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A numerical application of Bayesian optimization to the condition assessment of bridge hangers

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Abstract. Bridge hangers, such as those in suspension and cable-stayed bridges, suffer from cumulative fatigue damage caused by dynamic loads (e.g., cyclic traffic and wind loads) in their service condition. Thus, the identification of damage to hangers is important in preserving the service life of the bridge structure. This study develops a new method for condition assessment of bridge hangers. The tension force of the bridge and the damages in the element level can be identified using the Bayesian optimization method. To improve the number of observed data, the additional mass method is combined the Bayesian optimization method. Numerical studies are presented to verify the accuracy and efficiency of the proposed method. The influence of different acquisition functions, which include expected improvement (EI), probability-of-improvement (PI), lower confidence bound (LCB), and expected improvement per second (EIPC), on the identification of damage to the bridge hanger is studied. Results show that the errors identified by the EI acquisition function are smaller than those identified by the other acquisition functions. The identification of the damage to the bridge hanger with various types of boundary conditions and different levels of measurement noise are also studied. Results show that both the severity of the damage and the tension force can be identified via the proposed method, thereby verifying the robustness of the proposed method. Compared to the genetic algorithm (GA), particle swarm optimization (PSO), and nonlinear least-square method (NLS), the Bayesian optimization (BO) performs best in identifying the structural damage and tension force.

Keywords: additional mass method; bayesian optimization; bridge hanger; condition assessment; damage detection; finite element model

1. Introduction

Bridge hangers are important parts of suspension and cable-stayed bridges within the load path where forces are transmitted from the deck to the pylons and into the foundation. Changes in the physical properties of bridge hangers may lead to internal force changes in the entire bridge structure (Nazarian *et al*. 2016). The characteristics of light mass, high flexibility, and slight damping make bridge hangers vulnerable to damage (Ye *et al*. 2012). Specifically, the bridge hanger might accumulate damage inevitably with the increasing service time under cyclic and random loads such as wind, vehicle, pedestrian, and other types of load (Peng *et al*. 2022, Gobbato *et al*. 2014), due to a number of unforeseen reasons, such as material degradation, corrosion, overloading and fatigue, etc (Ding *et al*. 2020). Furthermore, the damage of bridge hangers includes anchor head damage, sheath damage and steel wire damage (Yao *et al*. 2021, Meng *et al*. 2019, Lin *et al*. 2017). Therefore, the evaluation of damage to the bridge hangers is necessary in hanger installation during bridge construction and hanger maintenance during operation periods.

To better evaluate the damage to the bridge hanger, it is necessary to establish an equilibrium equation with tension force. Russell et al. proposed an empirical equation based on experiments of the natural frequencies and tension (Russell and Lardner 1998). Ren *et al*. (2005) proposed empirical equations by fitting the experimental data to estimate hanger tension based on fundamental frequency only. Zui *et al*. (1996) proposed practical formulas to estimate hanger tensions with measured natural frequencies of first-order and second-order modes. Amabili *et al*. (2010) established a method for the calculation of hanger tension with frequency derived on the basis of the Rayleigh-Ritz method. Ceballos and Prato (2008) presented all the explicit analytical expressions for natural frequencies considering the bending stiffness of the cable and rotational restraint at the ends, which may be used to determine axial force. Evidently, the vibration-frequency equation can be obtained to evaluate damage based on the theory of structural dynamics, which derives a relationship between the tension force and the natural frequency of the hanger. However, the vibration-frequency equation includes many unknown damage parameters in the bridge hanger, such as frequency, tension, bending stiffness, and boundary constraint condition. Therefore, a method for parameter evaluation is needed to calculate these unknown parameters. In dynamic testing, only limit number of low order structural

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frequencies can be extracted. To identify both number structural parameters and tension force is a challenge, as only limit number of low order structural frequencies can be extracted in the dynamic testing. The number of observed data is usually less than the number of unknown parameters. The additional mass method (AMM) is developed to increase the number of observed data (Xie and Li 2014). In the AMM method, an additional mass is added to the structure. When the weight and the placement of additional mass is different, the frequencies of the structure also will change. The dynamic testing of the structure with additional mass are performed and more observed data can be obtained. AMM is a promising method for condition assessment of bridge hanger.

A number of damage detection methods have been explored (Hou *et al*. 2021, Pereira *et al*. 2021, Peng *et al*. 2021, Bayik *et al*. 2020, Pathirage *et al*. 2018). These methods can be divided into two categories. One consists of deterministic models, including but not limited to genetic algorithm (GA), and particle swarm optimization (PSO), and the other consists of uncertain models such as expectation maximum (EM) and Bayesian optimization (Ye *et al*. 2021, Avci *et al*. 2020). Li *et al*. (2018) used the Gaussian mixture model (GMM) in modeling the patterns of cable tension ratio, and the parameters of GMM are estimated by using the EM algorithm under maximum likelihood criteria. The EM algorithm may be prone to converge to local maximum, and selecting a suitable initial value is important (Meysam and Omid 2021). Koh and Shankar (2003) employed the GA approach to determine the unknown parameters of substructures without needing interface measurements. Cao *et al*. (2017) used the enhanced PSO to overcome difficulties in finite element (FE)-based suspension bridge optimization. Wang *et al*. (2011) presented a spectral element model updating procedure to identify damage in a structure using Guided wave propagation results. Two damage spectral elements are developed to model the local and global damage in one-dimensional homogeneous and composite waveguide, respectively. Xu *et al*. (2020) constructed a FE model to simulate structural behaviours of the offshore wind turbine system. The FE model and the test results were used to analyse the variation of the support condition of the monopile, through the FE model updating process using estimation of distribution algorithms. For the classic damage detection method, the unknown structural parameters are updated by means of the sensitivity method. In other words, they are very difficult to solve the damage identification problem due to the following two issues. On the one hand, a search that directly calculates the objective and constraint functions during the optimization process may be impossible because uncertain objective and probabilistic constraint functions of these problems are non-linear and implicit within a FE analysis program (Li and Law 2010). On the other hand, evaluation of the mean and variance values of the uncertain objective function is a challenging task (Hou *et al*. 2018). Obviously, the performance of the deterministic models, which can solve the problems with linear objective and constraint functions, in solving multi-objective problems remains unknown as hey have been only applied to single-objective problems (Do *et al*. 2021). However, for the damage identification problem of bridge hanger, its stiffness and tension force are unknown, that is, it's a multi-objective parameter identification problem.

