}$$

$$\mathbf{y}(\mathrm{k})=\mathbf{C}\mathbf{z}(\mathrm{k})+\mathbf{D}\mathbf{u}(\mathrm{t}), \tag{2}$$

where $\mathbf{A}$ is the state matrix, $\mathbf{B}$ the input influence matrix, $\mathbf{C}$ the output influence matrix, and $\mathbf{D}$ the direct transmission matrix (Juang 1994). From Eqs. (1) and (2), the Markov parameters of the system can be obtained as

$$\mathbf{Y}(\mathrm{k})=\mathbf{C}\mathbf{A}^{\mathrm{k}-1}\mathbf{B}, \tag{3}$$

where $\mathbf{Y}(\mathrm{k})$ is the system's Markov parameter.

The NExT-ERA approach begins by constructing a block Hankel matrix comprising Markov parameters, as follows

$$\mathbf{H}(\mathrm{k}-1)=\begin{bmatrix}\mathbf{Y}_{\mathrm{k}} & \mathbf{Y}_{\mathrm{k}+1} & \cdots & \mathbf{Y}_{\mathrm{k}+\beta-1}\\ \mathbf{Y}_{\mathrm{k}+1} & \mathbf{Y}_{\mathrm{k}+2} & \cdots & \mathbf{Y}_{\mathrm{k}+\beta}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{Y}_{\mathrm{k}+\alpha-1} & \mathbf{Y}_{\mathrm{k}+\alpha} & \cdots & \mathbf{Y}_{\mathrm{k}+\alpha+\beta-2}\end{bmatrix}, \tag{4}$$

where the observability matrix $\mathbf{P}_\alpha$ and the controllability matrix $\mathbf{Q}_\beta$ are given by

$$\mathbf{P}_\alpha=\begin{bmatrix}\mathbf{C}\\ \mathbf{CA}\\ \mathbf{CA}^2\\ \vdots\\ \mathbf{CA}_{\alpha-1}\end{bmatrix} \tag{5a}$$

$$\mathbf{Q}_\beta=\begin{bmatrix}\mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \cdots & \mathbf{A}^{\beta-1}\mathbf{B}\end{bmatrix} \tag{5b}$$

Observability matrix $\mathbf{P}_\alpha$ can be obtained from Eq. (4) for $k=1$ using singular value decomposition, as follows

$$\mathbf{H}(0)=\mathbf{P}_\alpha\mathbf{Q}_\beta=\mathbf{R}\Sigma\mathbf{S}^{\mathrm{T}}, \tag{6}$$

where matrixes $\mathbf{R}$ and $\mathbf{S}$ are orthogonal, and the rectangular matrix Σ is given by

$$\Sigma=\begin{bmatrix}\Sigma_n & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}, \Sigma_n=\mathrm{diag}[\sigma_1\ \sigma_2\ \cdots\ \sigma_{\mathrm{n}}] \tag{7}$$

From Eqs. (6) and (7), the observability matrix $\mathbf{P}_\alpha$ and the controllability matrix $\mathbf{Q}_\beta$ can be calculated as follows

$$\mathbf{P}_\alpha=\mathbf{R}_{\mathrm{n}}\Sigma_{\mathrm{n}}^{1/2},\quad \mathbf{Q}_\beta=\Sigma_n^{1/2}\mathbf{S}_{\mathrm{n}} \tag{8}$$

State matrix $\mathbf{A}$ can be obtained from Eqs. (4), for $k=2$, using singular value decomposition, and from Eq. (8) as follows

$$\mathbf{H}(1)=\mathbf{P}_\alpha\mathbf{A}\mathbf{Q}_\beta=\mathbf{R}_{\mathrm{n}}\Sigma_{\mathrm{n}}^{1/2}\mathbf{A}\Sigma_{\mathrm{n}}^{1/2}\mathbf{S}_{\mathrm{n}} \tag{9}$$

Finally, the state matrix $\mathbf{A}$, input influence matrix $\mathbf{B}$, and output influence matrix $\mathbf{C}$ can be expressed as

$$\mathbf{A} = \Sigma_n^{-1/2}\mathbf{R}_n^T\mathbf{H}(1)\mathbf{S}_n\Sigma_n^{-1/2} \tag{10a}$$

$$\mathbf{B} = \Sigma_n^{1/2}\mathbf{S}_n^T\mathbf{E}_r \tag{10b}$$

$$\mathbf{C} = \mathbf{E}_m^T\mathbf{R}_n\Sigma_n^{1/2}, \tag{10c}$$

where $\mathbf{E}_r = [\mathbf{I}_r\ \mathbf{0}_r\ \cdots \mathbf{0}_r]$, $\mathbf{E}_m = [\mathbf{I}_r\ \mathbf{0}_r\ \cdots \mathbf{0}_r]^T$, r denotes the number of inputs and m is the number of outputs.

