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Critical buckling analyses of nonlinear FG-CNT reinforced nano-composite beam

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Abstract. This paper investigates the effect of linear and non-linear distribution of carbon nanotube volume fraction in the FG-CNTRC beams on the critical buckling by using higher-order shear deformation theories. Here, the material properties of the CNTRC beams are assumed to be graded in the thickness direction according to a new exponential power law distribution in terms of the carbon nanotube volume fractions. The single-walled carbon nanotube is aligned and distributed in the polymeric matrix with different patterns of reinforcement; the material properties of the CNTRC beams are described by using the rule of mixture. The governing equations are derived through using Hamilton's principle. The Navier solution method is used under the specified boundary conditions for simply supported CNTRC beams. The mathematical models provided in this work are numerically validated by comparison with some available results. New results of critical buckling with the non-linear distribution of CNT volume fraction in different patterns are presented and discussed in detail, and compared with the linear distribution. Several aspects of beam types, CNT volume fraction, exponent degree (n), aspect ratio, etc., are taken into this investigation. It is revealed that the influences of non-linearity distribution in the beam play an important role to improve the mechanical properties, especially in buckling behavior. The results show that the *X-Beam* configuration is the strongest among all different types of CNTRC beams in supporting the buckling loads.

Keywords: nanotube; critical buckling; shear deformation; volume fraction; FG-CNTRC; beam

1. Introduction

Since the discovery of carbon nanotubes by Iijima (1991), nanostructures are increasingly used because of their superior electronic, thermal, and mechanical properties (Dresselhaus 2001). Other studies have shown that they have good properties so that they can be used for nano-electronics, nano-tools, and nanocomposite (Al-Furjan *et al*. 2020a, Alimirzaei *et al*. 2019, Berghouti *et al*. 2019). Because of the difficulties encountered in experimental methods for predicting nanostructure responses under different loading conditions, Molecular Dynamics (MD) simulations are used. This approach represents the dynamics of atoms or molecules of materials by a discrete solution of Newton's classical equations of motion. Jin and Yuan (2003) used MD and constant force approach and reported Young's modulus (SWCNTs) of approximately 1236 ± 7 GPa. Cornwell and Wille (1997) used the MD to obtain the Young's modulus of SWCNTs. The continuum mechanics Methods are often used to study some physical problems at the nano-scale (Ru 2000). Recently, the continuum mechanics approach has been widely used successfully to study the responses of nanostructures, such as statics (Wang and Cai 2006, Reddy and Pang 2008, Eltaher *et al*. 2019), the buckling (Chemi *et al*. 2018, Wang *et al*. 2006, Amara *et al*. 2010), free vibration (Zidour *et al*. 2012, Dihaj *et al*. 2018, Arefi *et al*. 2018), wave propagation (Hu *et al*. 2008), and thermo-mechanical analysis of CNT (Rafii-Tabar 2004). Recently, various research used the continuum mechanics approach (Allam *et al*. 2020, Al-Furjan *et al*. 2020c, d, Shariati *et al*. 2020, Hussain *et al*. 2019, Abualnour *et al*. 2019, Belbachir *et al*. 2019, Sahla *et al*. 2019, Semmah *et al*. 2019). On the other hand, the theory of nonlocal elasticity, advanced by Eringen (1972, 1983), is based on the assumption that the stress at one point is a function of deformations at all points of the continuum mechanics. Lu *et al*. (2007), Tounsi *et al*. (2013), Eltaher *et al*. (2016), Wu *et al*. (2018), Karami *et al*. (2018), Mehar *et al*. (2018b) and Shahsavari *et al*. (2019) used the nonlocal elasticity of the constitutive equations to study the vibrations, bending and buckling of the CNT.