In contrast to deterministic approaches, Bayesian optimization (BO) is a powerful sequential method to solve optimization problems with unknown objective and the BO has demonstrated its ability in solving combinatorial optimization problems (Jones *et al*. 1998). In addition, an appealing feature of the Bayesian approach is that not only the optimal values of the modal parameters can be obtained but also the associated estimation uncertainty can be quantified in the form of probability distribution (Kuok and Yuen (2016)). For example, Gregori *et al*. (2020) used the Bayesian optimization technique to optimize an entire catenary section, leading to a catenary configuration with an interaction force that has a standard deviation notably lower than that provided by the nominal catenary design. Zhang *et al*. (2021) presented quantile random forest with Bayesian optimization as a data-driven method for probabilistic prediction, which exhibited superior performance compared with other optimization algorithms and models in terms of accuracy and computational expense. The Bayesian optimization method can find the optimal parameters effectively and quickly, which makes it suitable for the global optimization of black-box objective functions that are expensive to evaluate (Liang 2019). Specifically, the loss is modeled as a function of unknown parameters in Bayesian optimization; then, a probabilistic surrogate model and acquisition function are employed to iteratively determine the unknown parameter setting in a new site (Shahriari *et al*. 2015). Therefore, this study proposes using the BO for solving damage identification problem as it does not require special forms of the uncertain objective functions. In this way, the non-linear and implicit nature of damage identification problem can be addressed by using the mean functions of the GP models as surrogates for the uncertain objective function.

This paper presents a new approach for the identification of tension force and condition assessment of bridge hangers via the Bayesian optimization and additional mass method. The bridge hanger is modeled with Euler beam elements using the finite element method (FEM), and the relationship between the tension force and structural frequencies is derived. The accuracy and efficiency of the proposed method is verified by comparison studies with Genetic algorithm and particle swarm optimization. We apply the AMM method to increase the number of observed frequencies for the identification of damage. The Bayesian optimization method is used to identify the tension force and the severity of the damage. Effects of acquisition functions, boundary conditions, and measurement noise on the identified results are also studied. Results show that the tension force and the severity of the damage can be identified via the proposed method under a noise environment.

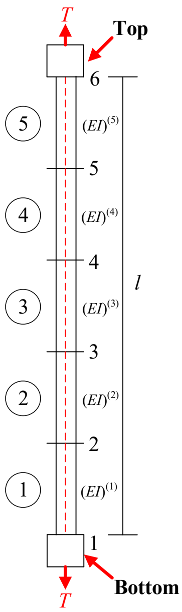


Fig. 1 FE model of bridge hanger

2. Methodology

2.1 Equation of motion and frequency

Based on the FEM, the hanger can be represented with several Euler beam elements as shown in Fig. 1.

The matrix form of the free vibration of the FE model can be expressed as follows (Xie *et al*. 2014)

$$M\ddot{U} + (K + TK_G)U = 0 \tag{1}$$

where M, K, and K_G are the global mass, stiffness, and geometric stiffness matrices, respectively; $\ddot{U}$ and $U$are the acceleration and displacement, respectively; and T is the tension force. The element's mass, stiffness, and geometric stiffness matrix can be expressed as follows (Chopra 2012, Spanos and Chen 1980, Yuen and Katafygiotis 2010)

$$M^{(i)} = \frac{\rho A l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{2}$$

$$K^{(i)} = \frac{EI}{l}\begin{bmatrix} 12/l^2 & 6/l & -12/l^2 & 6/l \\ 6/l & 4 & -6/l & 2 \\ -12/l^2 & -6/l & 12/l^2 & -6/l \\ 6/l & 2 & -6/l & 4 \end{bmatrix} \tag{3}$$

$$K_G^{(i)} = \frac{1}{l}\begin{bmatrix} 6/5 & l/10 & -6/5 & l/10 \\ l/10 & 2l^2/15 & -l/10 & -l^2/30 \\ -6/5 & -l/10 & 6/5 & -l/10 \\ l/10 & -l^2/30 & -l/10 & 2l^2/15 \end{bmatrix} \tag{4}$$

where $M^{(i)}$, $K^{(i)}$, and $K_G^{(i)}$ are the i-th element mass, stiffness, and geometric stiffness matrices, respectively; M, K, and K_G can be assembled from the element matrix; ρ is the mass density; E is Young's modulus of elasticity; I is the moment of inertia; and l is the length of the element.

Furthermore, the relationship between the hanger tension force T and the natural frequencies of the hanger f can be obtained by solving the following eigenvalue problem (Xie and Li 2014)

$$|K + TK_G - (2\pi f)^2 M| = 0 \tag{5}$$

where $|\bullet|$ is the determinant. Eq. (5) shows the relationship between frequency f, tension force T, and the global stiffness matrix K. Obviously, the dynamic characteristics of a hanger, such as frequency, are highly correlated to its mass density, Young's modulus of elasticity, the moment of inertia, the tension, length, and boundary conditions.

