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On bending characteristics of smart magneto-electro-piezoelectric nanobeams system

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Abstract. The content of this study focuses on bending of flexoelectric Magneto-Electro-Elastic (MEE) nanobeams inserted within the foundation of Winkler-Pasternak according to nonlocal elasticity theory. Applying Hamilton's principle, the nonlocal nanobeams' governing equations in the framework higher order refined beam theory are attained and resolved through adapting an analytical solution. A parametric research is demonstrated for studying the effects that magneto-electro-mechanical loadings, the nonlocal parameter, flexoelectric, as well as the aspect ratio all have on the deflection properties of nanobeams. A discovery lead to beam geometrical parameters, the boundary conditions, flexoelectricity and nonlocal parameter partake substantial effects on nanoscale beams' dimensionless deflection.

Keywords: piezoelectric nanobeam; bending; flexoelectric; nonlocal elasticity theory; magneto-electric

1. Introduction

In recent years, Magneto-Electro-Elastic (MEE) composite containing the piezoelectric, the prime MEE mainly utilized in 1970s and piezo-magnetic phase were discovered. There are also various methods to determine the natural properties and behavior of mentioned materials such as the differential quadrature method. It important to know the natural characteristics of the material before any initiation of application as it helps understanding each material better. In general, nano structure materials need to be handled and understood very well in order to achieve high level of precision, efficiency and optimization (Van den Boomgaard *et al*. 1974). MEE nanomaterials such as $NiFe_2O4$-PZT, $BiFeO_3$, $BiTiO_3$-$CoFe_2O_4$ and their discovery on nanostructures had become an important part of studies done by Zheng *et al*. (2004), Wang *et al*. (2008), Prashanthi *et al*. (2012) and Altabey (2017). Potentially, for this to become a major reason in order for nanostructure to be used in many more applications, each nanostructure's mechanical behavior and performance must be studied further and features must be adequately recognized prior to any novel design proposal. These further studies could also allow more development of knowledge on the existing discovered features. Overall, this could help on comprehending every aspects of each material prior to any big production and application into society. The traditional mechanic continuum theories are discovered for the purpose of predicating the behavior of structures up to a minimum size and yet the theories are found to give inaccurate predictions. In addition to that, an additional parameter is added to the modelling of the continuum which is a size parameter via the nonlocal theories. The studied models based on nonlocal elasticity theory which are utilized in many studies are can be found in Eringen (1968, 1972, 2002, 2006). Eltaher *et al*. (2013) studies have aided on the development of the models in this paper. In their study, nonlocal elasticity as well as Timoshenko beam theory were well examined therefore, according to an elastic medium, the stability behavior of SWCNT are explained. Furthermore, a study was conducted on Winkler and Pasternak parameter which aided on discovering aspect ratio of the SWCNT as well as nonlocal parameter. In additional to that, a research on nonlinear free vibration of SCNTs according to strains Eringen's nonlocal elasticity theory was conducted via Mohammadhassani *et al*. (2014a) which helped analyzing the models further. According to modified couple stress theory, an illustration of the vibration-analysis of micro composite thin beam was provided by Ehyaei and Akbarizadeh (2017).

A unified formulation for modeling inhomogeneous nonlocal beams is developed by Barati (2017). By integrating Euler Bernoulli beam model as well as nonlocal elasticity theory, free vibration evaluation of chiral double-walled carbon nanotube embedded in an elastic medium was studied by was studied by Dihaj *et al*. (2018).