Recently, a new class of composite materials known as Functionally Gradient Materials (FGM) has drawn considerable attention from researchers (Udupa *et al*. 2014, Avcar 2015, 2019, Refrafi *et al*. 2020, Nikbakht *et al*. 2019, Boutaleb *et al*. 2019, Menasria *et al*. 2020, Matouk *et al*. 2020, Tlidji *et al*. 2019, Gafour *et al*. 2020, Zghal *et al*. 2020). Frikha *et al*. (2016) present a 2-node, 4 DOF/node beam element based on higher-order shear deformation theory for axial-flexural-shear functionally graded material. Zghal and Dammak (2020) investigates the vibrational behavior of beams made of functionally graded materials using a mixed formulation elaborated within a double field of displacements and stresses.

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Many studies have been realized in recent years on composite materials reinforced by carbon nanotube. Mellouli *et al*. (2020) used the modified first-order shear deformation theory (modified FSDT) and the meshfree Radial Point Interpolation Method (RPIM) to examine the free vibration of Functionally Graded Carbon Nanotubes-Reinforced (FG-CNTRC). Mallek *et al*. (2019a) presented a geometrically nonlinear finite shell element to predict nonlinear dynamic behavior of piezolaminated FG-CNTRC shell. Zghal *et al*. (2018) analyzed the non-linear bending of nanocomposites reinforced by graphene-nanotubes with finite shell element and membrane enhancement. Various research exists in the literature, in terms of vibration analysis (Hajlaoui *et al*. 2019a, Bourada *et al*. 2020, Mehar *et al*. 2017, 2018a, 2020a, b), buckling analysis (Boulal *et al*. 2020, Kolahchi *et al*. 2015, 2017, Bousahla *et al*. 2020, Al-Furjan *et al*. 2020b, Mehar and Panda 2019, Karami and Karami 2019, Jamali *et al*. 2019) and bending analysis (Mehar *et al*. 2018a, Hajlaoui *et al*. 2019b, Mallek *et al*. 2019b, 2020) of FG-CNT. Also, there are other studies on the mechanical behavior of laminated composites such as Katariya and Panda (2016, 2020), Katariya *et al*. (2017a, b, 2018).

Buckling does not mean failure of the structure; in general, it is the state of geometrical instability of the structure induced by various in-plane forces. Buckling is one of the main modes of failure of the structural components when subjected to in-plane compressive stresses caused by thermal/mechanical loads. As a result, the analysis of vibration and buckling of laminated structures made of composite and/or hybrid materials becomes significant (Panda and Katariya 2015). Zhang *et al*. (2015) investigated in the buckling behavior of FG-CNT reinforced composite thick skew plates under different compressions by using element-free IMLS-Ritz method. It is observed that the increase of skew angle is likely to pronounce the influence of geometric and other parameters on the buckling load intensity factors. Kiani (2017) studied the buckling loads and mode shapes of rectangular FG-CNTRC reinforced composite plates subjected to parabolic edge compression. It is verified that, by usage of a proper distribution of CNTs in a matrix, the buckling loads of the plate may be enhanced significantly. Bensattalah *et al*. (2019) studied the critical buckling of a Single-Walled Carbon Nanotube (SWCNT) embedded in Kerr's medium. It is obtained that the maximum value of the buckling load occurs in the case of Pasternak elastic foundations more than these values of Winkler's and Kerr's foundations.

This previous research focused mostly on the study of the distribution of CNTs in FG (beams, plates or shells) by the uniform distribution or by the linear distribution. In this paper, we focus on the nonlinear distribution of CNTs in FG-CNTRC beams. The critical buckling analysis was performed with the consideration of CNT law distribution. In this regard, the higher-order shear deformation theories are used to accurately formulate the influences of the transverse shear stress distribution along the beam thickness. It is assumed that the CNT volume fraction distribution law is exponential. The effect of different parameters on the buckling analysis such as aspect ratios, volume fraction, and order of the exponent of volume fraction law distribution are investigated.