2.2 Measured frequencies with the additional mass method

In dynamic testing, extracting high-order structural frequencies is a challenge. For example, the structure is under ambient excitation and the ambient responses usually contain only low-frequency structural information. To obtain further information about the hanger, we apply AMM in which further mass is added to a different location on the structure. The weight of the mass is known. After the additional mass is placed on the structure, the mass matrix and frequencies of the structure will be changed but the rotational inertia of the added masses is negligible (Xie and Li 2014). Therefore, the tension force and condition of the structure are the same as those before the additional mass was added. By changing the location and weight of the mass, we can obtain new structural frequencies for the identification of damage.

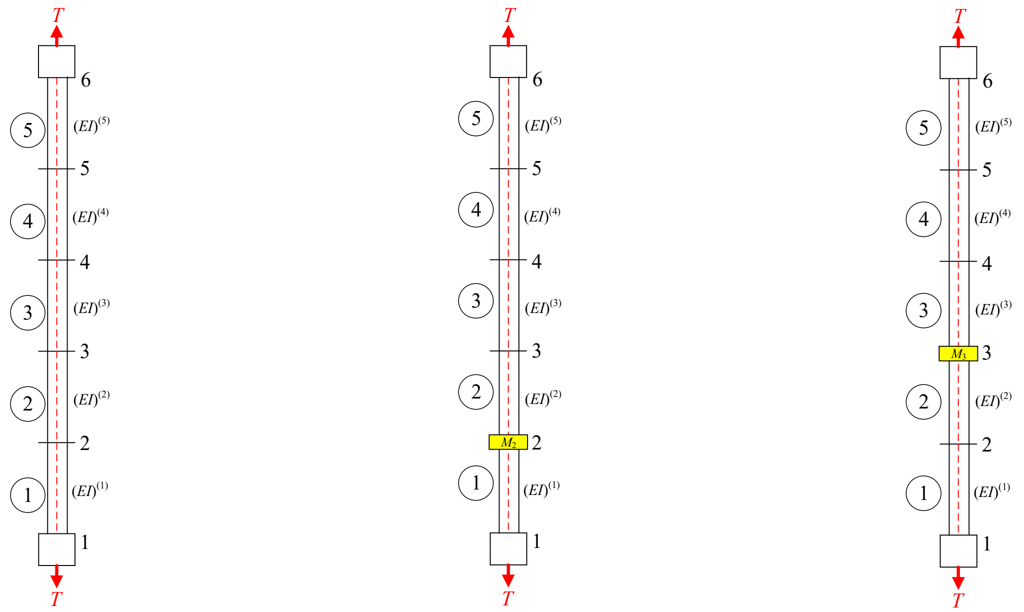
As shown in Eq. (5), mass M is the only determined value. Therefore, the frequency can be obtained by placing different masses at various positions on the hanger in accordance with AMM, which can be expressed as (Xie and Li 2014)

$$\begin{cases} |K + TK_G - (2\pi f_1)^2 M_1| = 0 \\ |K + TK_G - (2\pi f_2)^2 M_2| = 0 \\ |K + TK_G - (2\pi f_3)^2 M_3| = 0 \\ \vdots \\ |K + TK_G - (2\pi f_m)^2 M_m| = 0 \end{cases} \tag{6}$$

where M_1, and f_1 are the original mass matrix and frequency of the hanger without additional mass, respectively; $M_2, \cdots, M_m$ are the mass matrices with additional mass at different locations of the hanger; and $f_2, \cdots, f_m$ are the corresponding frequencies. The experimental process is presented in Fig. 2.

2.3 The identification of damage by means of maximum likelihood estimation

Let $\mathcal{Y} = \{[f_1, f_2, \cdots f_n] \in R^n\}$ be a set of frequencies obtained from AMM, where n is the number of observations/measured frequencies. The number of observations n is equal to the sum the number of frequencies from AMM. The measured i-th frequency of the hanger can be expressed as a mathematical representation of unknown structural parameters and tension force $\boldsymbol{\theta}$, that is



(a) Original beam (b) Additional mass at Node 2 (c) Additional mass at Node 3

Fig. 2 Procedure of AMM

$$f_i = \hat{f}_i(\boldsymbol{\theta}) + \boldsymbol{\varepsilon} \tag{7}$$

where $\hat{f}_i(\boldsymbol{\theta})$ is the frequency from the FE model; and ε is the residual term, which can be used to represent the difference between the measured and calculated frequencies. The residual term ε is assumed to have normal distribution with mean value μ and variance σ.

Based on the observed data $\mathcal{Y} = \{[f_1, f_2, \cdots f_n] \in R^n\}$, the likelihood function $p(\mathcal{Y}|\boldsymbol{\theta})$ can be expressed as (Yuen 2010)

$$p(\mathcal{Y}|\boldsymbol{\theta}) = \frac{1}{\sqrt{(2\pi\sigma)^n}} exp\left[-\frac{1}{2\sigma^2} J(\boldsymbol{\theta})\right] \tag{8}$$

where

$$J(\theta) = \sum_{i=1}^{n} \eta_i^2 = \sum_{i=1}^{n} \frac{\left(f_i - \hat{f}_i\right)^2}{\hat{f}_i^2} \tag{9}$$

The maximum likelihood estimation (MLE) of unknown parameters $\boldsymbol{\theta}$ can be achieved by maximizing the likelihood function over the parameter space as

$$\boldsymbol{\theta_{ML}} = \boldsymbol{argmax\, p\, (\mathcal{Y}|\theta)} \tag{10}$$