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According to 3rd order beam model, buckling and vibration of piezomagnetic and piezoelectric nanobeams are successfully verified by Barati (2017). Subsequently, according to a meshless approach, Roque *et al*. (2011) studied the vibration, buckling as well as bending, free of Timoshenko nanobeams. Embedded within the nonlocal component relevance of Eringen, majorities of papers which have been published, are in search for enlarging nonlocal beam models for nano structures. To verify bending which is mentioned in investigations done by Peddieson *et al*. (2003), Wang (2005), Wang *et al*. (2008) and Yang *et al*. (2010), the proposal on nonlocal Euler Bernoulli beam theory along with Timoshenko beam theory was used which are both also widely used and accepted. A considerable amount of time was allocated to study the small-size agents in SWCNTs which were investigated via Murmu *et al*. (2009), Youcef *et al*. (2015). As a result of that, Akbas (2018) studied the wave propagation of functionally graded anisotropic nanoplates laying on Winkler-Pasternak foundation. Apart from that, Mohammadhassani *et al*. (2012) analyzed the propagation of elastic waves within thermally impacted embedded carbon-nanotube-reinforced composite beams through many different shear deformation plate theories. Another important note is that a free vibration analysis for a piezoelectric nanobeam applying the nonlocal elasticity theory is verified by Kaghazian *et al*. (2017). In addition to that, large amplitude forced vibration for functionally graded nano-composite plate with piezoelectric layers laying on nonlinear elastic foundation was investigated by Khorami *et al*. (2017). Besides that, Zenkour (2016) investigated the thermo-magneto-electro-elastic evaluation for a functionally graded nanobeam incorporated along with functionally graded piezomagnetic layers. Furthermore, Zemri *et al*. (2015) investigated the wave propagation analysis for smart strain gradient piezo-magneto-elastic nonlocal beams. Subsequently according to modified couple stress theory, the static analysis of laminated piezo-magnetic size-dependent curved beam was checked by Chemi *et al*. (2015). Moreover, Bouadi *et al*. (2018) studied the analytical solutions for magneto-electro-elastic beams. Sobhy (2013) investigated the thermal buckling analysis of single-layered graphene sheets laying on an elastic medium. Not to mention that by referring to silica aerogel foundation, bending of electro-mechanical sandwich nanoplate was examined by Besseghier *et al*. (2015). A study by Sobhy (2013) demonstrates the impact of variables on nanostructures. These variables could be for instance, thickness ratio, parameter, porosity index, plate aspect ratio, foundation characteristics and applied voltage which all affect the bending behavior of sandwich nanoplates. Additionally, Altabey (2017) presented a three unknown normal along with shear deformations whereby nonlocal beam theory for bending analysis was utilized in this problem. Consequently, a group of scientists further researched buckling and bending of the FG according to Euler Bernoulli along with nonlocal Timoshenko beam theories (Mohammadhassani *et al*. 2013a, b, c, 2014b, 2015). They described that the power-law exponent happens to have a wide impact on the behaviors from the FG nanobeam (Semmah *et al*. 2019). This paper studied the bending of MEE nanobeams according to nonlocal elasticity theory, governing equations of a nonlocal nanobeam on Winkler-Pasternak substrate are derived via Hamilton's principle. Galerkin's approach is integrated for the purpose of resolving the governing equations (Civalek and Demir 2011, Elmerabet *et al*. 2017). It is essential to take into consideration the impacts of variable factors for instance flexoelectric effect, magnet potential, length scale variable, Winkler-Pasternak variables, applied voltage and nonlocal parameter on deflection characteristics of a nanobeam are investigated (Daie *et al*. 2011, Prashanthi *et al*. 2012, Şimşek *et al*. 2013, Tounsi *et al*. 2013, Kheroubi *et al*. 2016, Karami *et al*. 2017, Kadari *et al*. 2018, Ghorbanpour Arani and Zamani 2019, Shariati *et al*. 2020c).

2. Theory and formulation

The refined shear deformable beam's displacement field may be defined through

$$u_x(x,z) = u(x) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{1}$$

$$u_z(x,z) = w_b(x) + w_s(x) \tag{2}$$

where u happens to be the axial mide-plane displacement while w_b and w_s signify the bending as well as shear components for transverse displacement, respectively. Additionally, $f(z)$ is the shape function which represents shear stress/strain distribution via thickness of beam that in turn, has a trigonometric essence for the given study, hence why it is not necessary for a shear correction factor (Shariati *et al*. 2019, 2020a, b).

$$f(z) = z + h_0 - \tan[m(z+h_0)], \qquad m = 0.03 \tag{3}$$

The suggested beam model's non-zero strains can be articulated as such

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2} \tag{4}$$

$$\gamma_{xz} = \mathrm{g}(z)\frac{\partial w_s}{\partial x} \tag{5}$$

where $g(z) = 1 - df(z)/dz$.

According to Maxwell's equation, a relation amongst the electric field (Ex, Ez) as well as the electric potential (ϕ) and magnet field (Qx, Qz) along with the magnet potential (ψ), may get obtained as Ke *et al*. (2014).

$$E_x = -\phi_{,x} = \cos(\xi z)\frac{\partial \phi}{\partial x} \quad Q_x = -\psi_{,x} = \cos(\xi z)\frac{\partial \psi}{\partial x} \tag{6}$$

$$\begin{gathered} E_z = -\phi_z = \xi\sin(\xi z) - \frac{2v}{h} \\ Q_z = -\psi_z = \xi\sin(\xi z) - \frac{2v}{h} \end{gathered} \tag{7}$$

whereby $\xi = \pi/h$. Additionally, V is the external electric that is being pertained on the nanobeam.