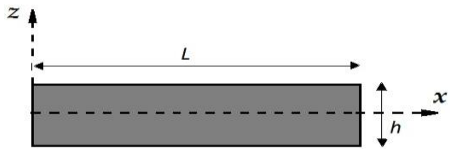


Fig. 1 Geometry of carbon nanotube-reinforced composite CNTRC beam

2. Geometrical and properties of FG-CNTRC beams

The geometry of the FG-CNTRC beams for that subjected to an axial compressive force is shown in Fig. 1. The CNTRC beams are presented simply supported and made of SWCNTs embedded in the isotropic matrix.

Table 1 shows the mathematical exponential expression of the CNT volume fraction for each configuration of distribution. Where n is the exponent degree of volume fraction carbon nanotube equation and V_{cnt}^{*} is the volume fraction of CNTs, which can be obtained from the equation

$$V_{cnt}^{*} = \frac{W_{cnt}}{W_{cnt} + (\rho^{cnt}/\rho^{p})(1 - W_{cnt})} \tag{1}$$

in which W_{cnt} is the mass fraction of the CNT, ρ^{cnt} is the density of CNT and ρ^{P} is the density of polymer in CNTRC beams. In the case of (n = 0) the distribution of CNT is uniform in the matrix (UD-CNTs) and in the case of (n = 1) the distribution of CNT is linear distribution in the matrix. The CNT volume fraction varies through the thickness of the direction of the nanocomposite beams.

According to the rule of mixture model, Young's modulus and shear modulus of the FG-CNTRC material are determined as follows Shen (2009)

$$E_{11} = \eta_1 V_{cnt} E_{11}^{cnt} + V_p E^p \tag{2a}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{cnt}}{E_{22}^{cnt}} + \frac{V_p}{E^p} \tag{2b}$$

$$G_{12} \frac{\eta_3}{G_{12}} = \frac{V_{cnt}}{G_{12}^{cnt}} + \frac{V_p}{G^p} \tag{2c}$$

where V_{cnt} and V_p are the CNT volume fractions and polymer according to the following relationship $V_{cnt} + V_p = 1$. E_{11}^{cnt}, E_{22}^{cnt} and G_{12}^{cnt} are the Young modulus and shear modulus of SWCNTs, respectively. Furthermore, E^p and G^p indicate the corresponding properties of the isotropic matrix. Yas and Samadi (2012) based on the rule of mixture and MD simulations given by Han and Elliott (2007), Yas and Samadi (2012) predicted three efficiency parameters which are used to capture the size dependent properties of the CNTRC beams. Table 2 shows the efficiency parameters related to the volume fraction (V_{cnt}^{*}).

The mass density and Poisson's ratio of the FG-CNTRC

Table 1 Cross sections of different types of carbon nanotube reinforcement and their law of CNT volume fraction distribution

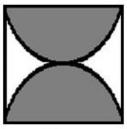
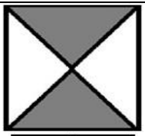
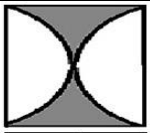
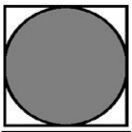
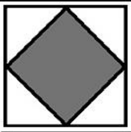
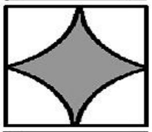
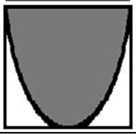
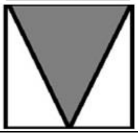
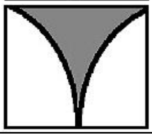
	$0 < n < 1$	$n = 1$	$n > 1$	V_{cnt}
X-beam				$(n+1)(2\frac{\|z\|}{h})^n V_{cnt}^*$
O-beam				$(n+1)(1-2\frac{\|z\|}{h})^n V_{cnt}^*$
V-beam				$(n+1)(\frac{1}{2}+\frac{z}{h})^n V_{cnt}^*$

Table 2 The CNT efficiency parameters (η)

Case	η_1	$\eta_2 = \eta_3$
$V_{cnt}^* = 0.12$	1.2833	1.0566
$V_{cnt}^* = 0.17$	1.3414	1.7101
$V_{cnt}^* = 0.28$	1.3238	1.7380

beams are evaluated by using the rule of mixture, the results of which are, respectively.

$$\rho = V_{cnt}\rho^{cnt} + V_p\rho^p \tag{3}$$

$$\nu_{12} = V_{cnt}\nu_{12}^{cnt} + V_p\nu^p \tag{4}$$

3. Equations of motion

The displacements field of this theory of any point in the beam along the x and z axes, denoted by $u(x,z,t)$ and $w(x,z,t)$ respectively as follows Şimşek (2010).