The solution is equal to minimize the weight sum-square error between the measured frequencies obtained from AMM and the calculated frequencies from FEM as follows the Eq. (11). The solution of Eq. (11) can be obtained from Bayesian optimization.

$$\boldsymbol{\theta_{ML}} = \underset{\boldsymbol{\theta}}{\boldsymbol{argmin}} \boldsymbol{J(\theta)} = \underset{\boldsymbol{\theta}}{\boldsymbol{argmin}} \sum_{i=1}^{n} \frac{\left(\boldsymbol{f_i} - \boldsymbol{\hat{f}_i(\theta)}\right)^2}{\boldsymbol{\hat{f}_i(\theta)^2}} \tag{11}$$

2.4 Maximum likelihood estimation with the Bayesian optimization

Bayesian optimization is a method of applying method of the Bayesian theorem, which updates the prior distribution to obtain draws from the posterior distribution. In Bayesian optimization, the prior distribution and objective function can be represented by the Gaussian process (GP), which is affordable to evaluate and is used to propose points in the search space where sampling is likely to yield an improvement (Williams and Rasmussen 2006). The damage evaluation problems can be solved by Bayesian optimization, which can be cast as an optimization problem by maximizing or minimizing an objective function with the unknown parameters (Wang *et al.* 2019). In Bayesian optimization, the Gaussian process model is constructed with additional training data and the hyperparameters in the Gaussian process model is optimized. The Bayesian optimization searches for the minimum value that is predicted from Gaussian process regression (Cai and Peng 2002).

Specially, in each iteration of the Bayesian optimization (BO), the acquisition function is a measure of searching the promising point in the input variable space. This point is generated by maximizing the acquisition function and will be used to refine the GP model by optimizing the hyperparameters for the uncertain objective and probabilistic constraint functions (Do *et al.* 2021). Once the convergence condition is reached, the unknown parameters can be obtained. The maximum likelihood estimation of unknown structural parameters is obtained from the Bayesian optimization method. In other words, the objective function $J(\theta)$ can be sampled at $\theta_t = \underset{\theta}{argmax} u(\theta|\mathcal{D}_{1:t-1})$, where u is the acquisition function and $\mathcal{D}_{1:t-1} = \{(\theta_1, J(\theta_1)), \ldots, (\theta_{t-1}, J(\theta_{t-1}))\}$ are the t-1 samples drawn from the objective function so far. In this study, the expected improvement (EI) method is used to identify the acquisition function, which is expressed as (Vereecken *et al.* 2020)

$$\mathrm{EI}(\theta) = \mathbb{E}\, max(J(\theta) - J(\theta^+), 0) \tag{12}$$

where $J(\theta^+)$ is the value of the best sample so far; and θ^+ is the location of that sample, i.e., $\theta^+ = \underset{\theta \in \{\theta_1, \theta_2 \ldots, \theta_t\}}{argmin}\ J(\theta_i)$. The EI can be evaluated analytically under the GP model (Wan and Ni 2020)

$$\mathrm{EI}(\theta) = \begin{cases} (\mu(\theta) - J(\theta^+) - \xi)\Phi(Z) + \sigma(x)\phi(Z) & \text{if } \sigma(x) > 0 \\ 0 & \text{if } \sigma(x) = 0 \end{cases}$$
$$Z = \begin{cases} \dfrac{(\mu(\theta) - J(\theta^+) - \xi)}{\sigma(\theta)} & \text{if } \sigma(\theta) > 0 \\ 0 & \text{if } \sigma(\theta) = 0 \end{cases} \tag{13}$$

where $\mu(\theta)$ and $\sigma(\theta)$ are the mean and standard deviation of the GP posterior predictive at θ, respectively; $\Phi(\bullet)$ and $\phi(\bullet)$ are the cumulative distribution function and probability density function of the standard normal distribution, respectively; and ξ determines the amount of exploration during optimization, where higher values lead to further exploration.

Therefore, the Bayesian optimization procedure includes the following steps (Wan and Ni 2020):

Step 1: A sampler set is generated and the GP model is trained.

Step 2: The next sampling point θ_t is found by optimizing the acquisition function over the GP: $\theta_t = \underset{\theta}{argmax}\, u(\theta|\mathcal{D}_{1:t-1})$.

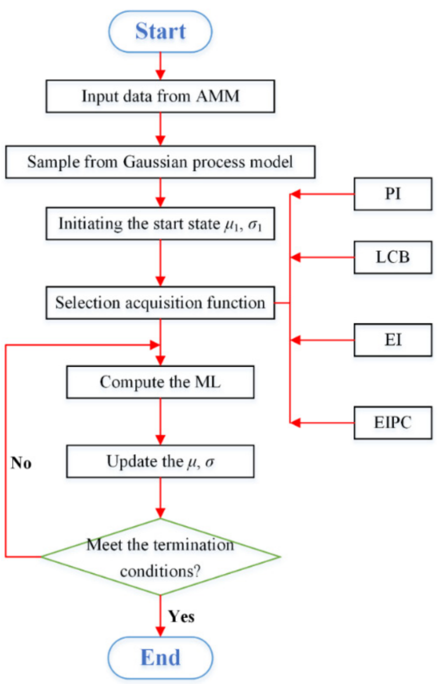


Fig. 3 Flowchart of the damage identification based on BO and AMM

Step 3: A possibly noisy sample $J(\theta_t)$ is obtained from Eq. (11).

