Smart Structures and Systems, *Vol. 31, No. 2 (2023) 199-212*
https://doi.org/10.12989/sss.2023.31.2.199

Comparing type-1, interval and general type-2 fuzzy approach for dealing with uncertainties in active control

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(*Received June 16, 2022, Revised September 22, 2022, Accepted September 29, 2022*)

Abstract. Nowadays fuzzy logic in control applications is a well-recognized alternative, and this is thanks to its inherent advantages. Generalized type-2 fuzzy sets allow for a third dimension to capture higher order uncertainty and therefore offer a very powerful model for uncertainty handling in real world applications. With the recent advances that allowed the performance of general type-2 fuzzy logic controllers to increase, it is now expected to see the widespread of type-2 fuzzy logic controllers to many challenging applications in particular in problems of structural control, that is the case study in this paper. It should be highlighted that this is the first application of general type-2 fuzzy approach in civil structures. In the following, general type-2 fuzzy logic controller (GT2FLC) will be used for active control of a 9-story nonlinear benchmark building. The design of type-1 and interval type-2 fuzzy logic controllers is also considered for the purpose of comparison with the GT2FLC. The performance of the controller is validated through the computer simulation on MATLAB. It is demonstrated that extra design degrees of freedom achieved by GT2FLC, allow a greater potential to better model and handle the uncertainties involved in the nature of earthquakes and control systems. GT2FLC outperforms successfully a control system that uses T1 and IT2 FLCs.

Keywords: active control; general type-2 fuzzy logic controller; nonlinear benchmark structures; seismic vibration

1. Introduction

Reducing structural responses against natural hazards such as earthquake and strong winds, to enhance safety and providing conditions for serviceability and maintenance is one of the goals of researchers. Control tools are very effective in achieving these goals. Structural control systems are classified into several categories including passive, active, semi-active, and hybrid systems. In addition to classical methods, intelligent methods are also used to control the structures. Some of these algorithms, such as optimal control, pole positioning, H2, H∞ and etc., are based on mathematical methods, and some others such as fuzzy and neural algorithms, are intelligent algorithms (Mohammadi and Haghighipour 2017, Braz-Cesar and Barros 2018).

Fuzzy logic, introduced by Zadeh (1965), the father of fuzzy logic, enables the use of linguistic directions as a basis for control. The main advantages of using fuzzy control can be its robustness against the uncertainties and errors in the various parts of the control system such as data, loads, structure model, measurements and etc. Another important feature of this method is the ability to use it in nonlinear systems. Due to the nature of nonlinear behavior of structures, this method can be useful for structural control.

One of the main constraints of type-1 fuzzy logic systems (T1FLS) is their inability to consider uncertainty in fuzzy rules, to overcome this deficiency, Zadeh (1975) introduced more general kinds of fuzzy sets which their membership function grades are themselves fuzzy. The two most widely studied of these are interval-valued fuzzy sets and general type-2 fuzzy sets (Wu 2012, Tan and Chua 2007, Castillo *et al*. 2016, Hagras 2007). Because of the lack of basic calculation methods for type-2 fuzzy sets (T2FS) in their early days, type-2 fuzzy logic controllers (T2FLC) did not emerge until fairly recently. Things have changed a lot during the past decade, so that T2FlC (which is still an emerging field) now has the attention of the fuzzy systems community. Gradually, research on fuzzy type-2 systems was developed; as Mendel and John (2002) developed the basic concepts of type-2 fuzzy sets. Liang and Mendel (2000) proposed an effective computational method for calculating operators of type-2 fuzzy sets using the concept of upper and lower membership functions. Karnik *et al*. (Karnik *et al*. 1999 and Karnik and Mendel 2001) provided the entire foundation and framework for singleton type-2 fuzzy systems, including type-reduction and two very widely used algorithms for computing the type-reduced set (the KM and EKM algorithms). The comparison between T1 and T2 FLCs is necessary to show T2 FLC advantages. Ontiveros-Robles *et al*. (2018) presented a comparison in the robustness of IT2 and GT2 fuzzy logic controllers. They concluded, based on the realized experiments, the GT2 FLCs demonstrate better performance than IT2 FLCs in noisy environments. Mittal *et al*. (2020) presented a concise review on T2FLS and its most successful applications in the area of pattern

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http://www.techno-press.org/?journal=sss&subpage=7
ISSN: 1738-1584 (Print), 1738-1991 (Online)

Table 1 Fuzzy abbreviations and their definitions

Abbreviation	Definition
T1	Type 1
T2	Type 2
IT2	Interval type 2
GT2	General type 2
FS	Fuzzy set
FLS	Fuzzy logic system
FLC	Fuzzy logic controller
MF	Membership function
FOU	Footprint of uncertainty
COS	Center of sets (type reduction)
KM	Karnik-Mendel (algorithms)
LMF	Lower membership function
UMF	Upper membership function

recognition, classification and control along with its comparison with T1FS.

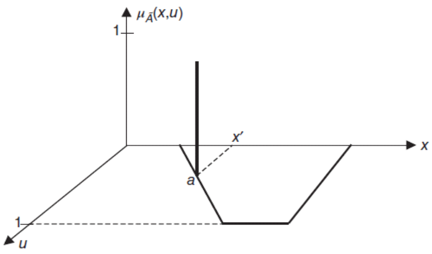
However, despite the ability of IT2FLC method to deal with issues of high uncertainty, research on the use of these systems in the field of control of structures has been very limited. Golnargesi *et al*. (2018) studied the seismic control of structures using ATMD with an interval type-2 fuzzy controller. In their research, they showed that, an interval type-2 fuzzy controller significantly reduces the structural response compared with type-1 fuzzy controller. Bathaei *et al*. (2018) investigated the performance of a semi-active tuned mass damper (TMD) with adaptive magneto-rheological (MR) damper using type-1 and type-2 fuzzy controllers for seismic vibration mitigation of an 11-degree of freedom building model. Paul *et al*. (2018) applied a bidirectional active control with type-2 fuzzy PD and PID system to compensate the unknown uncertainties of a two-story building. In Azadvar *et al*. (2018) research, an interval type-2 fuzzy system has been used to reduce damage in a structure equipped with MR dampers. Ramezani *et al*. (2019) used type-1 and type-2 fuzzy logic controllers to determine the damping ratio of TMD. It was concluded that T2 FLC has robust performance against uncertainties compared to other controlling systems. Therefore, it can be used in actual applications more confidently. Al-Ghazali and Shariatmadar (2021) used IT2FLC for higher performance of hybrid active control used in adjacent buildings interconnected by viscous dampers considering soil-structure interaction. Hosseini Lavassani and Shangapour (2022) used efficiently an interval type-2 fuzzy controller for hybrid control of a high-rise building including Soil–Structure interaction under near-Field and far-field ground motions. They concluded that the IT2FLC is very effective in reducing structural responses,

The growth of interest in type-2 fuzzy logic has not fully manifested itself in real-world applications using general type-2 fuzzy sets. The emphasis has been on interval type-2 fuzzy sets, thus not taking advantage of the more general representation, recently, there has been a growing interest in using general T2 FSs, because they have more design degrees of freedom than IT2 FSs and, therefore, have the potential to outperform a system that uses IT2 FSs. The complex nature of the uncertainty encountered in the real-world problems indicates that generalized type-2 fuzzy logic is needed in real-world devices and applications, in particular in problems of structural control that is the case study in this paper. To the best of the author's knowledge, at this moment, there is not any researches on general T2FLC applied to control of structures under seismic vibrations. This paper describes an α-plane representation of GT2 FS, and how it can be used to solve structural control problems. The main purpose of this paper is to make GT2 FLSs more accessible to civil engineers and to expedite the use of general type-2 fuzzy sets (GT2 FSs) for structural control. There has to be many reported successful applications of GT2FLC in order to fully manifest itself in real-world applications. This research can add another example of success to the history of GT2 fuzzy application to help its way of progress and evolution.