For the Eq. (5), displacement components u and w are associated with displacements along x and z directions, respectively. Besides, u_0 is the axial displacement, w_b and w_s are the bending and shear components of transverse displacement along the mid-plane of the beam.

The strains associated with the displacements in Eq. (5) are

$$\begin{cases} u(\text{x,z,t}) = u_0 - z\dfrac{\partial w_b}{\partial x} - f(z)\dfrac{\partial ws}{\partial x} \\ w(x,z,t) = w_b(x,t) + w_s(x,t) \end{cases} \tag{5}$$

$$\begin{cases} \varepsilon_x = \dfrac{\partial u}{\partial x} = \dfrac{\partial u_0}{\partial x} - z\dfrac{\partial^2 w_b}{\partial x^2} - f(z)\dfrac{\partial^2 w_s}{\partial x^2} \\ \gamma_{xz} = g(z)\dfrac{\partial w_s}{\partial x} \end{cases} \tag{6a}$$

where

$$f(z) = \text{z-}\ \frac{5}{4}z\left(1 - \frac{4z^2}{3h^2}\right), \qquad g(z) = 1 - \frac{df(z)}{dz} \tag{6b}$$

The linear elastic constitutive relations for normal stress and shear stress are

$$\begin{cases} \sigma_x = Q_{11}(z)\varepsilon_x \\ \tau_{xz} = Q_{55}(z)\gamma_{xz} \end{cases} \tag{7a}$$

where

$$\begin{cases} Q_{11}(z) = \dfrac{E_{11}(z)}{1-\nu^2} \\ Q_{55}(z) = G_{12}(z) \end{cases} \tag{7b}$$

For the case of buckling loads act as external forces, virtual displacement principle takes the form Kiani *et al*. (2011)

$$\delta U + \delta V = 0 \tag{8}$$

in which δU is the virtual strain energy of the beam which calculated by

$$\begin{aligned} \delta U &= \int_0^L \int_{-h/2}^{h/2} (\sigma_x \delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz}) dz dx \\ &= \int_0^L (N\frac{d\delta u_0}{dx} - M_b\frac{d^2\delta u_b}{dx^2} - M_s\frac{d^2\delta u_s}{dx^2} + Q\frac{d\delta w_s}{dx}) \end{aligned} \tag{9}$$

where N, M_b, Ms and Q denote the stress resultants defined as

$$(N, M_b, M_s) = \int_0^L (1, z, f)\sigma_x dz, \qquad Q = \int_{-h/2}^{h/2} g\tau_{xz} dz \tag{10}$$

And δV is the virtual potential energy of the buckling loads due to the axial compressive (N_{x0}) forces which induce the geometrical instability state of the structure, the virtual potential energy can be written

$$\delta V = \int_0^L \left[N_{x0}\frac{d(w_b + w_s)}{dx}\frac{(\delta w_b + \delta w_s)}{dx}\right] dx \tag{11}$$

Substituting the expressions of δU and δV from Eqs. (9) and (11) in Eq. (8) and integrating by parts versus both space and time variables, and collecting the coefficients of