Step 4: The sample is added to previous samples $\mathcal{D}_{1:t} = \{\mathcal{D}_{1:t-1}, (\theta_t, J(\theta_t))\}$, and the GP is updated.

Step 5: Steps 2–4 are repeated until the convergence criterion is met.

Fig. 3 shows the flowchart of the damage identification combined with AMM and the BO method.

2.5 Performance of Bayesian optimization in the identification of damage

To evaluate the performance of Bayesian optimization, we adopt the mean absolute error (MAE) and mean absolute percentage error (MAPE) as measurement criteria. The MAE of a model with respect to a test set is the mean of the absolute values of the individual prediction errors in the test set (Ye *et al.* 2019). Each calculation error represents the difference between the true and optimized values expressed by

$$MAE = |\theta_{true} - \theta_{id}| \tag{14}$$

where θ_{true} is the true value; and θ_{id} is the optimized value.

The MAPE is a measure of the accuracy of the prediction of a forecasting method in statistics. It is defined by

$$MAPE = \frac{|\theta_{true} - \theta_{id}|}{\theta_{true}} \tag{15}$$

3. Numerical studies

Numerical studies are performed to illustrate the accuracy and efficiency of the proposed method for the identification of damage. The investigated Jiubao Bridge is located in Hangzhou, China, which can be seen in Fig. 4. The total length of the bridge (inclding approach bridge) is 1855 m, and its main bridge is 3 × 210 m. The approach bridge adopts the cantilever-section box beam and the main bridge employs a new structural system with the combination of beam and arch (Ye *et al.* 2019). The length of the shorter hanger is 4 m and the diameter of the cross section is 0.077 m. The elastic modulus and material density of the hanger are 1.9 × 10^8 kPa and 7850 kN/m^3, respectively. The physical parameter of the hanger such as length, cross section area, tension force, elastic modulus and material density is from a real bridge (Jiubao Bridge). The geometric stiffness caused by the tension force is considered. The dynamic characteristics of the hanger are from the linear-elastic stage. The hanger has fixed supports

Fig. 4 Jiubao Bridge

at the two ends. The tension force is calculated from a real bridge structure and then a tension force (1700 kN) is added along the longitudinal direction of the hanger. In this paper, the hanger is modelled with 10 Euler–Bernoulli beam elements. But for the convenience of calculation, two adjacent small beam elements are combined into one structural element. For example, the Euler–Bernoulli beam elements 1 and 2 have the same stiffness parameter. In the problem of identification, we only consider the five unknown stiffness parameters.

Table 1 Identified results with different number of elements

Unknown parameters	Actual value	Number of elements			
		10	20	30	50
θ_1	0.6	0.6057	0.6277	0.6264	0.6369
θ_2	1	0.9982	0.9808	0.9986	0.9907
θ_3	1	0.9905	0.9986	0.9992	0.9966
θ_4	1	0.9983	0.9867	0.9976	0.9974
θ_5	1	0.9954	0.9853	0.9476	0.9321

Table 2 Performance of different number of elements for identification of unknown parameters

Number of elements	MAE Value	MAPE Value
10	0.0233	0.0271
20	0.0763	0.0948
30	0.0834	0.1010
50	0.1201	0.1447

3.1 The identification of stiffness with the different acquisition functions

In the first case, the proposed method is used to identify the location of the damage and the severity of the hanger. The coefficient of element stiffness $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots \theta_5]$ is unknown and identified with the proposed method. The boundary conditions, tension force, and mass density are known. The stiffness of element 1 is assumed to suffer a 40% reduction. Therefore, the actual values of unknown structural parameters are $\boldsymbol{\theta} = [0.6,1,1,1,1]$.

In order to better apply BO method to identify damage parameters, we discuss the influence of BO's parameters, for example, the length of an element and lower bound of the unknown stiffness parameters, on identification results, which can be seen as Tables 1~4. It can be seen from the Tables 1 and 2, the performance of identification of unknown parameters will be worse with the increase of the number of elements. As can be seen from the Tables 3 and 4, the higher the lower bound of the unknown stiffness parameters, the better performance of identification of unknown parameters. Therefore, we choose the 10-number of elements and 0.5-lower bound of the unknown stiffness parameters to establish the BO.

We measured the first four frequencies of the original structure and added a different mass to various locations of the hanger, which can be seen as Fig. 2. We moved the mass from node 2 to node 3 and recorded the first four frequencies of the hanger under different situations. When the damage exist between elements 2 and 3, that is, damage exist node 3, the proposed method considered that the damage exist in element 2. The additional mass at nodes 2 and 3 are 20 kg and 40 kg, respectively. The measured frequencies (three sets, each one including the first four frequencies) are considered as observed data for the identification of damage. When there is no additional mass, the first four natural frequencies of the structure are 9.1889 Hz, 35.4540 Hz, 78.3913 Hz and 140.4781 Hz, respectively. When additional mass is 20 kg on node 4, the first four natural frequencies of the structure are 8.7433 Hz, 31.5241 Hz, 72.7179 Hz and 138.5541 Hz, respectively. When additional mass is 40 kg on node 6, the first four natural frequencies of the structure are 7.8280 Hz, 30.6632 Hz, 78.1728 Hz, and 128.3066 Hz, respectively.