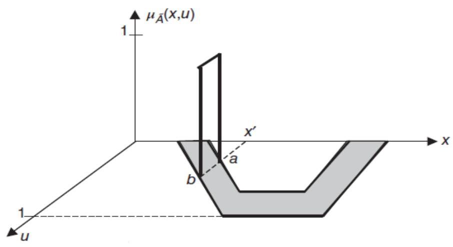
Yielding and nonlinear behavior is very likely to occur when buildings are subjected to severe wind or earthquake. The linear models do not include disturbances caused by large displacement or material nonlinearity and damage (Bozorgvar and Zahrai 2019). Therefore, the main objective of this work is to apply an intelligent controller; a general type-2 fuzzy logic controller, to the 9-story benchmark building defined by Ohtori *et al*. (2004) to handle the nonlinearity and uncertainties of the system. The effectiveness of controllers has been verified via numerical simulations based on MATLAB/Simulink programming software, which showed the superior performance of GT2 FLC over T1 and IT2 FLC in preventing damages of structural members and more effective in reducing the building responses. Instead of using ready toolboxes, codes have been written originally for T1, IT2 and GT2, which are very appropriate to reduce time cost and make fuzzy approach more suitable for civil engineering purposes.

2. General type-2 fuzzy logic systems

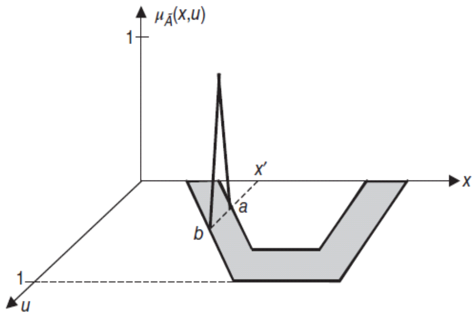
During the past 20 years great progress has been made in transitioning from type-1 (T1) fuzzy logic systems (FLSs) to interval type-2 (IT2) FLSs and, most recently, to general T2 FLSs (GT2 FLSs). Using IT2 FSs in an FLS has the potential to provide better (and certainly no worse) performance for a FLS than using T1 FSs, and using GT2 FSs in an FLS has the potential to provide better (and certainly no worse) performance for a FLS than using IT2 FSs, which is why there has been so much interest in IT2 FLSs and GT2 FLSs. Recall that an IT2 FS can be thought of as a blurred T1 FS that lets one account for membership function (MF) uncertainties; however, an IT2 FS weights all such uncertainties uniformly and can therefore be thought of only as a first-order fuzzy set uncertainty model. On the other hand, a GT2 FS also lets us account for MF uncertainties, but it weights all such uncertainties nonuniformly and can therefore be thought of as a second-order fuzzy set uncertainty model as depicted in Fig. 1. Both IT2 and GT2 FSs are parametric models. An IT2 FS is described by more parameters than a T1 FS, and a GT2FS is



(a) T1 FS



(b) IT2 FS



(c) GT2 FS

Fig. 1 Comparing the secondary MFs at x = x' for three types of FSs

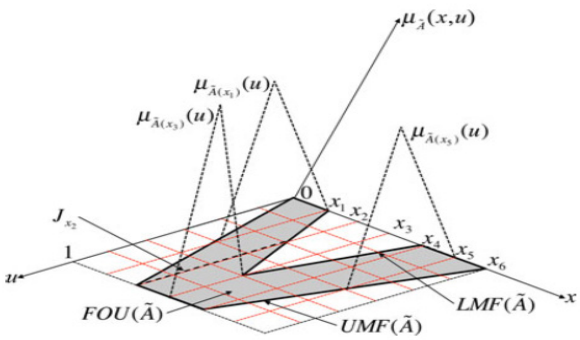


Fig. 2 Various elements of a GT2 FS

described by even more parameters. Each FS parameter in an FLS can be thought of as one of its design degrees of freedom; hence, an IT2 FLS has more design degrees of freedom than a T1 FLS, and a GT2 FLS has even more design degrees of freedom. It is the increase in the number of design degrees of freedom that offers the possibility of an IT2 FLS or a GT2 FLS outperforming a T1 FLS (John and Coupland 2007, Mendel 2014).

All computations that are needed for a GT2 FLS can be derived and performed again using only IT2 FS mathematics; this means that someone who is already familiar with IT2 FSs does not have to invest a lot of time to learn about GT2 FSs. In this paper, it has been assumed that the reader is familiar with the basics of fuzzy logic and type-1 and interval type-2 fuzzy logic systems, so that here the focus will be entirely on GT2FLS.

The MF of a GT2 FS is 3D as demonstrated in Fig. 2 with the x-axis called the primary variable, the y-axis called the secondary variable that is denoted u, and the z-axis called the MF value (or secondary MF value or secondary grade) that is denoted $\mu_{\tilde{A}}(x,u)$. A type-2 fuzzy set $\tilde{A}$ can be defined by its type-2 membership function $\mu_{\tilde{A}}(x,u)$ as

$$\tilde{A} = \int_{x\in X} \int_{u\in J_x} \frac{\mu_{\tilde{A}}(x,u)}{(x,u)} \qquad (1)$$

which is often referred to as the point-valued representation of a T2 FS and is useful as a starting point (Mendel 2017). where $x \in X$,$u \in J_x \subseteq [0,1]$and X represents the universe of the primary variable x of $\tilde{A}$. The secondary MF of $\tilde{A}$ is also called a vertical slice

$$\mu_{\tilde{A}}(x = x', u) \equiv \mu_{\tilde{A}}(x') = \int_{u\in J_{x'}} \frac{f_{x'}(u)}{u} \qquad (2)$$

where $0 \le f_{x'}(u) \le 1$. In the case of IT2FLS the secondary membership grades all equal 1, that is to say, for any $x = x'$, $f_{x'}(u) \equiv 1$. Note that the secondary MF is itself a T1 FS.

The footprint of uncertainty (FOU) is uncertainty in the primary membership grades of a type-2 MF, which consists of a bounded region; the upper MF (UMF) is a subset that has the maximum membership grade of the FOU, and the lower MF (LMF) is a subset that has the minimum membership grade of the FOU. The two-dimensional $\mu_{\tilde{A}}(x,u)$ is referred to as the footprint of uncertainty (FOU) of $\tilde{A}$

$$FOU(\tilde{A}) = \bigcup_{x\in X} J_x = \{(x,u) | u \in J_x \subseteq [0,1]\} \qquad (3)$$

where J_x is the primary membership of $\tilde{A}$; here the lower MF (LMF) $\underline{\mu}_{\tilde{A}}(x)$ and the upper MF (UMF) $\bar{\mu}_{\tilde{A}}(x)$

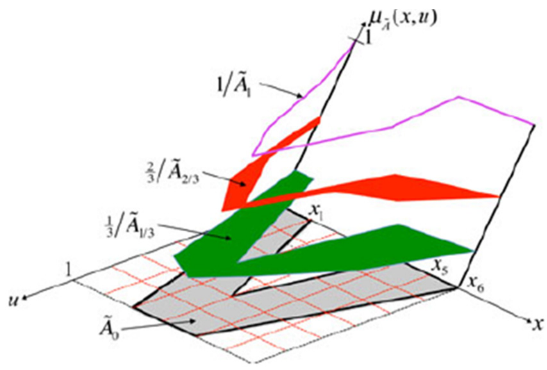


Fig. 3 GT2 FS horizontal-slice representation

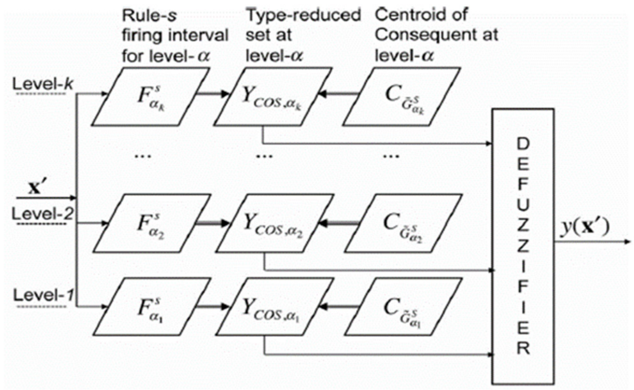


Fig. 4 GT2 Mamdani FLS computations for COS TR

comprise the FOU, where

$$\underline{\mu}_{\tilde{A}}(x) = LMF(\tilde{A}) = inf\{u|u \in [0,1], \mu_{\tilde{A}}(x,u) \geq 0\} \quad (4)$$

and

$$\bar{\mu}_{\tilde{A}}(x) = UMF(\tilde{A}) = sup\{u|u \in [0,1], \mu_{\tilde{A}}(x,u) \geq 0\} \quad (5)$$

There are four ways to mathematically represent a GT2 FS: (a) collection of points; (b) union of vertical slices, where each vertical slice is a T1 FS (a secondary MF); (c) union of wavy slices, where each wavy slice is an embedded T2 FS; and, (d) fuzzy union of horizontal slices as depicted in Fig. 3, where each horizontal slice resembles an IT2 FS raised to level α (Mendel 2014).