δu_0, δw_b and δw_s, the following equations of motion of the CNTRC beam can be written

$$\begin{cases} \delta u_0: \ \dfrac{dN}{dx} = 0 \\ \delta w_b: \ \dfrac{d^2 M_b}{dx^2} + N_{x0}\dfrac{d^2(w_b + w_s)}{dx^2} = 0 \\ \delta w_s: \ \dfrac{d^2 M_s}{dx^2} + \dfrac{dQ}{dx} + N_{x0}\dfrac{d^2(w_b + w_s)}{dx^2} = 0 \end{cases} \tag{12}$$

The stress resultants can be written in the form of material stiffness components and displacements as follows

$$\begin{cases} N = A_{11}\dfrac{du_0}{dx} - B_{11}\dfrac{d^2 w_b}{dx^2} - B_{11}^s\dfrac{d^2 w_s}{dx^2} \\ M_b = B_{11}\dfrac{du_0}{dx} - D_{11}\dfrac{d^2 w_b}{dx^2} - D_{11}^s\dfrac{d^2 w_s}{dx^2} \\ M_s = B_{11}^s\dfrac{du_0}{dx} - D_{11}^s\dfrac{d^2 w_b}{dx^2} - H_{11}^s\dfrac{d^2 w_s}{dx^2} \\ Q = A_{55}^s\dfrac{dw_s}{dx} \end{cases} \tag{13}$$

where stiffness components are given as

$$\begin{cases} (A_{11}, B_{11}, D_{11}) = \displaystyle\int_{-h/2}^{h/2} (1, z, z^2) dz \\ (B_{11}^s, D_{11}^s, H_{11}^s) = \displaystyle\int_{-h/2}^{h/2} (f(z), zf(z), f^2(z)) dz \\ A_{55}^s = \displaystyle\int_{-h/2}^{h/2} Q_{55}[g(z)] dz \end{cases} \tag{14}$$

Substituting expression from Eq. (13) of stress resultants into Eq. (12), it is obtained

$$\begin{cases} A_{11}\dfrac{d^2 u_0}{dx^2} - B_{11}\dfrac{d^3 w_b}{dx^3} - B_{11}^s\dfrac{d^3 w_s}{dx^3} = 0 \\ B_{11}\dfrac{d^3 u_0}{dx^3} - D_{11}\dfrac{d^4 w_b}{dx^4} - D_{11}^s\dfrac{d^4 w_s}{dx^4} + N_{x0}\dfrac{d^2(w_b + w_s)}{dx^2} = 0 \\ B_{11}^s\dfrac{d^3 u_0}{dx^3} - D_{11}^s\dfrac{d^4 w_b}{dx^4} - D_{11}^s\dfrac{d^4 w_s}{dx^4} + \dfrac{d^2 w_s}{dx^2} \\ + N_{x0}\dfrac{d^2(w_b + w_s)}{dx^2} = 0 \end{cases} \tag{15}$$

4. Analytical solution

The Navier solution method is used under the specified boundary conditions for a simply supported CNTRC beams. The solution of the displacement variables satisfying the boundary conditions can be expressed

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m\, cos(\lambda x) e^{iwt} \\ W_{bm}\, sin(\lambda x) e^{iwt} \\ W_{sm}\, sin(\lambda x) e^{iwt} \end{Bmatrix} \tag{16}$$

where U_m, W_{bm} and W_{sm} are arbitrary parameters to be determined, ω is the eigen frequency associated with mth eigenmode, and $\lambda = m\pi/L$.

Substituting the expressions of u_0, w_b and w_s from Eq. (16) into Eq. (15), the analytical solutions can be determined from the following equations

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \begin{bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{17a}$$

where

$$\begin{aligned} &S_{11} = A_{11}\lambda^2; S_{12} = -B_{11}\lambda^3; S_{13} = -B_{11}^s\lambda^3 \\ &S_{22} = D_{11}\lambda^4 + N_{x0}\lambda^2 \\ &S_{23} = D_{11}^s\lambda^4 + N_{x0}\lambda^2 \\ &S_{33} = H_{11}^s\lambda^4 + A_{55}^s\lambda^2 + N_{x0}\lambda^2 \end{aligned} \tag{17b}$$

Clearly, the solution of Eq. (17a) gives the eigenvalue of critical buckling load.