The proposed Bayesian optimization method is used for the identification of damage. In the Bayesian optimization, the new sample is generated by means of an expected improvement acquisition function and the candidate in the

Table 3 Identified results with different lower bound of the unknown stiffness parameters

Unknown parameters	Actual value	Lower bound of the unknown stiffness parameters					
		0	0.1	0.2	0.3	0.4	0.5
θ_1	0.6	0.6592	0.6584	0.6451	0.6289	0.6083	0.6057
θ_2	1	0.9781	0.9542	0.995	0.9972	0.9788	0.9982
θ_3	1	0.9845	0.9998	0.9933	0.9837	0.9911	0.9905
θ_4	1	0.9647	0.9334	0.987	0.9838	0.9999	0.9983
θ_5	1	0.9379	0.9987	0.9241	0.9973	0.9958	0.9954

Table 4 Performance of different lower bound of the unknown stiffness parameters for identification of unknown parameters

Lower bound	MAE Value	MAPE Value
0	0.194	0.2335
0.1	0.1723	0.2112
0.2	0.1457	0.1758
0.3	0.0669	0.0862
0.4	0.0427	0.0482
0.5	0.0233	0.0271

Table 5 Identified results with different acquisition functions

Unknown parameters	Actual value	Type of acquisition function			
		EI	PI	LCB	EIPC
θ_1	0.6	0.6057	0.6381	0.6218	0.6446
θ_2	1	0.9982	0.9959	0.997	0.998
θ_3	1	0.9905	1	0.9943	0.9883
θ_4	1	0.9983	0.9904	0.9978	0.9755
θ_5	1	0.9954	0.9305	0.9764	0.9717

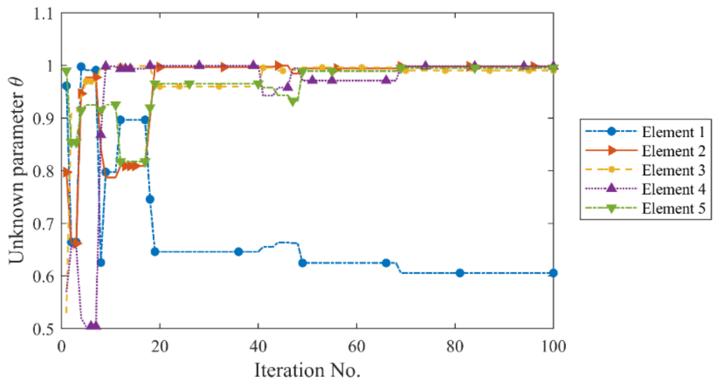


Fig. 5 Identification of unknown parameters with iterations in case 1

acquisition function is 100,000. The upper and lower bounds of the unknown parameters are 1 and 0.5, respectively. Fig. 5 shows the identified unknown parameters with iterations. In Bayesian optimization, we add a new bit of training data in each iteration and the new training data is selected from the acquisition function. Bayesian optimization reaches local minimum at 70 and 100. However, as the iteration continues, more samplers are used to search for the global minimum. Finally, a converged solution can be achieved. Bayesian optimization takes approximately 70 iterations to obtain results. The final results identified are found in Table 5. The five unknown parameters of $\theta_1 \sim \theta_5$ are 0.6057, 0.9982, 0.9905, 0.9983, and 0.9954. The five locations of damage in the hanger (40%, 0%, 0%, 0%, and 0% of stiffness reduction at elements 1, 2, 3, 4, and 5, respectively) were successfully identified. The results show some small discrepancies between the actual value and the identified value. The reasons are as follows: In each iteration of the Bayesian optimization, the Gaussian process model is constructed with additional training data and the hyperparameters in the Gaussian process model is optimized. The Gaussian process model is used to represent the objective function. The discrepancy between the Gaussian process model and the objective function may cause poor results of identified convergence.

In Bayesian optimization, the new sample is generated by an acquisition function. The choice of acquisition function may influence the identified results. Several acquisition functions such as EI, probability-of-improvement (PI), lower confidence bound (LCB), and expected improvement per second (EIPC) have been proposed to improve the performance in Bayesian optimization (Shiraiwa *et al.* 2020). In the following study, the different acquisition functions are used in Bayesian optimization. Fig. 6 shows the evaluation of the objective function with different acquisition functions. In Fig. 6, the blue dash-dot line with the point marker shows the value of the objective function with iteration when the EI method is used to select new samples. The results from LCB, PI and EIPC methods are shown in the orange solid line with a triangle-right marker, the yellow dash line with a point marker, and the dotted line with a triangle-up marker, respectively. All the results converge after 10 iterations. Among four acquisition functions, the identified unknown

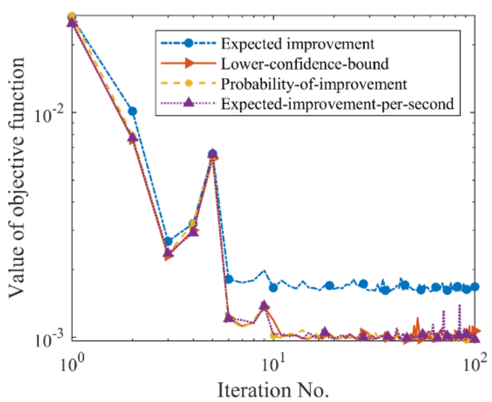


Fig. 6 Evaluation of objective function with different acquisition functions in case 1

Table 6 Performance of different acquisition functions for identification of unknown parameters

Acquisition function	MAE Value	MAPE Value
EI	0.0233	0.0271
PI	0.1213	0.1467
LCB	0.0563	0.0708
EIPC	0.1111	0.1408

parameters by Bayesian optimization with EI are closer to the actual value than the other three acquisition functions. Furthermore, the results of the MAE and MAPE for Bayesian optimization with four acquisition functions are plotted in Table 6. As shown in this figure, among the four acquisition functions, the MAE and MAPE of EI is the smallest and those of PI are the largest. This result means that among the four covariance functions, the EI acquisition function performs best in identifying the unknown parameters.