Liu (2008), introduced a horizontal slice representation for a T2 FS, which is useful not only for theoretical studies but for computation as well. Because a horizontal slice is analogous to the α-cut of a T1 FS, it is called an α-plane representation for T2 FS. The α-plane representation for $\tilde{A}$ is

$$\tilde{A} = \bigcup_{\alpha \epsilon [0,1]} FOU(\tilde{A}_\alpha) \quad (6)$$

The α-plane representation is very interesting because each FOU $(\tilde{A}_\alpha)$ can be associated with an IT2 FS of level α, with MF being equal to FOU $(\tilde{A}_\alpha)$. This suggests that operations that involve T2 FSs can be performed using readily available operations for IT2 FSs. In α-plane representation for a T2 FS, a GT2 fuzzy system is an aggregation of k horizontal-slice fuzzy systems, as in Fig. 4, where aggregation occurs by means of defuzzification. The idea of aggregating horizontal-slice fuzzy systems was proposed originally in Wagner and Hagras (2010), and was expounded upon in Mendel (2014).

This research will be based upon α-plane representation of GT2 Mamdani fuzzy system that uses center of sets (COS) type reduction and average of end points defuzzification with triangular secondary MFs. It should be clear that COS type-reduction and average of end points defuzzification requires many fewer computations than other methods; hence, will be greatly favored for a GT2 Mamdani fuzzy system (Mendel *et al*. 2009). Proposed GT2 FLS calculation steps are summarized as follows:

(1) The optimal number of α-planes (Liu 2008) is chosen to be 5 (kmax = 5, alpha = 0.2, 0.4, 0.6, 0.8 and 1). All of the following computations are performed for x = x', l = 1…, M and k = 1…, kmax:

(2) For α = α_k α-cuts of triangular secondary MF should be computed using the following equations:

$$\tilde{A}_\alpha(x) = [a_\alpha(x), b_\alpha(x)] \quad (7)$$

$$a_\alpha(x) = \underline{\mu}_{\tilde{A}}(x) + w[\bar{\mu}_{\tilde{A}}(x) - \underline{\mu}_{\tilde{A}}(x)]\alpha \quad (8)$$

$$b_\alpha(x) = \bar{\mu}_{\tilde{A}}(x) - (1-w)\,[\bar{\mu}_{\tilde{A}}(x) - \underline{\mu}_{\tilde{A}}(x)]\alpha \quad (9)$$

$\tilde{A}_\alpha(x)$ would be FOU raised to level α_k. w is the apex indicator which value can range between [0,1], the choice of each value changes the secondary MF shape as depicted in Fig. 5.

(3) Because COS type-reduction is used, one does not need the entire 3D MFs of the consequents, instead, only the centroids of the consequents at α = α_k needs to specified and fixed. The approach is:

- It is assumed that the secondary MFs are all symmetric triangles.
- m^l is the apex of the triangle. m^l is chose arbitrarily for each of the consequents, and fixed.
- m_1^l is the location of the left vertex, and m_2^l is the location of the right vertex at α = 0, once computed and fixed.
- The approximate centroid of each α-plane is computed using the following formulas:

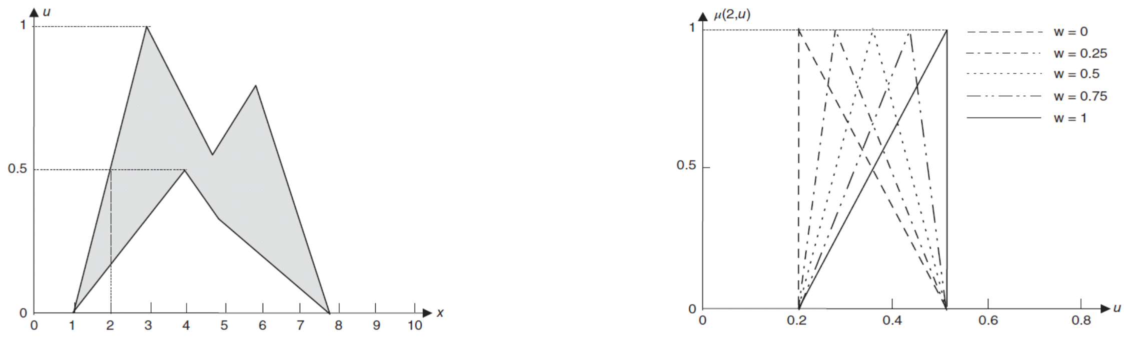
$$C_{l\alpha} \approx m_1^l + (m^l - m_1^l)\ \alpha_k \quad (10)$$

$$C_{r\alpha} \approx m_2^l - (m_2^l - m^l)\ \alpha_k \quad (11)$$

Only the left and right end-points achieved ($C_{l\alpha}$ and $C_{r\alpha}$) are needed for the next set of computations.

(4) Firing intervals for all rules, is computed using the product t-norm at each α-level.

(5) With the firing intervals and centroids of consequents computed for each α-level, using the KM algorithms, the type reduced set will be computed for each α-level $((y_{l,\alpha_k}(x'), y_{r,\alpha_k}(x'))$.



(a) Example FOU (b) Secondary MFs

Fig. 5 Example FOU and Secondary MFs at $x = 2$ for five values of w

(6) The average of end points is computed using the following formula:

$$y(x') = \frac{\sum_{k=1}^{k_{max}} \alpha_k \left[\left(y_{l,\alpha_k}(x') + y_{r,\alpha_k}(x') \right)/2 \right]}{\sum_{K=1}^{K_{max}} \alpha_k} \quad \alpha_k \qquad (12)$$

3. Numerical study

3.1 Structural model

Third-generation benchmark control problems for seismically excited nonlinear buildings are an effort to evaluate the developed control strategies in order to apply them in field applications and systematically evaluate the performance of various control strategies, especially in the case of nonlinear building structures. The 9-story benchmark building used for this study was designed for the Los Angeles region as defined by Ohtori *et al*. (2004) The 9-story benchmark structure is 45.75 m by 45.73 m in width and 37.19 m in height. The bays are 9.15 m on center, in both directions, with five bays each in the N–S and E–W directions. The lateral load resisting system of the building is comprised of steel perimeter MRFs with simple framing on the furthest south E–W frame. The interior bays of the structure contain simple framing with composite floors. The details of this structure are shown in Fig. 6. For more details refer to the problem definition paper Ohtori *et al*. (2004).