Through the extended Hamilton's principle, the governing equations may be developed by

$$\int_0^t \delta(\Pi_S - \Pi_W)\mathrm{d}t = 0 \tag{8}$$

wherein Π_S is identified as the total strain energy, Π_W happens to be the work done through external applied forces. To calculate the first variation of strain energy Π_S, the following is applied

$$\delta\Pi_S = \int \sigma_{ij}\delta\varepsilon_{ij}dv = \int \sigma_x\delta\varepsilon_x + \sigma_{xz}\delta\gamma_{xz} \tag{9}$$

Substituting Eqs. (1)-(2) into Eq. (6) yields

$$\begin{aligned}\Pi\mathrm{s} = &\int_0^a (N\frac{\partial\delta u}{\partial x} - M_b\frac{\partial^2\delta w_b}{\partial x^2} - M_s\frac{\partial^2\delta w_s}{\partial x^2} + Q\frac{\partial\delta w_s}{\partial x})dx\\ &\int_0^a\int_{-h/2}^{h/2}\Big[-D_x\cos(\xi z)\frac{\partial\delta\phi}{\partial x} + D_z\,\xi\sin(\xi z)\,\delta\phi - B_x\\ &\cos(\xi z)\frac{\partial\delta\psi}{\partial x} + B_z\xi\sin(\xi z)\,\delta\varphi\psi\Big]dxdz\end{aligned} \tag{10}$$

whereby the moments and forces shown in the equation above are expressed as such

$$\begin{aligned}(N, M_b, M_s) &= \int_A (1, z, f)\sigma_i dA\\ i &= (x, y, xy)\\ Q_i &= \int_A \mathrm{g}\sigma_i dA\ , i = (xz, yz)\end{aligned} \tag{11}$$

The work done from the applied forces, through the first variation, may be expressed as

$$\begin{aligned}\delta\Pi_w = \int_0^l [(-N_x^0)\frac{\partial w}{\partial x}\frac{\partial\delta w}{\delta x} - k_w\delta(w^b + w^s)\\ + k_p\frac{\partial^2(w_{b+}w_s)}{\partial x^2} + f_{13}\delta(w^b + w^s)]dx\end{aligned} \tag{12}$$

where k_w, k_p happen to be the linear, shear coefficients for medium, N^E, N^B electric and magnet, loading respectively.

$N_x^0 = N^E + N^B$,

$$N^E = -\int_{-h/2}^{h/2} e_{31}\frac{2V}{h}\mathrm{d}z \tag{13}$$

$$N^B = -\int_{-h/2}^{h/2} e_{31}\frac{2\Omega}{h}\mathrm{d}z \tag{14}$$

kw, *kp* and *cd* are in fact Winkler, Pasternak as well as damping constants.

The Euler-Lagrange below are the equations which were gathered by implementing Eqs. (10)-(13) in Eq. (8) wherein the coefficients for ∂u, ∂w^b, ∂w^s, $\emptyset$, ψ are equal to zero.

$$\frac{\partial N_x}{\partial x} = 0 \tag{15}$$

$$\begin{aligned}\frac{\partial^2 M_b}{\partial x^2} + (-N^E - N^B)\nabla^2(w^b + w^s)\\ -k_p\nabla^2(w^b + w^s) + k_w(w^b + w^s) = 0\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial^2 M_s}{\partial x^2} - \frac{\partial Q}{\partial x} + (-N^E - N^B)\nabla^2(w^b + w^s) - k_p\nabla^2\\ (w^b + w^s) + k_w(w^b + w^s) + f_{13}\delta(w^b + w^s) = 0\end{aligned} \tag{17}$$

$$\int_{-h/2}^{h/2}\Big[\cos(\xi z)\frac{\partial D_x}{\partial x} + \xi\sin(\xi z)\,D_z\Big]\mathrm{d}z = 0 \tag{18}$$

$$\int_{-h/2}^{h/2}\Big[B_x\cos(\xi z)\frac{\partial\delta\psi}{\partial x} + B_z\xi\sin(\xi z)\,\delta\psi\Big]\mathrm{d}z = 0 \tag{19}$$

2.1 Nonlocal elasticity theory

The nonlocal theory may get extended for the piezoelectric nanobeams by

$$\sigma_{ij-}(ea)^2\nabla^2\sigma_{ij} = \left[c_{ijkl}\varepsilon_{kl} - e_{mij}E_m - q_{nij}H_n\ \right] \tag{20}$$

$$D_{ij-}(ea)^2\nabla^2 D_{ij} = [e_{ikl}\varepsilon_{kl} + k_{im}\,E_m + d_{in}H_n] \tag{21}$$

$$B_{i-}(ea)^2\nabla^2 B_i = [q_{ikl}\varepsilon_{kl} + d_{im}\,E_m + \varkappa_{in}H_n] \tag{22}$$

$$p_{i-}(ea)^2\nabla^2 p_i = \left[\varepsilon_0\chi_{ij}E_j + e_{ikl}\varepsilon_{kl}\ \right] \tag{23}$$

Also, χ_{ij} happens to be the relative dielectric susceptibility while f_{ijkl} is the flexoelectric coefficient. Furthermore, $e_0 a$ is nonlocal parameter that has been introduced for the purpose of depicting the size-dependency of nanostructures.