Table 3 Comparison of dimensionless critical buckling loads for CNTRC beam ($L/h = 15$, $V_{cnt}^* = 0.12$)

Theory	*UD-beam*	*O-beam*	*X-beam*
Tagrara *et al.* (2015)	0.0985	0.0575	0.1291
Wattanasakulpong and Ungbhakorn (2013), FSDT	0.1032	0.0604	0.1367
Wattanasakulpong and Ungbhakorn (2013), TSDT	0.0985	0.0575	0.1291
Yas and Samadi (2012)	0.0986	0.0588	0.1288
Present	0.0984	0.0576	0.1289

Table 4 Dimensionless critical buckling loads for CNTRC beam

	$n = 1$		$n = 2$	
V_{cnt}^*	*X-beam*	*O-beam*	*X-beam*	*O-beam*
0.12	0.1289	0.0576	0.1445	0.0382
0.17	0.1979	0.0864	0.2223	0.0564

5. Numerical results and discussion

Static buckling behaviors of FG-CNTRC beams are investigated. The FG-CNTRC beams made of (polymetyl-methacrylate, referred as PMMA) as matrix, where PMMA is an isotropic material with $\upsilon^p = 0.3$, $\rho^p = 1190$ Kg/m^3, and $E^p = 2.5$ GPa. The (10, 10) single-walled CNTs SWCNT are selected as reinforcements. The adopted material properties for SWCNT are $\upsilon^{cnt} =$ 0.19, $\rho^{cnt} =$ 1400 Kg/m^3, $E_{11}^{cnt} =$ 600 GPa, $E_{22}^{cnt} =$ 10 GPa and $G_{12}^{cnt} =$ 17.2 GPa. The previous parameters are employed at ambient temperature.

For buckling analysis, the following dimensionless are employed

$$\bar{N} = \frac{N_{x0}}{A_{110}} \tag{18}$$

Table 5 Dimensionless critical buckling for *X-beam* pattern in exponential distribution

		n							
V_{cnt}^{*}	L/h	0.5	1	1.5	2	2.3	2.5	3	3.5
0.12	10	0.1860	0.1999	0.2098	0.2174	0.2211	0.2234	0.2283	0.2325
	15	0.1166	0.1289	0.1377	0.1445	0.1479	0.1500	0.1544	0.1581
	20	0.0768	0.0862	0.0932	0.0986	0.1013	0.1029	0.1064	0.1094
0.17	10	0.2928	0.3152	0.3308	0.3426	0.3484	-	-	-
	15	0.1788	0.1979	0.2118	0.2223	0.2275			
	20	0.1158	0.1303	0.1410	0.1493	0.1534	-	-	-

Table 6 Dimensionless critical buckling for *O-beam* pattern in exponential distribution

		n							
V_{cnt}^{*}	L/h	0.5	1	1.5	2	2.3	2.5	3	3.5
0.12	10	0.1299	0.1056	0.0878	0.0743	0.0677	0.0639	0.0556	0.0491
	15	0.0739	0.0576	0.0463	0.0382	0.0344	0.0322	0.0276	0.0240
	20	0.0460	0.0352	0.0279	0.0228	0.0204	0.0190	0.0162	0.0140
0.17	10	0.2021	0.1628	0.1340	0.1123	0.1018	-	-	-
	15	0.1118	0.0864	0.0689	0.0564	0.0505			
	20	0.0688	0.0522	0.0410	0.0332	0.0296	-	-	-

where A_{110} of beam made of pure matrix material and $\bar{N}$ is the critical mechanical load at which the structure buckles.

The present study aims to analyze the critical buckling of FG-CNTRC beam. In this work, the results of the dimensionless critical buckling obtained by the methods used in this study were compared with that calculated by Tagrara *et al*. (2015), Wattanasakulpong and Ungbhakorn (2013) and Yas and Samadi (2012). Table 3 shows this comparison and it is very clear that the present results show a good agreement with those obtained by them.