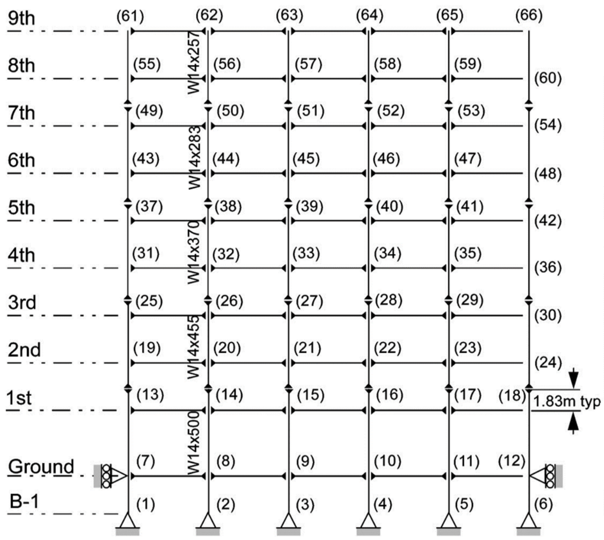


Fig. 6 Nine-story benchmark building, north–south moment-resisting frame

3.2 Nonlinear model

During large seismic events, structural members can yield, resulting in a nonlinear response behavior that may be significantly different than a linear approximation. To better represent the nonlinear behavior, a bilinear hysteresis model, as shown in Fig. 7, is used to model the plastic hinges, the points of yielding, of the 9-story building structural members. The bilinear bending properties are predefined for each structural member. These plastic hinges, which are assumed to occur at the moment resisting column–beam and column–column connections, introduce a material nonlinear behavior of these structures. For more details refer to the problem definition paper Ohtori *et al*. (2004).

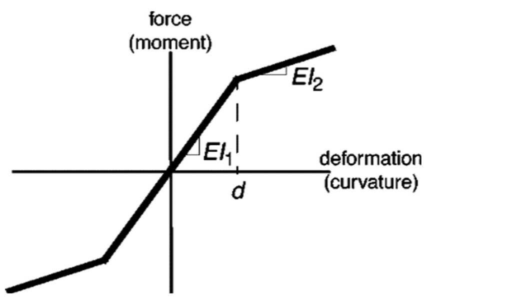
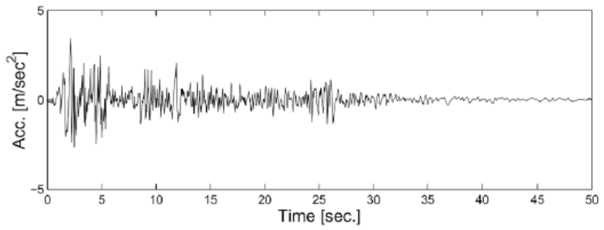


Fig. 7 Bilinear hysteresis model for structural member bending

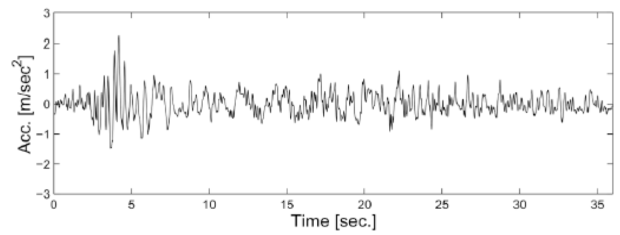
3.3 Ground motion records

To investigate the effectiveness of the control system, two far-field and two near-field historical ground motion records are selected:

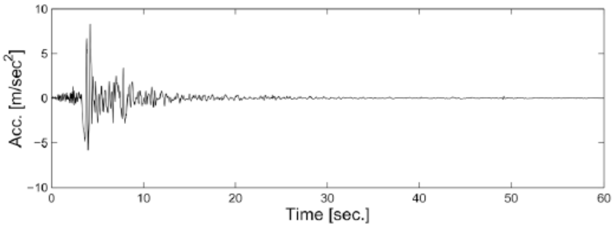
(1) El Centro: The N–S component recorded at the Imperial Valley Irrigation District substation in El Centro, California, during the Imperial Valley, California, earthquake of May 18, 1940.



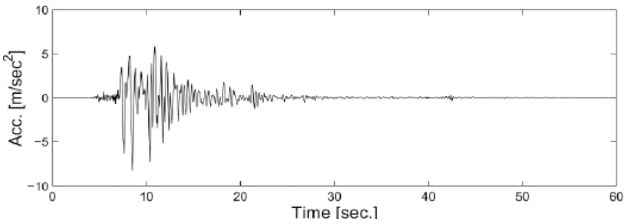
(a) El Centro



(b) Hachinohe



(c) Northridge



(d) Kobe

Fig. 8 Time histories of the near- and far-field historical earthquake records used in the benchmark study

(2) Hachinohe: The N–S component recorded at Hachinohe City during the Tokachi-oki earthquake of May 16, 1968.
(3) Northridge: The N–S component recorded at Sylmar County Hospital parking lot in Sylmar, California, during the Northridge, California earthquake of January 17, 1994.
(4) Kobe: The N–S component recorded at the Kobe Japanese Meteorological Agency station during the Hyogo-ken Nanbu earthquake of January 17, 1995.

The absolute peak acceleration of the earthquake records is 3.417, 2.250, 8.2676, and 8.1782 m/s^2, respectively. The earthquake records are shown in Fig. 8. Additionally, this benchmark study will consider various levels of each of the earthquake records including: 0.5, 1.0, and 1.5 times the magnitude of El Centro and Hachinohe; and 0.5 and 1.0 times the magnitude of Northridge and Kobe. This is a total of ten earthquake records to be considered in the evaluation of each control strategy.

3.4 Control strategy

Design of a control system includes specifications and location of the sensors, specification and location of the control devices, and a controller to determine the control action from the measured responses. Relative displacement and velocity information of different floors have been used as feedback to the FLC measured by sensors located on each floor. The size of the actuators is limited to provide a maximum control force of 1,000 kN where actuators with this capacity are readily available. Each actuator is implemented in the structure using a chevron brace configuration, in which the actuator is horizontal and rigidly attached between two consecutive levels of the building. Thus, the actuators placed on the first level will produce equal and opposite control forces on the first and second floors. Although there are multiple control devices acting on some floors, it is assumed that all control devices on a single floor experience the same inputs and respond in the same way. For the 9-story building, a total of 12 actuators have been used, three at the first, two at second, and one on each of the other floors.

To design a fuzzy system, inputs, outputs, membership functions, and fuzzy rules must be determined. These parameters can be determined by the knowledge of an expert or by conventional methods (Wu and Mendel 2019, Bernal et al. 2021, Cuevas et al. 2022). In this research, the general structure of the system, including input and output variables, the number and type of membership functions and fuzzy rules, are determined based on the knowledge and experience of authors. In some cases, system was modified iteratively, while trying to obtain optimality. The input values correspond to the relative displacement and velocity of each floor of the structure and the output values are related to the amount of force applied to the structure. The abbreviation's description of fuzzy variables is shown in Table 2 and specifications of T1FLC and T2FLC have been given in Table 3.

Table 2 Fuzzy linguistic variables

NL	**Large negative value**
N	Negative value
Z	Zero value
P	Positive value
PL	Large positive value

Table 3 Specifications of T1 and T2 FLC

	T1	T2
Fuzzification	singleton	singleton
Aggregation	Product t-norm	Product t-norm
Fuzzy Inference	Mamdani	Mamdani
Type Reducer	---------	Center of sets
Defuzzification	Center of sets	average

Table 4 Fuzzy associative memory

Relative velocity	Relative displacement		
	P	Z	N
P	NL	NL	NL
Z	P	Z	N
N	PL	PL	PL

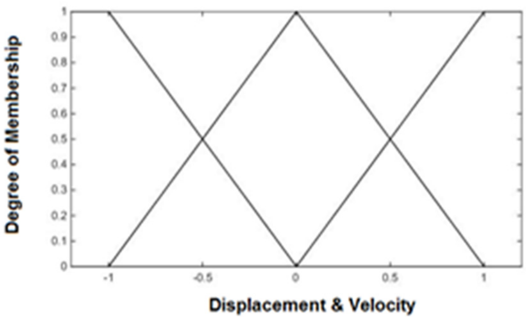


Fig. 9 T1 MFs for input variables

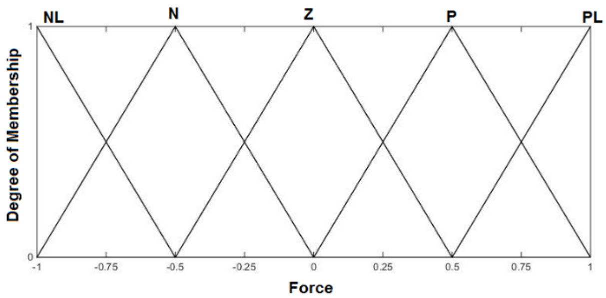


Fig. 10 T1 MFs for output variable

The fuzzy inference engine consists from set of rules which are given in Table 4. The rule-base is determined in a way that the control force pushes the structure toward the equilibrium point in each step of analysis. It can be noted that using T2 FSs to represent the inputs and outputs can lead to a smaller rule base, because MF uncertainties, represented by the FOUs of T2 FSs, let the T2 MFs cover the same range as T1 FSs, but with a smaller number of terms. This rule reduction helps decreasing computation and time cost. The membership functions chosen for the input and output variables are triangular shaped, as illustrated in Figs. 9-12. Notice that inputs and the output variables are normalized to [−1, 1] by the use of scale factors.