Where ∇^2 is the Laplacian operator, the stress relations may be expressed through

$$(1 - \mu\nabla^2)\sigma_{xx} = (1 - \lambda^2\nabla^2)[C_{11}\varepsilon_{xx} - e_{31}\,E_z - q_{31}H_z] \tag{24}$$

$$(1 - \mu\nabla^2)\sigma_{xz} = (1 - \lambda^2\nabla^2)[C_{55}\gamma_{xz} - e_{15}\,E_x - q_{15}H_x] \tag{25}$$

$$(1 - \mu\nabla^2)D_x = (1 - \lambda^2\nabla^2)[e_{15}\,\gamma_{xz} + k_{11}E_x + d_{11}H_x] \tag{26}$$

$$(1 - \mu\nabla^2)D_z = (1 - \lambda^2\nabla^2)[e_{31}\varepsilon_{xx} + k_{33}E_z + d_{33}H_z] \tag{27}$$

$$(1 - \mu\nabla^2)B_x = (1 - \lambda^2\nabla^2)[q_{15}\gamma_{xz} + d_{11}E_x + \varkappa_{11}H_x] \tag{28}$$

$$(1 - \mu\nabla^2)B_z = (1 - \lambda^2\nabla^2)[q_{31}\varepsilon_{xx} + d_{33}E_z + \varkappa_{33}H_x] \tag{29}$$

Integrating Eq. (21) through the cross-section area of nanobeam gives the following nonlocal relations for a refined beam model by: Additionally, normal forces alongside moments because of electrical field may be defined through Hakim *et al.* (2011), Mohammadhassani *et al.* (2014c), Toghroli *et al.* (2014), Aghakhani *et al.* (2015)

and Shahabi *et al.* (2016)

$$M_x = \int_{-h/2}^{h/2} \left(e_{31}\frac{2V}{h} + q_{31}\frac{2V}{h} \right) Z dz \tag{30}$$

$$\int_{-h/2}^{h/2} (1-\mu\nabla^2)\,\{D_x\}\cos(\xi z)\,dz = F_{11}^{e}\left\{\frac{\partial \phi}{\partial x}\right\} \tag{31}$$

$$\int_{-h/2}^{h/2} (1-\mu\nabla^2)\,\{B_x\}\cos(\xi z)\,dz = R_{11}^{e}\left\{\frac{\partial \psi}{\partial x}\right\} \tag{32}$$

$$(1-\mu\nabla^2)\int_{-h/2}^{h/2} \xi\sin(\xi z)\,D_z dz = [A_{31}\left(\frac{\partial u}{\partial x}\right) - E_{31}\nabla^2 w] \tag{33}$$

$$(1-\mu\nabla^2)\int_{-h/2}^{h/2} \xi\sin(\xi z)\,B_z dz = \left[G_{31}\left(\frac{\partial u}{\partial x}\right) - Q_{31}\nabla^2 w\right] \tag{34}$$

in which

$$\begin{aligned}&\{A_{11}, B_{11}, B_{11}^{s}, D_{11}, D_{11}^{s}, H_{11}^{s}\}\\ &= \int_{-\frac{h}{2}}^{\frac{h}{2}} c11\,(1, z, f, z^2, fz, f^2)dz\end{aligned} \tag{35}$$

$$(A_{31}, E_{31}) = \int_{-h/2}^{h/2} e_{31}\xi\sin(\xi z)\,\{1, z, f\}dz \tag{36}$$

$$(F_{11}, F_{33}) = \int_{-h/2}^{h/2} \{s_{11}\,cos^2(\xi z)\,, s_{33}\xi^2\,sin^2(\xi z)\}dz \tag{37}$$

$$(R_{11}, R_{33}) = \int_{-h/2}^{h/2} \{d_{11}\,cos^2(\xi z)\,, d_{33}\xi^2\,sin^2(\xi z)\}dz \tag{38}$$

$$(G_{31}, Q_{31}) = \int_{-h/2}^{h/2} q_{31}\xi\sin(\xi z)\,\{1, z, f^2\}dz \tag{39}$$

$$(\varkappa_{11}, \varkappa_{33}) = \int_{-h/2}^{h/2} \{\varkappa_{11}\,cos^2(\xi z)\,, \varkappa_{33}\xi^2\,sin^2(\xi z)\}dz \tag{40}$$

The governing equations for the nonlocal strain gradient nanoplate under electrical field in terms of the displacement may be derived through replacing Eqs. (30)-(34) into Eqs. (15)-(18) as such Shariati (2014).