The FG-CNTRC beams based on the polymer PMMA as a matrix and SWCNT (10, 10) as a nano reinforcement have been studied by several researchers such as Yas and Samadi (2012), Wattanasakulpong and Ungbhakorn (2013) and Tagrara *et al*. (2015). These researchers used three CNT volume fractions (0.12, 0.17 and 0.28). In the linear distribution the maximum of CNT volume fractions reaches to $2V_{cnt}^{*}$ (0.24, 0.34 and 0.58). In this work the maximum of CNT volume fractions in the nonlinear distribution reach to$(n+1)V_{cnt}^{*}$. For this reason, we have chosen the degree of exponent (n) in which the CNT volume fraction does not exceed the value 0.56 which is the maximum value in the linear distribution.

The effects of the nature of distribution (linear or parabolic distribution), two beam patterns, on the dimensionless critical buckling loads of the FG-CNTRC beams are shown in Table 4. In this case, aspect ratio *L*/*h* is set to 15. It is clear that the value of the dimensionless critical buckling loads obtained in parabolic distribution is higher than that obtained in a linear distribution. Increasing of CNT volume fraction leads to an increase in the critical buckling loads, and the maximum value of critical buckling

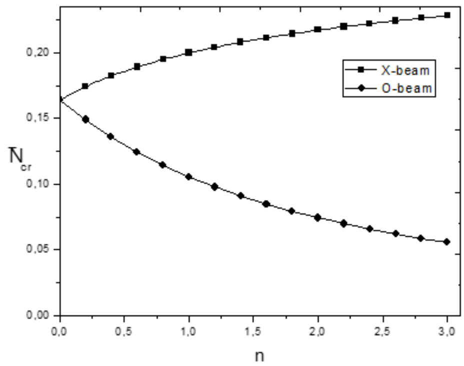


Fig. 2 Effect of degree of exponent *n* and the type of distribution on the critical buckling loads ($L/h = 10,\ V_{cnt}^{*} = 0.12$)

loads among the different configurations is that obtained in the *X-beam* configuration.

Tables 5 and 6 provide the dimensionless critical buckling load of FG-CNTRC beam in different aspect ratio and volume fraction with two patterns (*X-beam* and *O-beam*), respectively. These tables indicate that by increasing the aspect ratio, the dimensionless critical buckling would decrease. But by increasing the volume fraction, the dimensionless critical buckling load would increase. On the other hand, when the degree of exponent (n) increase, the dimensionless critical buckling load increase in *X-beam* pattern (Table 5) and a decrease in *O-beam* pattern (Table 6). Table 5 confirms the influence of the degree of exponent (n) in the *X-beam* pattern. In addition, the non-linearity of the distribution of carbon nanotube volume fraction is an important role to improve the strength and stiffness of FG-

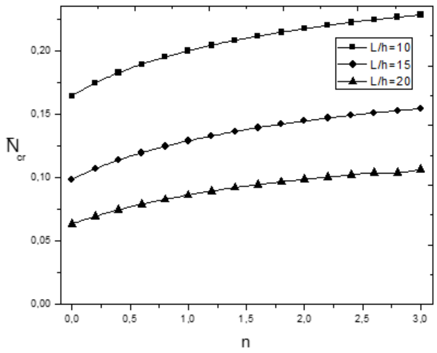


Fig. 3 Effect of degree of exponent *n* and the aspect ratio on the critical buckling loads

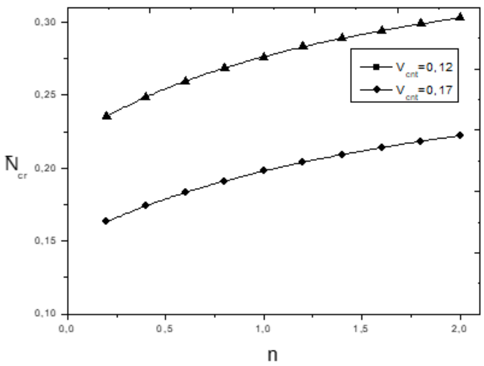


Fig. 4 Effect of degree of exponent (n) and the volume fraction on the dimensionless critical buckling (*X-beam*, L/h = 15)

