Smart Structures and Systems, *Vol. 30, No. 1 (2022) 1-15*
https://doi.org/10.12989/sss.2022.30.1.001

NNDI decentralized evolved intelligent stabilization of large-scale systems

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(*Received May 22, 2021, Revised October 19, 2021, Accepted March 2, 2022*)

Abstract. This article focuses on stability analysis and fuzzy controller synthesis for large neural network (NN) systems consisting of several interconnected subsystems represented by the NN model. Advanced and fuzzy NN differential inclusion (NNDI) for stability based on the developed algorithm with H infinity can be designed based on the evolved biological design. This representation is constructed using sector linearity for NN models. Sector linearity transforms a non-linear model into a linear model based on proposed operations. New sufficient conditions are realized in the form of LMI (linear matrix inequalities) to ensure the asymptotic stability of the trans-Lyapunov function. This transforms the nonlinear model into a linear model based on multiple rules. At last, a numerical case study with simulations is derived as illustration to prove its feasibility in real nonlinear structures.

Keywords: intelligent algorithm; large-scale system; neural fuzzy control; numerical modelling

1. Introduction

The numerical models in physical and design frameworks are usually highly measurable or have computational power. However, data preparation is a prerequisite when applying these models for different purposes which can require quite large computational resources. Therefore, you need to find a strategy that can reduce the computing resources required. Large frameworks allow these methods to control the program in some way. Much work has been done in demonstrating, researching, improving, and controlling large frameworks. Several methods have currently been developed to check the robustness and suitability of large scale or nonlinear systems.

For example, many provided approaches for the application in computer science, engineering, system, nonlinear structures, and etc. (Casciati 1997, Bontempi *et al*. 2003, Battaini *et al*. 2004, Xiao *et al*. 2020, Xu *et al*. 2020, 2021, Sun *et al*. 2021, Zhang *et al*. 2021a, b, Zhou *et al*. 2021a, b, c, d, Kong *et al*. 2021, Hu *et al*. 2021, Guo *et al*. 2021a, b, Du *et al*. 2021, Li *et al*. 2021a, b, c, Tian *et al*. 2021, Liu *et al*. 2021a, b, c, d, e, Lei *et al*. 2021, Wu *et al*. 2021, Sheng *et al*. 2022, Lv *et al*. 2022, Liu *et al*. 2022, Cao *et al*. 2021a, b, 2022, Mou *et al*. 2022, Cai *et al*. 2022, Zhang *et al*. 2022a, b, Zhang *et al*. 2022a, b, Zhong *et al*. 2022, Zheng *et al*. 2022, Li *et al*. 2022, Du *et al*. 2022, Xie *et al*. 2022, Yin *et al*. 2022a, b, Shang *et al*. 2022a, b, Qi *et al*. 2022). The novel merits of these methods for unraveling complex nonlinear frameworks have led to their use for recognizable proof and control and complex computing issues (Eswaran and Reddy 2016, Hu *et al*. 2018, Berg *et al*. 2019, Ying *et al*. 2019, Lee and Juang 2012, Battista and Varela 2019, Chen *et al*. 2019, Zhen *et al*. 2021, Zhao *et al*. 2021a, b, Li *et al*. 2021a, b, c, Shang *et al*. 2022a, b, Zhong *et al*. 2022). With the wide usage of frequency domain models, the problem of signal-power integrity in mixed-domain transient simulations has been solved. However, it is still difficult to verify the robustness of large nonlinear frames using model-based NN control. The framework has not been thoroughly reviewed. With this in mind, the NNDI (nerual network differential inclusion) represented in this study was used to check the robustness of large nonlinear frameworks. In this article, we will consider a large NN framework consisting of multiple interconnected subsystems. The basic feature of the control frame is its strength. There are reports on the reliability of dynamic neural network frameworks (see Panteley and Loria 1998, Sontag and Wang 1995, Sontag 1988, Tsai *et al*. 2012a, b, c, 2015, Chen *et al*. 2019, 2020a, b, c, Chen 2011a, b, 2014a, b, Adeli and Jiang 2006, Zhou *et al*. 2015, and Resources). Literature studies show that the issue of adaptation to large-scale neural network frameworks remains unclear. Therefore, to ensure the reliability of the large asymptotic NN framework, the power norm associated with the Lyapunov strategy is derived. In this regard, based on a distributed control scheme, several discreet controllers from Takagi Sugeno (TS) have created a balanced large-scale NN framework consisting of interconnected subsystems, which can be seen as an extension of the method used to study the robustness of large-scale NN frameworks. The Lyapunov function provides new and sufficient conditions in the form