$$[A_{11}\frac{\partial^2 u}{\partial x^2} - B_{11}\frac{\partial^3 w^b}{\partial x^3} - B_{11}^{s}\frac{\partial^3 w^s}{\partial x^3} + A_{31}\frac{\partial \phi}{\partial x} + G_{31}\frac{\partial \psi}{\partial x}] = \frac{\partial N_x}{\partial x} \tag{41}$$

$$\begin{aligned}&\Big[[B_{11}\frac{\partial^3 u}{\partial x^3} - D_{11}\frac{\partial^4 w^b}{\partial x^4} - D_{s}^{11}\frac{\partial^4 w^s}{\partial x^4}] + E_{31}\frac{\partial^2 \phi}{\partial x^2} + Q_{31}\frac{\partial^2 \psi}{\partial x^2}\\ &+ k_p\nabla^2(w^b + w^s) - k_w(w^b + w^s) - \mu[(-N^E - N^B)\\ &+ k_p\nabla^2(w^b + w^s) - k_w(w^b + w^s)]\Big] = \frac{\partial^2 M_b}{\partial x^2}\end{aligned} \tag{42}$$

$$\begin{aligned}&[A_{31}\frac{\partial u}{\partial x} - E_{31}\frac{\partial^2 w^b}{\partial x^2} - Q_{31}\frac{\partial^2 w^s}{\partial x^2} + F_{11}\frac{\partial^2 \phi}{\partial x^2} + R_{11}\frac{\partial^2 \psi}{\partial x^2}\\ &-F_{33}\phi - R_{33}\psi = 0\end{aligned} \tag{43}$$

$$\begin{aligned}&[A_{31}\frac{\partial u}{\partial x} - E_{31}\frac{\partial^2 w^b}{\partial x^2} - Q_{31}\frac{\partial^2 w^s}{\partial x^2} + F_{11}\frac{\partial^2 \phi}{\partial x^2} + \varkappa_{11}\frac{\partial^2 \psi}{\partial x^2}\\ &-F_{33}\phi - \varkappa_{33}\psi = 0\end{aligned} \tag{44}$$

3. Solution procedure

The displacement quantities are actually portrayed in a certain way, for the purpose of satisfying the aforementioned boundary conditions. With that in mind, the displacement quantities are displayed as such

$$u = \sum_{n=1}^{\infty} U_n \frac{\partial X_n(x)}{\partial x} e^{i\omega_n t} \tag{46}$$

$$w_b = \sum_{n=1}^{\infty} W_{bn}\,X_n(x)e^{i\omega_n t} \tag{47}$$

$$w_s = \sum_{n=1}^{\infty} W_{sn}\,X_n(x)e^{i\omega_n t} \tag{48}$$

$$\emptyset = \sum_{n=1}^{\infty} \emptyset_n\,X_n(x)e^{i\omega_n t} \tag{49}$$

$$\psi = \sum_{n=1}^{\infty} \psi_n\,X_n(x)e^{i\omega_n t} \tag{50}$$

where U_{mn}, W_{bmn}, W_{zmn}, $\emptyset$, ψ happen to be the unknown coefficients for the different boundary conditions ($\alpha = m\pi/a,\ \beta = n\pi/b$)

$$[K]\begin{Bmatrix} u_n \\ w_{bn} \\ w_{sn} \\ \phi \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q_n(1 + \mu\frac{n^2\pi^2}{L^2}\ ) \\ Q_n\left(1 + \mu\frac{n^2\pi^2}{L^2}\right) \\ 0 \\ 0 \end{Bmatrix} \tag{51}$$

where [*K*], [*F*] get identified as the stiffness, loading matrixes for nanobeam, respectively.

$$\begin{aligned}&K_{1,1} = A_{11}\alpha_1, \quad K_{1,2} = B_{11}\alpha_2, \quad K_{1,3} = B_{11}^{s}\alpha_2\\ &K_{1,4} = A_{31}\alpha_3, \quad K_{1,5} = G_{31}\alpha_3\end{aligned} \tag{52}$$

$$\begin{aligned}&K_{2,1} = B_{11}\alpha_{11}\\ &K_{2,2} = -D_{11}\alpha_7 + k_w\alpha_5 - k_p\alpha_6\\ &\qquad + \mu\big[\big(-N^E - N^B + k_p\big)\alpha_6 - k_w\alpha_5]\\ &K_{2,3} = D_{11}^{s}\alpha_7 + k_w\alpha_5 - k_p\alpha_6\\ &\qquad + \mu\big[\big(-N^E - N^B + k_p\big)\,\alpha_6 - k_w\alpha_5]\\ &K_{2,4} = E_{31}\alpha_6, \quad K_{2,5} = Q_{31}\alpha_6, \quad K_{3,1} = B_{11}^{s}\alpha_{11}\\ &K_{3,2} = -D_{11}^{s}\alpha_7 + \mu[(-N^E - N^B + k_p)\alpha_6 - k_w\alpha_5]\\ &K_{3,3} = -H_{11}^{s}\alpha_7 + k_w\alpha_5 - k_p\alpha_6 + \mu\big[\big(-N^E - N^B + k_p\big)\\ &\qquad \alpha_6 - (k_w + \frac{e_{31}}{2k_{33}})f_{13}\alpha_5]\end{aligned} \tag{53}$$