2.2 System identification for dynamic responses as measured by gyroscope

The linear transformation $\mathbf{T}$ between the dynamic responses $\mathbf{V}$ as measured by another type of sensor and the dynamic responses $\mathbf{Y}$ as measured by an accelerometer exists, as defined in an earlier study (Sim *et al*. 2011).

$$\mathbf{V} = \mathbf{TY} \tag{11}$$

Thus, the block Hankel matrix for dynamic responses $\mathbf{V}$ as measured by the gyroscope, can be expressed as

$$\begin{aligned}\mathbf{H}(k-1) &= \begin{bmatrix} \mathbf{V}_k & \mathbf{V}_{k+1} & \cdots & \mathbf{V}_{k+\beta-1} \\ \mathbf{V}_{k+1} & \mathbf{V}_{k+2} & \cdots & \mathbf{V}_{k+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{k+\alpha-1} & \mathbf{V}_{k+\alpha} & \cdots & \mathbf{V}_{k+\alpha+\beta-2}\end{bmatrix} \\ &= \begin{bmatrix} \mathbf{TY}_k & \mathbf{TY}_{k+1} & \cdots & \mathbf{TY}_{k+\beta-1} \\ \mathbf{TY}_{k+1} & \mathbf{TY}_{k+2} & \cdots & \mathbf{TY}_{k+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{TY}_{k+\alpha-1} & \mathbf{TY}_{k+\alpha} & \cdots & \mathbf{TY}_{k+\alpha+\beta-2}\end{bmatrix} \\ &= [\backslash\mathbf{T}\backslash]\mathbf{P}_\alpha\mathbf{A}^{k-1}\mathbf{Q}_\beta = \mathbf{P}_{\alpha V}\mathbf{A}^{k-1}\mathbf{Q}_\beta,\end{aligned} \tag{12}$$

where $[\backslash\mathbf{T}\backslash]$ denotes the diagonal matrix of the linear transformation matrix. The observability matrix $\mathbf{P}_{\alpha V}$ for the dynamic responses $\mathbf{V}$ measured by the gyroscope is given by

$$\mathbf{P}_{\alpha V} = \begin{bmatrix} \mathbf{TC} \\ \mathbf{TCA} \\ \mathbf{TCA}^2 \\ \vdots \\ \mathbf{TCA}_{\alpha-1}\end{bmatrix} \tag{13}$$

Similar to the system identification process for the dynamic response $\mathbf{Y}$ measured by the accelerometers, the observability matrix $\mathbf{P}_{\alpha V}$ can be obtained using Eq. (12) for $k = 1$ using singular value decomposition as follows

$$\mathbf{H}(0) = \mathbf{P}_{\alpha V}\mathbf{Q}_\beta = \mathbf{TR}\Sigma\mathbf{S}_n^T \tag{14}$$

From Eq. (14), the observability matrix $\mathbf{P}_{\alpha V}$ and controllability matrix $\mathbf{Q}_\beta$ can be calculated using Eq. (15).

$$\mathbf{P}_{\alpha V} = \mathbf{TR}_n\Sigma_n^{1/2}, \quad \mathbf{Q}_\beta = \Sigma_n^{1/2}\mathbf{S}_n \tag{15}$$

State matrix $\mathbf{A}$ can be obtained from Eqs. (12) for $k = 2$ using singular value decomposition, and Eq. (15), as

$$\mathbf{H}(1) = \mathbf{P}_{\alpha V}\mathbf{A}\mathbf{Q}_\beta = \mathbf{TR}_n\Sigma_n^{1/2}\mathbf{A}\Sigma_n^{1/2}\mathbf{S}_n \tag{16}$$

Finally, the state matrix $\mathbf{A}_v$, input influence matrix $\mathbf{B}_v$, and output influence matrix $\mathbf{C}_v$ for the dynamic responses $\mathbf{V}$ measured by the alternative sensors can be expressed as

$$\mathbf{A}_v = \Sigma_n^{-1/2}\mathbf{R}_n^T\mathbf{H}(1)\mathbf{S}_n\Sigma_n^{-1/2} \tag{17a}$$

$$\mathbf{B}_v = \Sigma_n^{1/2}\mathbf{S}_n^T\mathbf{E}_r \tag{17b}$$

$$\mathbf{C}_v = \mathbf{E}_m^T\mathbf{TR}_n\Sigma_n^{1/2}, \tag{17c}$$

where $\mathbf{E}_r = [\mathbf{I}_r\ \mathbf{0}_r\ \cdots \mathbf{0}_r]$, $\mathbf{E}_m = [\mathbf{I}_r\ \mathbf{0}_r\ \cdots \mathbf{0}_r]^T$, r denotes the number of inputs, and m is the number of outputs.