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http://www.techno-press.org/?journal=sss&subpage=7
ISSN: 1738-1584 (Print), 1738-1991 (Online)

of LMIs to ensure the near-stability of control fuzzy systems. This representation is constructed using sector non-linearity. This transforms the nonlinear model into a linear model based on multiple rules (see Loria and Nesic 2003, Chen *et al*. 2004, 2022a, b, c, d, Cheng 2021, Hsiao *et al*. 2005a, b, Chiang *et al*. 2007a, b, Chen 2009a, b, Chen *et al*. 2009a, b, Hung *et al*. 2019 and Resources). The remain parts of this document is organized in the following: First, a brief review at the NN model and the framework is presented. According to the Lyapunov method, the reliability paradigm is defined to ensure the asymptotic strength of the large NN framework. At last, we gave an example to explain the results and draw some conclusions.

From the properties of NNDI, the procedure of the conversion of the *l*th isolated NN subsystem (1) is given based on the assumptions $g_2 v \leq T(v) \leq g_1 v,\ v < 0$ and $g_1 v \leq T(v) \leq g_2 v,\ v \geq 0$.

Subsequently, the min-max matrix $G(v^e, \Psi^e)$ is defined as follows

$$G(v^e, \Psi^e) = diag\left(g(T_\varsigma)\right), \qquad \varsigma = 1, 2, \cdots, R^e.$$

Moreover, on the basis of interpolation method and Eq. (1), we could obtain

$$\begin{aligned} x_l(k+1) = &\left[\sum_{q_1^{S_l}=1}^{2} \cdots \sum_{q_{R^{S_l}}^{S_l}=1}^{2} \hbar_{q_1^{S_l}}^{S_l}(k) \cdots \hbar_{q_{R^{S_l}}^{S_l}}^{S_l}(k) G(v^{S_l}, \Psi^{S_l}) \right. \\ &\left. \cdot \left(W^{S_l} \left[\cdots \left[\sum_{q_1^2=1}^{2} \cdots \sum_{q_{R^2}^2=1}^{2} \hbar_{q_1^2}^{2}(k) \cdots \hbar_{q_{R^2}^2}^{2}(k)\, G(v^2, \Psi^2) \left(W^2 \left[\sum_{q_1^1=1}^{2} \cdots \sum_{q_{R^1}^1=1}^{2} \hbar_{q_1^1}^{1}(k) \cdots \hbar_{q_{R^1}^1}^{1}(k) G(v^1, \Psi^1) \left(W^1 Z_l(k) \right) \right] \right) \right] \cdots \right] \right) \right] \\ = &\sum_{v^{S_l}} \cdots \sum_{v^1} \hbar_{v^{S_l}}^{S_l}(k) \cdots \hbar_{v^1}^{1}(k) G(v^{S_l}, \Psi^{S_l}) W^{S_l} \cdots G(v^1, \Psi^1) W^1 Z_l(k) = \sum_{v_l} \hbar_{v_l} H_{v_l}(W, \Psi) Z_l(k) \end{aligned}$$

2. System description

Consider a multiple neural-network (NN) system $\boldsymbol{N}$ consisting of L interconnected subsystems where the *l*th isolated framework with R^e $(e = 1, 2, \cdots, S_l)$ neurons for every layer with inputs $u_l(k) \sim u_l(k-q+1)$ and state variables $x_l(k) \sim x_l(k-p+1)$.