The control strategy was implemented in MATLAB through the author's m-file written in Simulink. No fuzzy logic toolboxes have been used either for T1 or IT2 or GT2 fuzzy operations. Time delay and sensor measurement noises has been considered to model the irrefutable uncertainties that occur in the real world. Each of the measured responses contains a (root-mean square) noise of 0.03V. The measurement noises are modeled as Gaussian rectangular pulse processes with a pulse width of 0.01 s and are modeled by Band-Limited White Noise block in Simulink. In the area of active control of structures, time delay consideration is an important parameter which must be taken into consideration for realistic numerical model. In active control, there is a time difference between measuring and applying force, and although today's actuators take action in a fraction of a second, but it is better to consider this time delay due to the issue of control stability. Time delay is applied by unit delay block in MATLAB-Simulink.

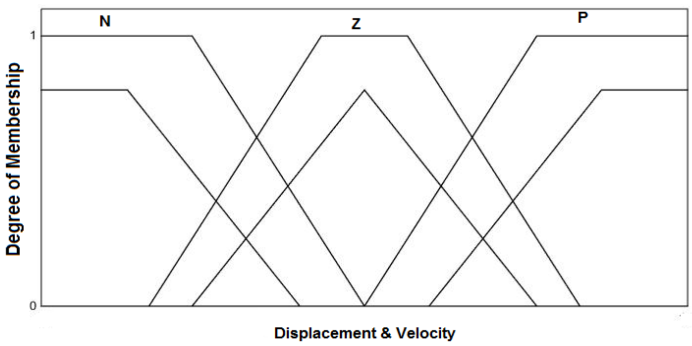


Fig. 11 T2 MFs for input variables

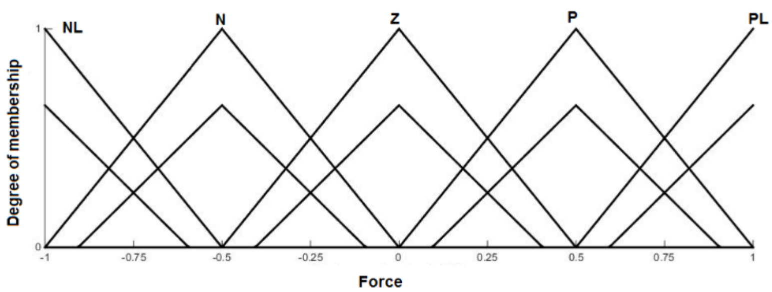


Fig. 12 T2 MFs for output variable

Table 5 Summary of evaluation criteria for the nonlinear benchmark problem

Interstory drift ratio	Level acceleration	Base shear	Normed interstory drift ratio
$J1 = max\left\{\frac{max_{t,i}\frac{\lvert d_i(t)\rvert}{h_i}}{\delta^{max}}\right\}$	$J2 = max\left\{\frac{max_{t,i}\lvert\ddot{x}_{ai}(t)\rvert}{\ddot{x}_a{}^{max}}\right\}$	$J3 = max\left\{\frac{max_{t,i}\lvert\sum_i m_i\ddot{x}_{ai}(t)\rvert}{F_b{}^{max}}\right\}$	$J4 = max\left\{\frac{max_i\frac{\lVert d_i(t)\rVert}{h_i}}{\lVert\delta^{max}\rVert}\right\}$
Normed level acceleration	Normed base shear	Ductility	Dissipated energy
$J5 = max\left\{\frac{max_i\lVert\ddot{x}_{ai}(t)\rVert}{\lVert\ddot{x}_a{}^{max}\rVert}\right\}$	$J6 = max\left\{\frac{\lVert\sum_i m_i\ddot{x}_{ai}(t)\rVert}{\lVert F_b{}^{max}\rVert}\right\}$	$J7 = max\left\{\frac{max_{t,j}\frac{\lvert\varphi_j(t)\rvert}{\varphi_{yj}}}{\varphi^{max}}\right\}$	$J8 = max\left\{\frac{max_{t,j}\frac{\int dE_j}{F_{yj}.\varphi_{yj}}}{E^{max}}\right\}$
Plastic connections	Normed ductility	Control force	Control device stroke
$J9 = max\left\{\frac{N_d^c}{N_d}\right\}$	$J10 = max\left\{\frac{max_j\frac{\lVert\varphi_j(t)\rVert}{\varphi_{yj}}}{\lVert\varphi^{max}\rVert}\right\}$	$J11 = max\left\{\frac{max_{t,i}\lvert f_i(t)\rvert}{w}\right\}$	$J12 = max\left\{\frac{max_{t,i}\lvert y_i^a(t)\rvert}{x^{max}}\right\}$
Control power	Normed control power		
$J13 = max\left\{\frac{max_t[\sum_i p_i(t)]}{\dot{x}^{max}W}\right\}$	$J14 = max\left\{\frac{\sum_i\frac{1}{t_f}\int_0^{t_f}p_i(t)}{\dot{x}^{max}W}\right\}$		

3.5 Evaluation criteria

The performance of the controller is investigated based on the evaluation criteria (J1−J14) specified for the nonlinear benchmark buildings, which are briefly presented in Table 5. The evaluation criteria are divided into three categories: Building responses, building damage and control devices. These three categories have both peak-and normed-based criteria. Small values of the evaluation criteria are generally more desirable. These criteria are calculated as a ratio of the controlled and uncontrolled responses. The first three criteria are based on peak inter story drift ratio (J1), level acceleration (J2) and base shear (J3), The next three criteria are based on normed building responses. (J7) and (J8) are related to the ductility factor and dissipated energy of the curvatures at the end of members. The ninth evaluation criterion (J9) is the ratio of the plastic connections sustained by the structure, while controlled and uncontrolled. The tenth evaluation criterion (J10) is the normed ductility factor. The third category is related to the control devices. This category assesses the required performance of the devices. Peak criterion (J11), (J12), and (J13) show control force, control device stroke, and power used for control and the fourteenth evaluation criterion (J14) is a measure of the total power required for the control of the structure. For more details refer to the problem definition paper Ohtori *et al*. (2004).

4. Numerical results and discussion

The performance of the fuzzy controller is checked according to the evaluation criteria specified (J1–J14) for 9 story nonlinear benchmark building. In order to evaluate the effectiveness of the proposed control system in managing the uncertainties governing the structure, the uncontrolled structure response and controlled structure equipped with T1 and IT2 fuzzy controller, have been investigated too. In Table 5 evaluation criteria has been reported for T1, IT2 and GT2 FLC. By reviewing Table 5, it can be concluded that with a same FLC construction (Rule base and MFs) and control features (number and place of sensors and actuators), GT2FLC showed a better performance in almost all criteria and almost never worse. To better demonstrate the evaluation criteria reduction of IT2 and GT2 controllers, ratio of two corresponding values is summarized in Table 6. The results show the ability of the GT2 fuzzy controller to reduce J1 to J6 (building response) for far field earthquakes up to 20% and for near field up to 6% in contrast with T1 fuzzy controller and in contrast with IT2 fuzzy controller for far field earthquakes up to 15% and for near field up to 6%. In comparison to the uncontrolled structure response, according to Table 6 GT2 fuzzy controller reduced J1 to J6 for far field earthquakes up to 40% and for nearfield up to 10% in most cases.