By comparing Eqs. (10) and (17), the only difference between the two results is the output influence matrix. Their relationship is expressed as follows

$$\mathbf{C}_v = \mathbf{E}_m^T\mathbf{TR}_n\Sigma_n^{1/2} = \mathbf{TC} \tag{18}$$

Finally, the linear transformation matrix T can be calculated as

$$\mathbf{T} = \mathbf{C}_v\mathbf{C}^+, \tag{19}$$

where superscript + indicates the Moore-Penrose pseudo inverse (Moore 1920).

2.3 Linear transformation matrix-based mode shape transformation

The linear transformation matrix-based mode shape transformation can be performed using the state matrix $\mathbf{A}$, output influence matrix $\mathbf{C}$, and linear transformation matrix $\mathbf{T}$ based on Eq. (18), as

$$\mathbf{A} = \mathbf{\Psi}[\mu_i]\mathbf{\Psi}^{-1} \tag{20}$$

$$f_i = \frac{|\ln(\mu_i)/\Delta T|}{2\pi} \tag{21}$$

$$\mathbf{\Phi}_v = \mathbf{C}_v\mathbf{\Psi} = \mathbf{TC\Psi} = \mathbf{T\Phi}, \tag{22}$$

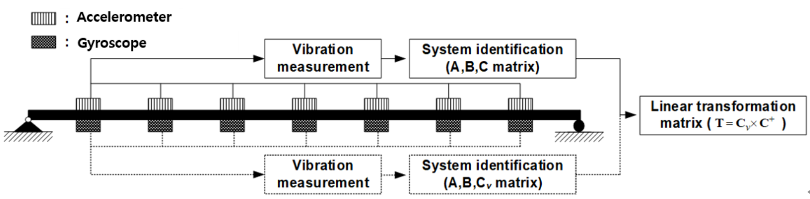

Fig. 1 Pre-measurement for calculating the linear transformation matrix

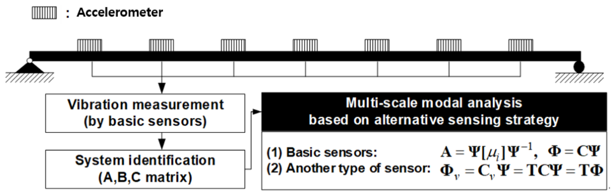

Fig. 2 Multi-scale modal analysis based on an alternative sensing strategy using only the basic sensors

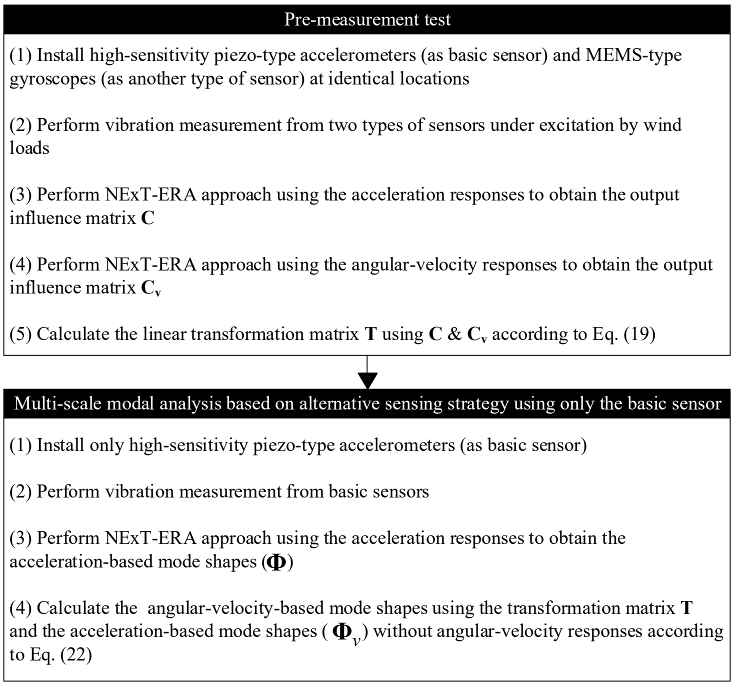

Fig. 3 Multi-scale modal analysis based on an alternative sensing strategy using only the basic sensors

where f_i is the i-th natural frequency, $\mathbf{\Phi}$ is the mode shape of the dynamic sensor, and $\mathbf{\Phi}_v$ is v the mode shape of another type of dynamic sensor.