$$
\begin{aligned}
&K_{3,4} = F_{11}\alpha_6, \quad K_{3,5} = R_{11}\alpha_6, \quad K_{4,1} = A_{31}\alpha_3\\
&K_{4,2} = -E_{31}\alpha_6, \quad K = -Q_{31}\alpha_6\\
&K_{4,4} = F_{11}\alpha_6 - F_{33}\alpha_5, \quad K_{4,5} = R_{11}\alpha_6 - R_{33}\alpha_5\\
&K_{5,1} = A_{31}\alpha_3, \quad K_{5,2} = -E_{31}\alpha_6\ , \quad 5 = -Q_{31}\alpha_6\\
&K_{4,4} = F_{11}\alpha_6 - F_{33}\alpha_5, \quad K_{4,5} = \varkappa_{11}\alpha_6 - \varkappa_{33}\alpha_5\\
&F_{1,1} = N\alpha_3, \quad F_{2,2} = M_b(1-\mu\alpha_6)\\
&F_{3,3} = M_s(1-\mu\alpha_6) - Q(1-\mu\alpha_3)
\end{aligned}
$$

in which

$$
\begin{aligned}
&\alpha_1 = \int_0^a X'(x)X''(x)\,dx, \quad \alpha_2 = \int_0^a X(x)X'''(x)\,dx\\
&\alpha_7 = \int_0^a X(x)X''''(x)\,dx, \quad \alpha_5 = \int_0^a X(x)\,X(x)dx\\
&\alpha_3 = \int_0^a X(x)\,X'(x)dx, \quad \alpha_{11} = \int_0^a X'(x)X'''(x)\,dx \qquad (54)\\
&\alpha_6 = \int_0^a X(x)X''(x)\,dx
\end{aligned}
$$

The uniform load is actually supposed how leading to bending and consequently is expressed by the form below

$$q_{dynamics} = \sum_{n=1}^{\infty} Q_n \sin\left[\frac{n\pi}{L}x\right] \sin\omega t \qquad (55)$$

$$Q_n = \frac{2}{L}\int_{x_0-c}^{x_0+c} \sin\left[\frac{n\pi}{L}x\right] q_x dx \qquad (56)$$

where Q_n is the Fourier coefficients while $q(x) = q_0$ is the uniform load density while x_0 is the centroid coordinate. Additionally, in the instance of concentrated point load the expression for the harmonic load intensity may get expressed as shown below

$$q(x) = p\delta(x - x_0)\sin\omega t \qquad (57)$$

$$Q_n = \frac{2p}{L}\sin\left[\frac{n\pi}{L}x_0\right] \qquad (58)$$

in which δ is the Dirac delta.

4. Numerical results and discussions

This particular portion of the paper analyzes the bending of piezoelectric nanobeams. Table 2 displays the material properties (Ramirez *et al*. 2006), the study's accuracy, is validated for contrasting the bending of this particular model with that of Arefi (2019). Arefi *et al*. (2016) for numerous nonlocal parameters which are portrayed in Table 3. The nanobeam's length is presumed as L = 10 nm. Additionally, the dimensionless deflection is taken into account through.

$$W = 100\frac{C_{11}I}{q_0 L^4} \qquad (59)$$

Fig. 2 investigates the nonlocal parameter effects versus various magnet potential. While doing so, it was discovered how increasing the value of magnet potential results

Table 1 The admissible functions $X_m(x)$ Sobhy (2013)

	Boundary conditions	The functions X_m
	At $x = 0,\ a$	$X_m(x)$
SS	$X_m(0) = X''_m(0) = 0$	$Sin(\alpha x)$
	$X_m(a) = X''_m(a) = 0$	