The second category of the evaluation criteria assesses the building damage. These criteria have been added because of the nonlinear character of this benchmark structure. Both ends of each element are considered in these criteria to assess yielding. Evaluation criteria *J*8 and *J*9 only have meaning for structures undergoing plastic deformations and are, therefore, undefined (should not be calculated or reported) when the uncontrolled building remains elastic, presented with dash lines in Table 6.

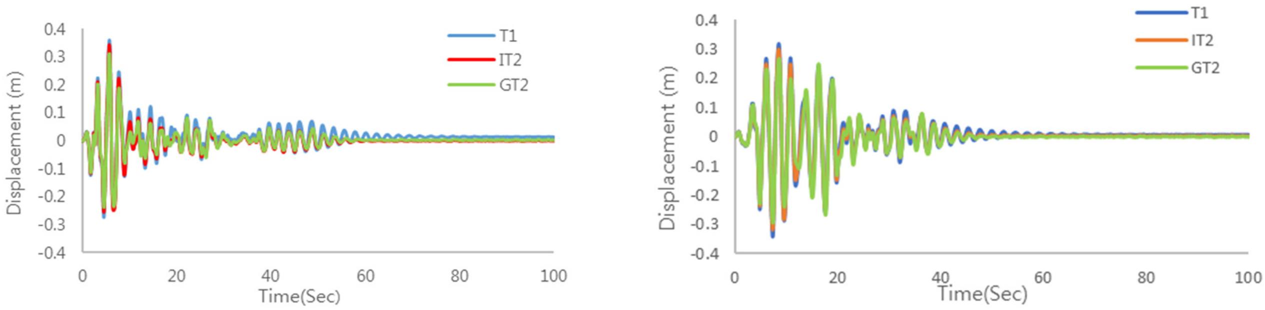
The evaluation criteria for the ductility factor (J7) is reduced up to 30% for far field and 15% for near field in contrast with T1, 20% for far field and 7% for near field in contrast with IT2 and up to 60% and 10% for far and near field respectively in contrast with uncontrolled structure. From Table 6, dissipated energy of the curvatures at the end of members (J8) and the ratio of the plastic connections sustained by the structure while controlled and uncontrolled (J9) is obtained almost zero for Hachinohe earthquake in case of type-2 controller and that is about 100%

Table 6 Earthquake evaluation criteria for 9-story benchmark structure

Earthquake		El Centro			Hachinohe			Northridge		Kobe	
Intensity		0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	0.5	1.0
J1	T1	0.872	0.917	0.937	0.841	0.888	0.886	0.914	0.985	0.842	0.961
	IT2	0.696	0.830	0.921	0.701	0.816	0.823	0.908	0.962	0.815	0.952
	GT2	0.725	0.785	0.824	0.659	0.716	0.742	0.865	0.961	0.697	0.951
J2	T1	0.797	0.858	0.877	0.866	0.858	0.899	0.923	0.983	0.993	0.945
	IT2	0.656	0.811	0.858	0.769	0.810	0.935	0.946	1.003	0.947	0.975
	GT2	0.691	0.694	0.770	0.851	0.695	0.870	0.886	0.956	0.884	0.914
J3	T1	0.836	0.982	0.983	0.847	0.937	0.981	0.977	1.040	0.919	0.985
	IT2	0.754	0.967	0.973	0.715	0.910	0.969	0.988	1.003	0.891	1.012
	GT2	0.728	0.935	0.977	0.666	0.813	0.970	0.976	0.989	0.817	0.979
J4	T1	0.702	0.882	0.997	0.922	0.981	1.002	0.822	0.807	0.524	1.085
	IT2	0.548	0.691	1.028	0.835	0.910	0.940	0.775	0.771	0.740	1.128
	GT2	0.525	0.634	0.893	0.820	0.866	0.824	0.668	0.886	0.569	1.280
J5	T1	0.684	0.742	0.747	0.862	0.917	0.930	0.824	0.838	0.799	0.884
	IT2	0.795	0.642	0.666	1.189	0.854	0.900	0.796	0.822	0.743	0.864
	GT2	1.022	0.649	0.607	1.546	0.882	0.860	0.752	0.774	0.877	0.814
J6	T1	0.697	0.810	0.846	0.915	0.961	0.971	0.807	0.829	0.769	0.897
	IT2	0.583	0.744	0.790	0.844	0.920	0.949	0.759	0.794	0.708	0.865
	GT2	0.557	0.684	0.757	0.836	0.877	0.920	0.686	0.747	0.640	0.828
J7	T1	0.823	0.871	0.942	0.844	0.807	0.913	0.919	1.010	0.791	0.954
	IT2	0.720	0.764	0.886	0.704	0.698	0.953	0.891	0.953	0.805	1.029
	GT2	0.678	0.662	0.791	0.639	0.550	0.878	0.815	0.966	0.632	0.950
J8	T1	------	0.752	0.744	------	0.456	0.851	0.670	0.890	0.543	0.828
	IT2	------	0.582	0.648	------	0.112	0.770	0.607	0.855	0.448	0.818
	GT2	------	0.310	0.531	------	0.002	0.652	0.440	0.785	0.238	0.700
J9	T1	------	0.857	1.014	------	0.579	0.817	0.905	1.053	0.923	0.987
	IT2	------	0.776	0.986	------	0.211	0.803	0.905	1.053	0.923	1.000
	GT2	------	0.653	0.942	------	0.035	0.775	0.889	1.053	0.615	0.987
*J*10	T1	0.699	0.898	1.063	0.923	1.082	1.069	1.002	0.815	0.725	0.986
	IT2	0.552	0.882	1.030	0.836	0.899	0.948	0.816	0.764	1.006	1.022
	GT2	0.517	0.603	0.965	0.820	0.708	0.885	0.720	1.110	0.752	0.947
*J*11	T1	0.007	0.011	0.011	0.006	0.010	0.010	0.011	0.011	0.010	0.011
	IT2	0.006	0.009	0.011	0.006	0.006	0.009	0.011	0.011	0.006	0.011
	GT2	0.011	0.011	0.011	0.010	0.011	0.011	0.011	0.011	0.011	0.011
*J*12	T1	0.182	0.191	0.201	0.173	0.175	0.164	0.172	0.160	0.235	0.229
	IT2	0.159	0.179	0.196	0.144	0.160	0.165	0.171	0.157	0.227	0.231
	GT2	0.148	0.163	0.181	0.131	0.141	0.155	0.163	0.158	0.194	0.243
*J*13	T1	0.004	0.006	0.008	0.003	0.005	0.007	0.008	0.010	0.008	0.014
	IT2	0.004	0.006	0.007	0.003	0.005	0.007	0.008	0.011	0.009	0.013
	GT2	0.005	0.008	0.012	0.004	0.008	0.011	0.011	0.014	0.015	0.022
$J14\times 10^{-4}$	T1	1.000	2.000	3.000	1.000	2.000	4.000	2.000	2.000	1.000	2.000
	IT2	2.000	3.000	4.000	3.000	3.000	5.000	3.000	2.000	2.000	2.000
	GT2	3.000	4.000	5.000	4.000	5.000	8.000	4.000	4.000	3.000	3.000