3.2 Stiffness and tension force identification with different boundary conditions

The tension force of a bridge hanger usually varies with its condition. For example, the tension force decreases when the bridge hanger is damaged. Performing the damage identification with tension force identification may be a promising task. The proposed Bayesian optimization approach is used to identify the coefficients of the element stiffness $\boldsymbol{\theta}$ and coefficients of tension force α. In Bayesian optimization, the expected improvement acquisition function is used and the candidate in the acquisition function is the same as that in case study 1.

To simulate the damage of the bridge hanger, the stiffness of elements 2 and 4 are assumed to be reduced by 10% and 20%, respectively. Furthermore, the tension force decreases from 1700 kN to 1275 kN. Thus, the actual values of unknown structural parameters are $\boldsymbol{\theta} = [1,0.9,1,0.8,1]$ and the coefficients of tension force α are 0.75. Similar to that in case 1, the hanger has pinned supports at each end. The material density, cross-section size, and length of the hanger are the same as those used in case 1.

Fig. 7 shows the identified unknown parameters with iterations. The identified result of Element 1~5 is shown with a blue dash-dot line with a point marker, an orange solid line with a triangle-right marker, a yellow dash line with a point marker, a purple dotted line with a triangle-up marker, and a green dash-dot line with a triangle-down marker, respectively. The identified result of tension force is shown with an aqua line with a triangle-left marker. The identified tension force reaches its actual value after 40 iterations and the identified coefficients of the stiffness of elements 2 and 4 take approximately 100 iterations. The changes of frequencies are more sensitive to the tension force than the local damage. The identified results of the tension force converge to the actual value faster than the identified results of the local damages. The final identified coefficients of the stiffness of the unknown element $\theta_1 \sim \theta_5$ are 0.993, 0.917, 0.982, 0.811, and 0.973. The damages (10% in element 2 and 20% in element 4) are successfully identified. The final identified coefficient of tension force α is 75.7% and the tension force decreases from 1700 kN to 1271 kN. These results show that the detection of damage and tension force can be identified from the proposed Bayesian optimization-based method for the identification of damage.

The boundary condition of a hanger is usually available because they should be determined in the construction stage. When the boundary conditions are unknown, the boundary of the hanger can be represented with a set of springs (Wang *et al*. 2014). The set of springs is used to restrain the deformation of the boundary degrees of freedom. The stiffness parameters of springs are unknown but can be identified with the proposed method. We assumed that the boundary condition of the hanger is available.

The types of boundary condition of a hanger can be pinned support, fixed support, and a combination of pinned and fixed supports at each end. In the following section, the proposed Bayesian optimization approach is carried out to identify the condition of a hanger with different supports. A hanger with fixed supports at the two ends as well as a hanger with a pinned support at the bottom and a fixed support at the top are also studied. The same unknown and input parameters for Bayesian optimization are used.

Fig. 8 shows the value of the objective function with iterations. All the results converge after 100 iterations. The final identified results of the coefficient of the element's stiffness and tension force are plotted in Table 7. Evidently,

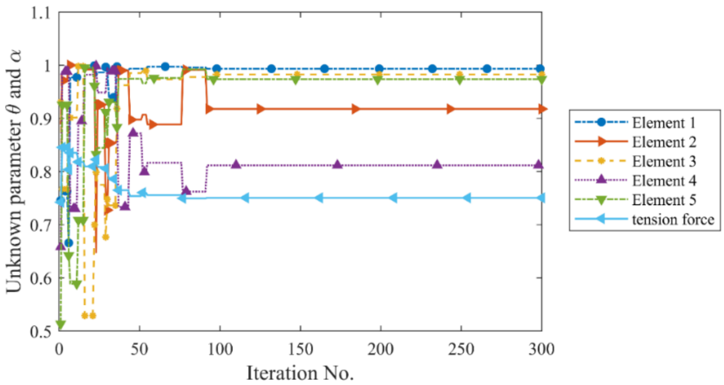


Fig. 7 Identification of unknown parameters with iterations in case 2

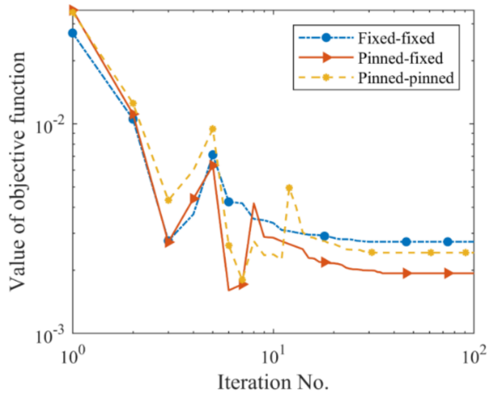


Fig. 8 Evaluation of objective function with different boundary conditions

Table 7 Identified results with different boundary conditions

Unknown parameters	Actual value	Boundary conditions		
		Pinned-pinned	Fixed-fixed	Fixed-pinned
θ_1	1	0.993	0.999	0.999
θ_2	0.9	0.917	0.886	0.867
θ_3	1	0.982	0.992	0.998
θ_4	0.8	0.811	0.82	0.83
θ_5	1	0.973	0.973	0.968
α	0.75	0.753	0.752	0.757

Table 8 Performance of different boundary conditions for identification of unknown parameters

Optimization method	MAE Value	MAPE Value
Pinned-pinned	0.083	0.0886
Fixed-fixed	0.072	0.0792
Fixed-pinned	0.105	0.1185

the performance in identifying the unknown parameters of a hanger with fixed supports at the two ends based on the Bayesian optimization is better than that of the other two types of boundary conditions. Specifically, the MAE and MAPE of parameter identification with Bayesian optimization in the study of a hanger with fixed supports at the two ends are smallest, and those of a hanger with a pinned support at the bottom and a fixed support at the top are largest, as illustrated in Table 8.