2.4 Alternative sensing strategy for a multi-scale modal analysis

Once the linear transformation matrix **T** is calculated from the premeasurement test using another type of dynamic sensor, as shown in Fig. 1, the mode shapes related to the accelerometers and gyroscope can be identified simultaneously using the dynamic responses measured by the accelerometers, as shown in Fig. 2. Therefore, an alternative sensing strategy would be useful when the vibration intensity is relatively weak. Because an accelerometer has better sensitivity than a gyroscope, it can measure relatively weak vibration signals that cannot be measured with a gyroscope. Under ambient vibration with a weak vibration intensity, the proposed method has the advantage of being able to obtain the angular velocity signal-based mode shape using only the acceleration signal.

In previous studies (Park *et al*. 2016, Qu *et al*. 2017, 2019, Yang *et al*. 2023), only one type of mode shape corresponding to the response measured from one type of sensor was considered, whereas the proposed approach can simultaneously extract two types of mode shapes based on the response measured from an accelerometer using a linear transformation matrix. The procedure for performing such multi-scale modal analysis based on the proposed approach is as follows.

(a) Perform a pre-measurement test with two types of sensors (accelerometer and gyroscope).
(b) Calculate the linear transformation matrix based on Eq. (19), where the $\mathbf{C}_v$ can be calculated based on NExT-ERA method using the responses measured from the accelerometer.
(c) Newly measured responses using only accelerometer.
(d) Simultaneously extract two types of mode shapes corresponding to the accelerometer and gyroscope based on the responses measured from the accelerometer using the linear transformation matrix (Eq. (22)).

3. Experimental validation

Experimental validation was performed according to the flowchart depicted in Fig. 3. This study aims to theoretically verify the proposed approach and confirm its feasibility. Therefore, we considered an idealized structure, which is dominant in flexural responses, rather than complex structures.

3.1 Pre-measurement for calculating the linear transformation matrix

To confirm a feasibility of the proposed approach, a pre-measurement test was conducted before the alternative sensing test. In the experiment, high-sensitivity piezo-type accelerometers (model name: PCB393B12) and MEMS-type gyroscopes (model name: LPY503AL), which have a relatively high noise level and low sensitivity compared to the accelerometers were utilized as the accelerometer and gyroscope, respectively. The model properties of the test structure are shown in Table 1, and five accelerometers and gyroscopes were established on the structure, as shown in Fig. 4. Ambient vibration tests were conducted under excitation by wind loads. The vibration measurements were performed for 100 min at a sampling frequency of 100 Hz. Figs. 5 and 6 depict the time-domain and frequency-domain responses as measured from the accelerometers and gyroscopes, respectively. Output-only system identification was performed using the NExT-ERA approach to obtain the observability matrix to extract the output influence matrix $\mathbf{C}$. Fig. 7 shows the estimated mode shapes extracted from the measured accelerations. Finally, the linear transformation matrix $\mathbf{T}$ was calculated using the output influence matrix $\mathbf{C}$ obtained from the acceleration responses and the output influence matrix $\mathbf{C}_v$ obtained from the angular-velocity responses in accordance with Eq. (19). To validate the estimated linear transformation matrix, new mode shapes were initially estimated using the linear transformation matrix and the acceleration-based mode shapes. Next, the new mode shapes converted with the linear transformation matrix, and the mode shapes extracted from the measured angular velocities were compared to each other, as shown in Fig. 8. The two mode shapes mostly coincided with each other.

Table 1 Structural model properties

Parameter	Value
Damping ratio	0.01
Mass density (ρ)	7850 kg m^{-3}
Poisson's ratio (v)	0.28
Elasticity modulus (E)	200 GPa
Length (L)	2.04 m
Width (w)	100 mm
Thickness (t)	10 mm

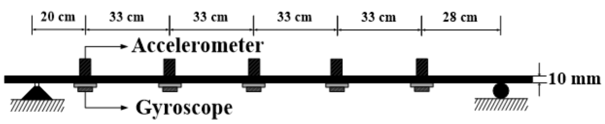

Fig. 4 Test model and sensor installation for the pre-measurement test

3.2 Multi-scale modal analysis based on an alternative sensing strategy

An alternative sensing test is conducted to confirm the feasibility of the proposed approach. As mentioned above, the proposed approach can perform multiscale modal analysis using accelerometers only once the linear transformation matrix $\mathbf{T}$ is calculated from the pre-measurement test. Thus, because the linear transformation matrix was estimated in Section 3.1, a modal analysis