Table 2 $BiTiO_3$-$CoFe_2O_4$ composite materials' material properties

Properties	$BiTiO_3$-$CoFe_2O_4$
Elastic (GPa)	c_{11} = 226, c_{12} = 125, c_{13} = 124, c_{33} = 216, c_{44} = 44.2, c_{66} = 50.5
Piezoelectric ($C \times m^{-2}$)	e_{31} = −2.2, e_{33} = 9.3, e_{15} = 5.8
Dielectric ($10^{-9}\ C \times V^{-1} \times m^{-1}$)	k_{11} = 5.64, k_{33} = 6.35
Piezomagnetic ($N \times A^{-1} \times m^{-1}$)	q_{15} = 275, q_{31} = 290.1, q_{33} = 349.9
Magnetoelectric ($10^{-12}\ Ns \times V^{-1} \times C^{-1}$)	s_{11} = 5.367, s_{33} = 2737.5
Magnetic ($10^{-6}\ Ns^2c^{-2}/2$)	$\varkappa_{11}$ = −297, $\varkappa_{33}$= 83.5
Mass density (10^3 Kg/m^3)	ρ = 5.55

Table 3 Comparing nanobeam's dimensionless deflections for magnetic potential and electric voltage

L/h	μ (nm^2)	$\psi = 0.001$	
10		Arefi (2019)	Present
	1	3.68	3.5781
	2	3.71	3.6482
	3	3.77	3.7302
	4	3.84	3.79011
	5	3.94	3.8952
L/h	μ (nm^2)	$\phi = 0.001$	
10		Arefi and Zenkour (2016)	Present
	0	3.68	3.59892
	1	1.3333	3.66921
	2	1.3645	3.74018
	3	1.3958	3.80234
	4	1.4270	3.92011

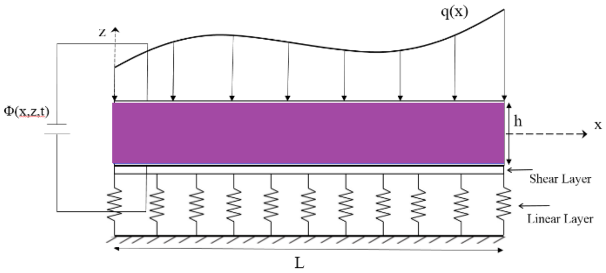


Fig. 1 Geometry of nanobeam laying on an elastic foundation

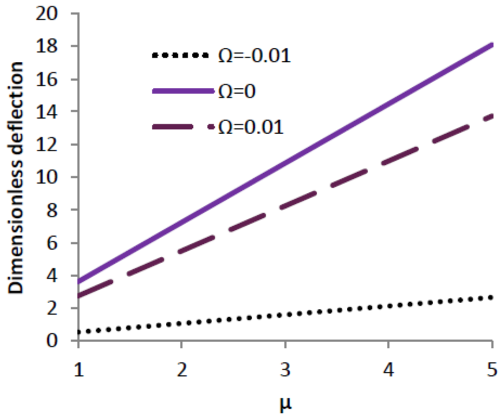
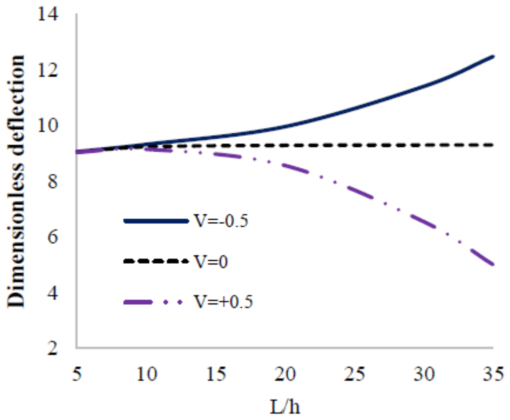
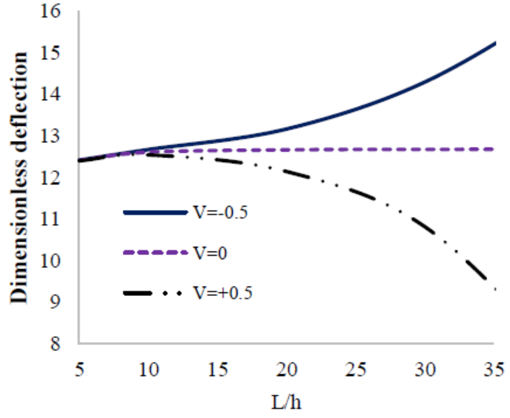


Fig. 2 Effects of nonlocal parameters upon the dimensionless deflection for the uniform load of numerous magnet potential parameters that are affected through flexoelectric (L/h = 10, V = 0, K_w = K_p = 20)