3.3 Stiffness and tension force identification with different measurement noise

The measurement noise cannot be ignored in engineering problems. Measurement noises in the frequencies are usually 1% and 2% (Xu *et al*. 2009). The measurement noise in the frequencies is added with the following equation

$$f_{i,noise} = f_i \times (1 + R * std) \tag{16}$$

where f_i is the measured i-th frequency without noise, $f_{i,noise}$ is the measured i-th frequency with noise, R is a random variable from standard normal random distribution, and std is the level of measurement noise.

The identification of damage with measurement noise is considered in studying the robustness of the proposed method. A hanger with a pinned support at each end is studied. The hanger is tested with the same procedure as that in case study 2. The measured first four frequencies are added with 1% and 2% random noise. The structural damage considered in this section is 15% stiffness reduction in element 1, 20% stiffness reduction in element 3, and 15% tension force reduction. Therefore, the actual values of unknown structural parameters are $\boldsymbol{\theta} = [0.85,1,0.8,1,1]$ and the coefficient of tension forceα is 0.85.

The proposed Bayesian optimization with the expected improvement acquisition function was conducted to identify the structural damage and tension force. The identification of the damage to the hanger with GA, PSO and NLS are also conducted. In GA, the population size is 30. The cross-over and migration fractions are 0.8 and 0.2, respectively. The maximum generation number is 500. In PSO, the swarm size and maximum generation number are 50 and 500, respectively. In NLS, the upper and lower bounds of the unknown parameters are 1 and 0.5, respectively. The final identified results of the coefficient of the stiffness of the element and the tension force are plotted in Tables 9 and 10, respectively. Furthermore, the results of the MAE and MAPE metrics for the identification of the damage with three parameter identification methods are shown in Tables 11 and 12. As illustrated in these tables, BO, GA and PSO have the same identification accuracy, but the computational efficiency of BO is better than PSO and GA. lthough, the NLS has the best computational efficiency

Table 9 Identified results with 1% measurement noise

Unknown parameters	Actual value	Results from BO	Results from GA	Results from PSO	Results from NLS
θ_1	0.85	0.855	0.858	0.856	0.996
θ_2	1	0.974	0.968	0.968	1.000
θ_3	0.8	0.781	0.766	0.768	0.702
θ_4	1	0.952	0.987	0.977	0.867
θ_5	1	0.961	0.952	0.936	1.000
α	0.85	0.851	0.853	0.854	0.861

Table 10 Identified results with 2% measurement noise

Unknown parameters	Actual value	Results from BO	Results from GA	Results from PSO	Results from NLS
θ_1	0.85	0.821	0.802	0.813	1.000
θ_2	1	0.996	0.993	0.985	0.879
θ_3	0.8	0.805	0.805	0.779	0.911
θ_4	1	0.969	0.978	0.937	0.604
θ_5	1	0.971	0.974	0.966	0.996
α	0.85	0.839	0.865	0.844	0.870

Table 11 Performance of different condition assessments with 1% measurement noise for identification of unknown parameters

Optimization method	MAE Value	MAPE Value	Computational Time/s
BO	0.138	0.1438	217.806
GA	0.138	0.1484	2214.650
PSO	0.161	0.1708	266.955
NLS	0.388	0.4402	1.009

Table 12 Performance of different condition assessments with 2% measurement noise for identification of unknown parameters

Optimization method	MAE Value	MAPE Value	Computational Time/s
BO	0.109	0.1173	227.050
GA	0.123	0.1354	2433.125
PSO	0.176	0.1888	278.312
NLS	0.802	0.860	1.297

than BO, GA and PSO, but is has the worst identification accuracy. Therefore, this result means that BO can capture a good solution in the identification of damage compared with GA, PSO and NLS.

In this study, a Bayesian optimization-based condition assessment method was proposed for bridge hangers. The measured frequencies from AMM were used as observed data for the condition assessment. The MLE of the tension force of and the severity of the damage to the hanger were detected by minimizing the differences between the measured frequencies and the frequencies calculated from the FEM method. Condition assessments with various types of boundary conditions, acquisition functions, and level of measurement noise were conducted. Based on this study, several conclusions could be drawn as follows:

- The tension force and severity of the damage can be identified based on the proposed method with the different acquisition functions (EI, PI, LCB, and EIPC), types of boundary conditions (pinned-pinned, fixed-fixed, and fixed-pinned) and levels of measurement noise (1% and 2%);
- Among the four acquisition functions, the identification of damage by Bayesian optimization with the EI acquisition function is closer to the measured data than the other three acquisition functions and the corresponding MAE and MAPE are smaller as well;
- With the different levels of measurement noise, the three parameter identification methods (BO, GA, and PSO) can also identify the structural damage and tension force, but the computational efficiency of BO is better than PSO and GA.

4. Conclusions

In this study, a Bayesian optimization-based condition assessment method was proposed for bridge hangers. The measured frequencies from AMM were used as observed data for the condition assessment. The MLE of the tension force of and the severity of the damage to the hanger were detected by minimizing the differences between the measured frequencies and the frequencies calculated from the FEM method. Condition assessments with various types of boundary conditions, acquisition functions, and level of measurement noise were conducted. Based on this study, several conclusions could be drawn as follows:

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- With the different levels of measurement noise, the three parameter identification methods (BO, GA, and PSO) can also identify the structural damage and tension force, but the computational efficiency of BO is better than PSO and GA.

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