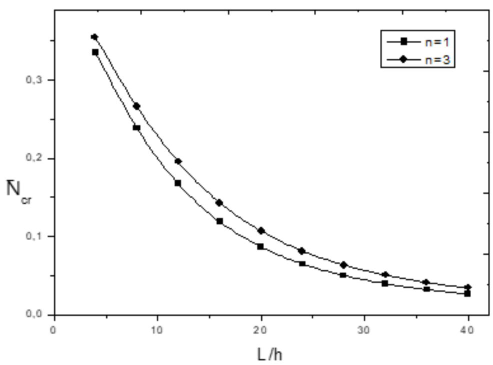


Fig. 5 Effect of degree of exponent (n) and the aspect ratio on the dimensionless critical buckling (*X-beam*, V_{cnt} = 0.12)

CNTRC beams.

Fig. 2 presents the critical buckling of FG-CNTRC beams with two distribution patterns of reinforcements, the Volume fraction of CNT is set equal to 0.12, and aspect ratio is set to 10. It is verified that *X-beam* pattern of CNT results in the maximum buckling load of the plate and *O-beam* pattern yields the minimum buckling load of the beam, because the increase of volume fraction exponent increases the critical buckling load in *X-beam*, but decrease in the *O-beam*.

In Fig. 3, the volume fraction of CNT is set equal to 0.12, and *X-beam* distribution is considered. The influence of the degree of exponent n and the aspect ratio in the increasing of critical buckling load in this work is very clear. It can be seen that the increase of aspect ratio and/or the degree of the exponent (n) increase the critical buckling loads.

Fig. 4 gives the dimensionless critical buckling load as a function of degree of the exponent (n) and the volume fraction with *X-beam* pattern of CNTs across the thickness. In this figure, the increase of the volume fraction and the degree of the exponent lead to increasing the dimensionless critical buckling load. On the contrary, in Fig. 5 the dimensionless critical buckling load decrease when the aspect ratio L/h increase. It is observed that the values of the dimensionless critical buckling load in the case of the non-linearity distribution (n = 3) is higher than the values that obtained in the case of the linearity distribution (n = 1). These results confirm the influence of the non-linearity distribution to improve the mechanical properties of FG-CNTRC beams.

6. Conclusions

In the present study, the buckling behavior of the beams made of functionally graded CNTRC is investigated. Material properties of tPMMA and (10, 10) SWCNT reinforcements are assumed to be simply supported beams boundary conditions, based on higher-order shear deformation theories, Hamilton principle's, Navier solution method. Distribution of CNTs across the beam thickness is assumed to be uniform in *UD-beam* pattern and non-uniform in two patterns (*X-beam* and *O-beam*). For this non-uniformity, the CNTs distributions are assumed to be graded in the thickness direction according to a new exponential power law distribution in terms of the carbon nanotube volume fractions. After performing comparison studies, parametric studies are done to explore the influences of the degree of the exponent, carbon nanotube volume fraction, aspect ratio and the type of configuration. The results obtained by the present method can be summarized as follows:

• The influences of non-linearity distribution in the beam play an important role to improve the mechanical properties, especially in buckling behavior.

• *X-Beam* configuration is the strongest among all different types of CNTRC beams in supporting the buckling loads.

• Increasing one of these parameters, CNT volume fraction, aspect ratio, and degree of the exponent (n), also increase the critical buckling loads. And therefore, makes the FG-CNTRC beams stiffer and more powerful.

• The influence of the degree of exponent (n) of CNT volume fraction law in the FG-CNTRC performance plays an important role to improve the mechanic properties of beams.

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