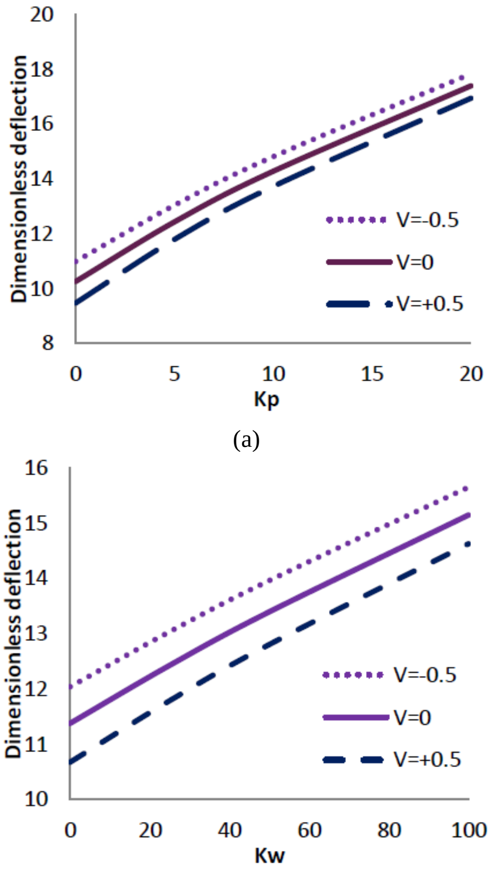


(a) $K_w = K_p = 0$



(b) $K_w = 25,\ K_p = 5$

Fig. 3 Effect of slenderness ratio versus dimensionless deflection for uniform load and numerous electric voltage with voltage without elastic foundation and with elastic foundation affected by flexoelectric (L/h = 10, Ω = 0)



(a)

(b)

Fig. 4 Effect of the Pasternak and Winkler foundation versus dimensionless deflection for uniform load and numerous electric voltage affected through flexoelectric (L/h = 10, $\mu = 2$, $\Omega = 0$)

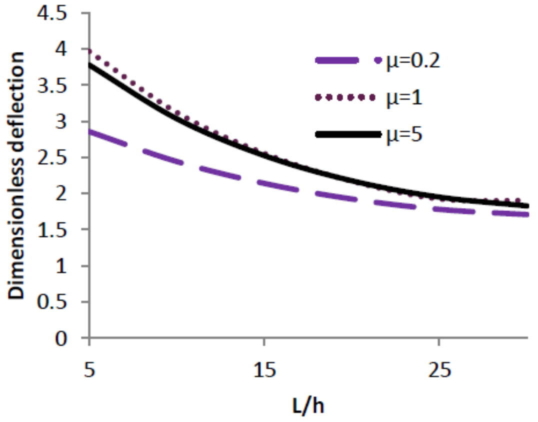


Fig. 5 Effect of the aspect ratio versus of dimensionless deflection for uniform load and numerous nonlocal parameter affected by flexoelectric for (L/h = 10, $\Omega = V = 0$, K_p = K_w = 20)

increase of dimensionless deflection so when nonlocal parameter arises the magnitude of deflection increase, and we understand from this subject that magnet potential plays a very important role when it comes to deflection.

Dimensionless deflection nanobeam with regards to slenderness ratio and nonlocal scale parameter $\mu = 2$ presented in Fig 3. It discovered; external electric voltage caused that increasing of bending of nanobeam and external voltage for negative values of nanobeam demonstrated a lessening effect for positive voltages. So, the axial tensile as well as the compressive forces accumulated in the nanobeams through the created positive along with negative voltages, respectively. In addition, it has been seen lightly, how the dimensionless deflection is approximately independent of slenderness ratio for zero electric voltages ($V = 0$) (Castrucci 2014).

The dimensionless deflection for nanobeams' variation in comparison to the Winkler and Pasternak parameters for numerous electric voltages as well as nonlocal parameters at L/h = 10 are shown on Fig. 4, respectively. It is found through this given figure how, regardless of the magnitude and sign of electrical voltage, the dimensionless deflection increases when there is an upsurge of Winkler and Pasternak parameters, So the increment in stiffens of the nanobeam. It must be mentioned how a constant electric voltage the increase of dimensionless defection with Pasternak parameter measurement alongside a higher rate than that from the Winkler parameter.

Fig. 5 presents the deflection variation for nanobeams with the aspect ratio. For this example, the aspect ratio varies between L/h = 10 to L/h = 30. These results indicate how the effect of aspect ratio softens the nanobeam. The results obtained from these statistics show how the maximum deflection rises when the nanobeams gradient index is increased as well. The findings gathered, clearly show how indicate how increasing the μ, increased the maximum deflection greatly.

5. Conclusions

This article studies the deflection of MEE nanobeams. Hamilton's principle was integrated to gain the nonlocal nanobeams' governing equations, based primarily on higher order refined beam theory. They were then solved through an analytical solution. A parametric study is given for investigating the nonlocal parameter's effects, along with aspect ratio and the magneto-electro-mechanical loadings on the deflection properties of nanobeams. The important discovery is the effects under dimensionless deflection.

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