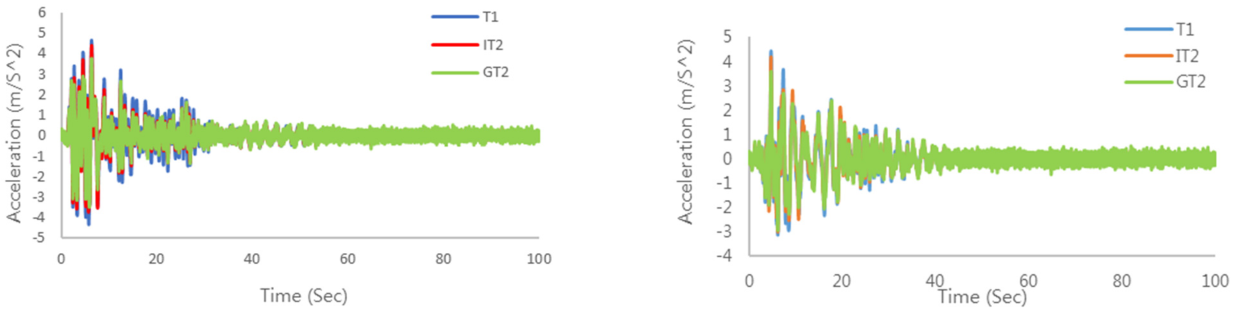
Table 7 Comparison of evaluation criteria for T1, IT2 and GT2 FLC

Improvement percentage		J1 Drift ratio	J2 Level acceleration	J3 Base shear	J7 Ductility	J8 Dissipated energy	J9 Plastic connections
Elcentro	IT2/T1	9.441	5.506	1.460	12.291	22.602	9.524
	GT2/IT2	5.493	14.422	3.357	13.251	46.798	15.789
	GT2/T1	14.416	19.134	4.768	23.914	58.823	23.810
Hachinohe	IT2/T1	8.148	5.580	2.900	13.502	75.366	63.636
	GT2/IT2	12.278	14.147	10.657	21.252	98.255	83.333
	GT2/T1	19.425	18.94	13.248	31.885	99.570	93.939
Northridge	IT2/T1	2.25	------	3.886	5.599	4.008	0
	GT2/IT2	0.043	4.64	2.2	------	8.152	0
	GT2/T1	2.29	2.65	6.001	4.376	11.83	0
Kobe	IT2/T1	0.855	------	------	------	1.18	------
	GT2/IT2	0.189	6.22	3.27	7.64	14.39	1.316
	GT2/T1	1.042	3.311	0.613	0.42	15.4	0



(a) El Centro earthquake (b) Hachinohe earthquake

Fig. 13 Displacement time history of the 9th story



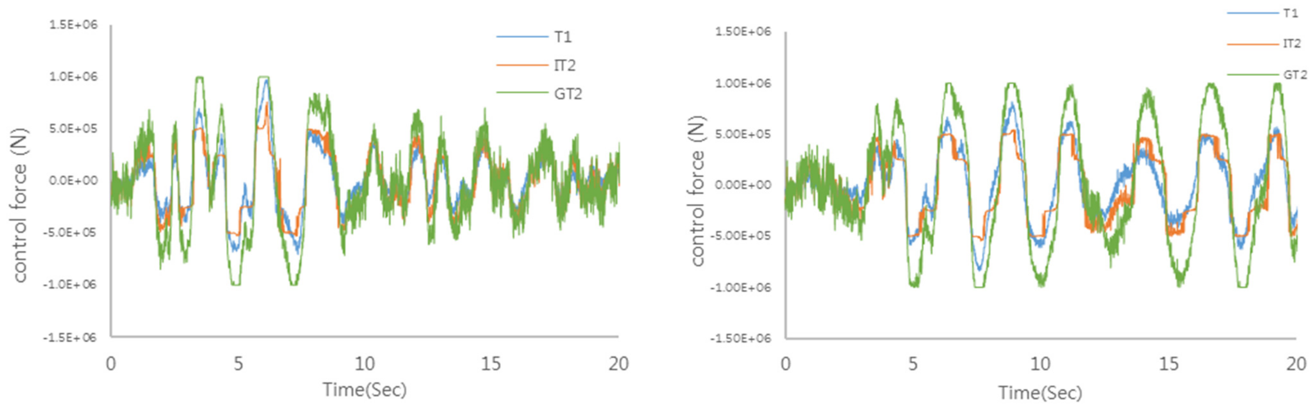
(a) El Centro earthquake (b) Hachinohe earthquake

Fig. 14 Acceleration time history of the 9th story

improvement. The tenth evaluation criterion (J10) is the normed ductility factor, improved about 40% for far field and 10% for Kobe earthquakes but comprise almost same answers for Northridge earthquake. The comparing results of the maximum curvature and energy dissipation at the ends of structural members prove that the structure with GT2FLC is very well capable of withstanding severe earthquakes.

The third category of criteria is related to the control devices. J11 indicating max control force is almost the same for T1 and T2 controllers because of maximum capacity of actuators. Control device stroke (J12) improved up to 16% for far field and had almost no changes for near field earthquakes in contrast to T1. J13 and J14 criterions indicating power required for the control increased up to 50% for GT2 FLC in contrast to T1.

To assess the effectiveness of the proposed controller, the time history of displacement and acceleration of the 9th floor are also examined. Graphs in Figs. 13-14 represent comparison of displacement and acceleration time history responses of 9th floor for T1, IT2 and GT2 control systems when subjected to far field earthquakes. It is observed



(a) El Centro earthquake (b) Hachinohe earthquake

Fig. 15 Control Force time history of the 9th story

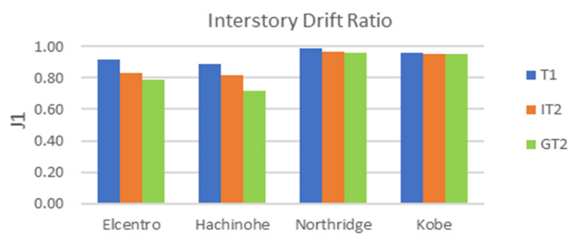


Fig. 16 Comparison of J1 for T1, IT2 and GT2 FlC for proposed earthquakes with unit intensity

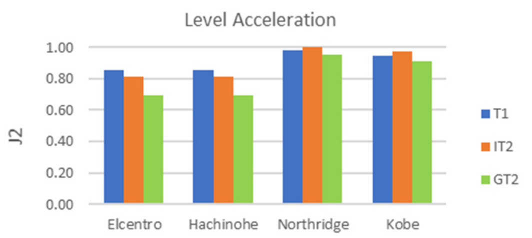


Fig. 17 Comparison of J2 for T1, IT2 and GT2 FLC for four proposed earthquakes with unit intensity

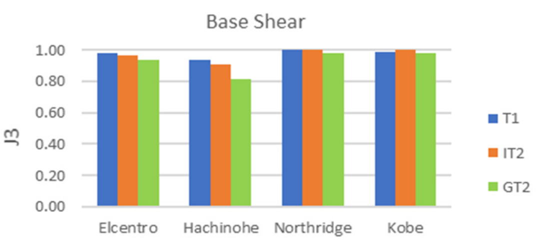


Fig. 18 Comparison of J3 for T1, IT2 and GT2 for four proposed earthquakes with unit intensity

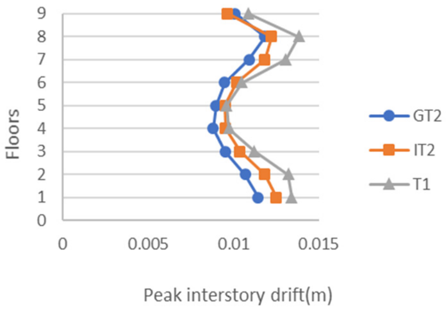
carefully that GT2FLC has been able to perfectly reduce the displacement and acceleration time history responses especially in peak condition, in comparison with two other FLCs. These figures also show the ability of the fuzzy controller to drive the system to the rest position and converge to zero which means the system is stable. Fig. 15 demonstrates the control force time history applied to the 9th story for far field earthquakes. As demonstrated, in peak condition GT2 FLC applies a higher control force (maximum capacity of actuators) but the amount of force applied by IT2 and T1 FLC are very close and in some cases even the maximum force applied by T1 FLC is more than IT2FLC.

The structural performance of a building is checked from two points of view, structural safety and residential comfort. Therefore, the peak inter-story drift (J1) and base shear force (J3) as indexes of building's damage and maximum level acceleration of stories (J2) as an index of comfort level are of importance to estimate the proposed control performance. To better demonstrate these important criteria, bar graphs in Figs. 16-18 have been illustrated. It is observed that with the use of the proposed controller, human comfort (J2) and safety level (J1, J3) of the structure are both very well guaranteed for far field earthquakes.