Fig. 5 Acceleration responses measured by five sensors: (a) time domain signal; (b) frequency domain signal

Fig. 6 Angular velocity responses measured by five sensors: (a) time domain signal; (b) frequency domain signal

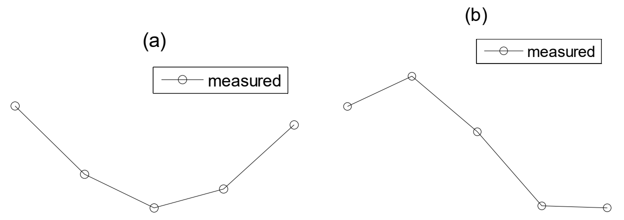

Fig. 7 Acceleration-based mode shapes: (a) first mode shape; (b) second mode shape

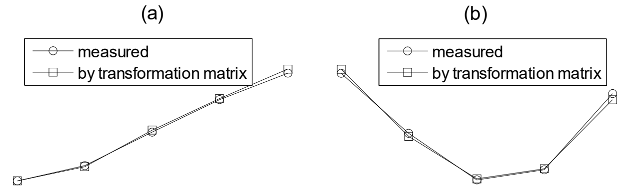

Fig. 8 Angular-velocity-based mode shapes and new angular-velocity-based mode shapes converted by the linear transformation matrix using acceleration-based mode shapes: (a) first mode shape; (b) second mode shape

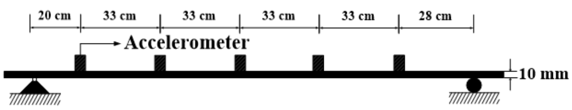

Fig. 9 Accelerometer installation for the multi-scale modal analysis based on an alternative sensing strategy

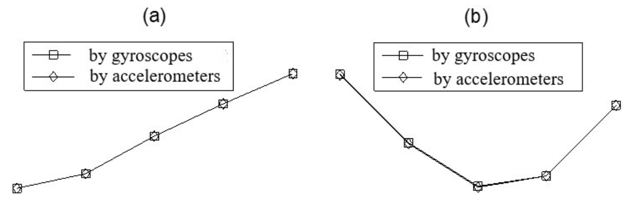

Fig. 10 Mode shapes estimated from the pre-measurement test (by gyroscopes) and the alternative sensing strategy (by accelerometers): (a) first mode; (b) second mode

related to the acceleration and angular velocity data could be simultaneously performed without the actual installation of the gyroscopes. To this end, the gyroscopes were removed from the test structure, and the accelerations were measured under excitation by wind loads, as shown in Fig. 9. Finally, the angular velocity-based mode shapes were calculated using the acceleration-based mode shapes and a linear transformation matrix. Fig. 10 shows a comparison of the angular-velocity-based mode shapes estimated from the pre-measurement test and the mode shapes transformed from the acceleration-based mode shapes based on the transformation matrix. The modal assurance criterion (MAC) values were 0.9999 and 0.9998 for the first and second modes, respectively. This indicates that the two mode shapes mostly coincide.

4. Conclusions

In this study, we propose a novel multiscale modal analysis approach based on a linear transformation matrix. The final goal of this study is to apply the proposed method to the multi-scale sensing-based damage assessment of a bridge-like structure capable of one-dimensional modeling. The proposed approach can estimate two different types of mode shapes based on a linear transformation matrix that relates the mode shapes estimated from the responses of the accelerometers to other mode shapes from the alternatives. The crucial advantages of the proposed approach are as follows: Assume that an accelerometer is relatively sensitive and can easily measure responses, even under small external forces. Next, assume that the gyroscope is relatively insensitive and can measure responses only when a large external force is applied. The responses were measured using an accelerometer and gyroscope under an artificially applied strong external force. In this study, we refer to this as a pre-measurement test. The measured responses A (accelerometer) and B (gyroscope) were used to calculate the linear transformation matrix. Once the linear transformation matrix is calculated from the premeasurement test, the two types of mode shapes related to the accelerometer and gyroscope can be simultaneously estimated from the vibration data measured from the accelerometer only.

To confirm the feasibility of the proposed approach, pre-measurement and alternative sensing tests were sequentially performed using accelerometers and gyroscopes. First, the linear transformation matrix is calculated using a pre-measurement test. Next, an alternative sensing test under wind load excitation was performed using accelerometers without gyroscopes. Finally, alternative sensing tests were sequentially performed using accelerometers as accelerometers and gyroscopes as gyroscopes. The MAC values were 0.9999 and 0.9998 for the first and second modes, respectively, indicating that the two mode shapes coincided well with each other.

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