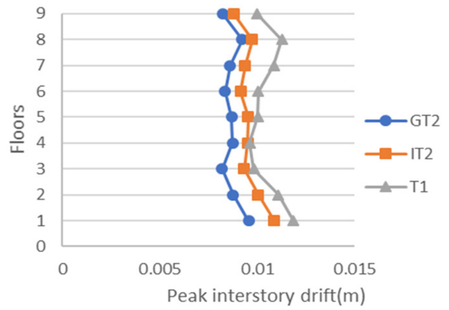
Figs. 19-20 demonstrate maximum drift and acceleration in all floors for El Centro and Hachinohe earthquakes. It can be examined that Peak inter-story drift improves for all floors and upper floors experience better improvement in peak acceleration by GT2 FLC. The important point is that by upgrading from T1 to GT2 FLC the procedure of changes in the structure behavior doesn't differ. In other words, with the same procedure better results are obtained.

Table 8 shows the computational time cost analysis results for different fuzzy controllers with same condition. It is shown that the complexity of calculations makes T2 controllers more time consuming which represents an implementation shortcoming.

In this paper, the maximum curvature at the ends of the structural members are considered as the main response of the structure for indicating the nonlinear behavior of the selected building. In control process, this response is compared with the yield curvature at the ends of the structural members in order to specify the plastic hinges and damages in the building. In Fig. 21, the ratio of the maximum curvature at the ends of the structural members to the yield curvature (φ_j/φ_{yj}) are summarized for the different control strategies. This ratio (called curvature ratio) represents the nonlinear behavior of the structure, and the control systems are supposed to reduce this ratio in order to prevent damages in structural members. For all earthquakes applying GT2 FLC results lower amounts. It

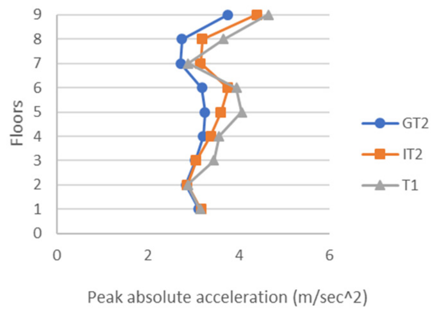


(a) El Centro earthquake

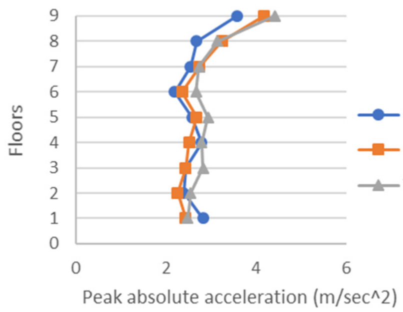


(b) Hachinohe earthquake

Fig. 19 Peak inter-story drift responses of floors



(a) El Centro earthquake



(b) Hachinohe earthquake

Fig. 20 Peak absolute acceleration responses of floors

Table 8 Computational time cost analysis results for different fuzzy controllers

	T1FLC	IT2FLC	GT2FLC
Time (sec)	127.5	184.72	301.6

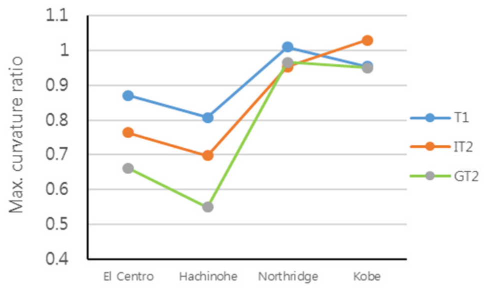


Fig. 21 Comparison of curvature for T1, IT2 and GT2 FLCs

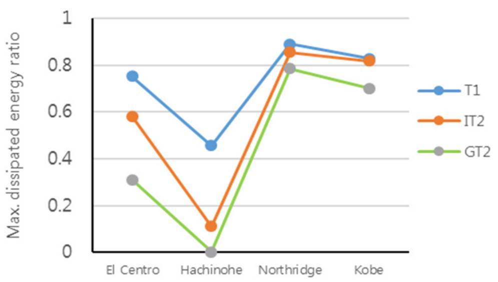


Fig. 22 Comparison of dissipated energy for T1, IT2 and GT2 FLCs

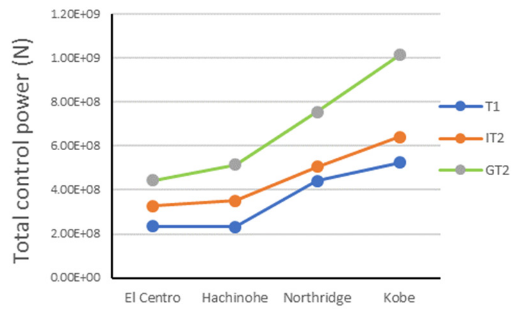


Fig. 23 Comparison of control power for T1, IT2 and GT2 FLC s

can be concluded that the structure with GT2 fuzzy controller is capable of withstanding the severe earthquakes better than the uncontrolled structure and structures equipped with T1 and IT2FLC.

Energy dissipation describes the irreversible processes occurring in a material subjected to loading cycles. Energy dissipation also represents the nonlinear behavior of the structure and the control systems are supposed to reduce this ratio in order to prevent damages in structural members. In Fig. 22, the ratio of the maximum dissipated energy (dE_j) to the yield curvature (φ_{yj}) and yield moment (F_{yj}) at the ends of the structural members are summarized for different fuzzy controllers. This amount is successfully reduced for all earthquakes by applying IT2 and GT2 FLCs and for Hachinohe reduces to about zero for GT2 controller.

In Fig. 23 the total control power is demonstrated for all fuzzy approaches. GT2 FLC takes about 2 times more power to operate. This issue can be considered a shortcoming for GT2 FLC application.

5. Conclusions

An important motivation for this research comes from the lack of applications using general type-2 fuzzy logic systems. Type-2 fuzzy logic is a growing research topic with much evidence of successful applications. This work can lead to the conclusion that the use of generalized type-2 fuzzy systems can be a good choice when there is a high level of uncertainty in the problem. The complexity of general type 2 fuzzy operations and lack of available toolboxes has always prevented engineers from using this powerful tool to deal with the complex nature of the uncertainty encountered in the real-world problems. Using the latest simplified relationships introduced by fuzzy experts and rewriting them from the perspective of structural control, a simplified scheme has been proposed that is also more efficient at much higher accuracy. And it is hoped that engineers will be inspired by this process to use general type-2 fuzzy for their future research in structural control problems.

This paper is the first study on the application of an GT2FLC in active control of structures. In order to evaluate the effectiveness of the proposed control system, a T1 and IT2FLC has been designed too. To demonstrate the performance advantages for type-2 systems over their type-1 counterparts, when uncertainties are present, four earthquake records with different intensity were chosen therefore the total of ten earthquake records were used in the simulations. High lighting the challenge faced by engineers to design effective control systems capable of reducing structural responses to unknown future earthquakes, far field and nearfield earthquakes were both considered. The efficiency of the fuzzy controller was checked through a number of evaluation criteria specified for the benchmark study. Certainly, the use of benchmark structures increases credibility and generality of the research.

By comparing the results of GT2 FLC with T1 and IT2 FLCs, it could be concluded that the GT2 FLC is very powerful in preventing damages of structural members and more effective in reducing the building responses. It should be noted that achieving these advantages was at the cost of higher computational time and more control power consumption. Finally, by considering the generality issues of the proposed controller, it can be concluded that the GT2FLC is capable of providing very encouraging results in the field of active structural control.

For future work, applying an optimization method for tuning GT2 FLC parameters is highly recommended. Using GT2 FLC with semi active control systems would be appropriate in reducing control power consumption. considering the advantages of GT2 FLC, makes the use of GT2FLC suitable for special structures such as bridges, telecommunication structures, power plants and etc.

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