In this way, the weight lattice for the eth layer is composed as W^e, and the transfer function vector of the *eth* layer could be derived as

$$\Psi^e(v) \equiv [T_1(v) \quad T_2(v) \quad \cdots \quad T_{R^e}(v)]^T, \\ e = 1, 2, \cdots, S_l$$

where $T_\varsigma(v)$ $(\varsigma = 1, 2, \cdots, R^e)$ are the transfer functions associated with $\Psi^e(v)$. Hence the last output of the *l*th isolated NN subsystem could be inferred based on the NNDI framework portrayal as follows

$$x_l(k+1) = \Psi^{S_l}\left(W^{S_l} \Psi^{S_l - 1} \left(W^{S_l - 1} \Psi^{S_l - 2} \left(\cdots\cdots \cdot \Psi^2 \left(W^2 \Psi^1 \left(W^1 Z_l(k) \right) \right) \cdots\cdots \right) \right) \right), \tag{1}$$

$$y(k+1) = A\big(a(k)\big) y(k), \\ A\big(a(k)\big) = \sum_{i=1}^{r_l} h_i\big(a(k)\big) A_i, \tag{2}$$

where $\sum_{i=1}^{r_l} h_i(a(k)) = 1$, $h_i(a(k)) \geq 0$ and positive integer r_l and $a(k)$ of $y(k) = [y_1(k) \quad y_2(k) \quad \cdots \quad y_n(k)]^T$.

where $H_{v_l}(W, \Psi) \equiv G(v^{S_l}, \Psi^{S_l}) W^{S_l} \cdots G(v^1, \Psi^1) W^1$, $\sum_{v^e} h_{v^e}^e = \sum_{q_1^e=1}^{2} \cdots \sum_{q_{R^e}^e=1}^{2} h_{q_1^e}^e \cdots h_{q_{R^e}^e}^e$, $h_1^e, h_2^e \in [0, \quad 1]$, $\sum_{q_\varsigma^e=1}^{2} h_{q_\varsigma^e}^e = 1$ under $h_{v_l} \geq 0$.

Herein, we rewrite the following NNDI representation

$$X_l(k+1) = \sum_{i=1}^{r_l} h_{il}(k)\{A_{il} X_l(k) + B_{il} U_l(k)\}$$

where $U_l^T(k) = [u_l(k) \quad \cdots \quad u_l(k-q+1)]$, $X_l^T(k) = [x_l(k) \quad \cdots \quad x_l(k-p+1)]$ and H_{il} is a constant matrix.

3. NNDI fuzzy controllers with EBA

As shown in the system, the design has been extended with N combined with a large number of TS controls to set NN. By specifying the fuzzy control system design procedure in Fig. 1, distributed control schemes use parallel distributed compensation (PDC) technology to synthesize groups of fuzzy controls to stabilize the system. The concept of the PDC system is that each rule is a distributed design of the corresponding rule of the T-S type model.

The *l*th fuzzy controller is in the following form

$$\text{Rule } j\text{: IF } x_{1l}(k) \text{ is } M_{j1l} \text{ and } \cdots \text{ and } x_{pl}(k) \text{ is } M_{jpl} \\ \text{THEN } u_l(k) = -F_{jl} X_l(k) \tag{3}$$

where $j = 1, 2, \cdots, J_l$, $X_l^T(k) = [x_l(k), x_l(k-1), \cdots,$

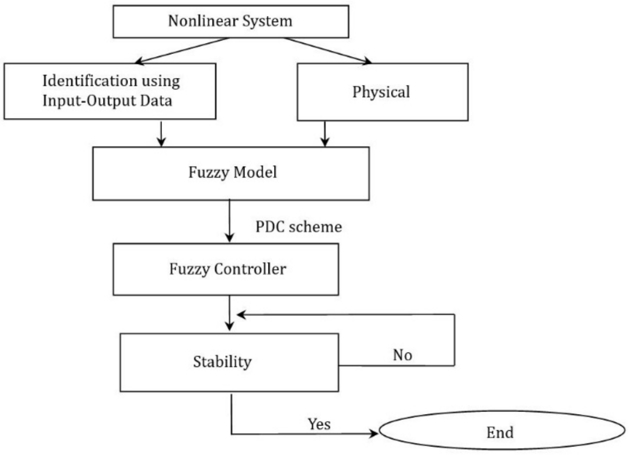

Fig. 1 Introduction the complete design procedure

$x_l(k-p+1)]$ and $M_{j\mu l}$ $(\mu = 1, 2, \cdots, p)$ are the fuzzy sets. Moreover, the IF-THEN fuzzy controller is inferred as follows

$$\begin{aligned} u_l(k) &= -\sum_{j=1}^{J_l} \bar{h}_{jl}(k) F_{jl} X_l(k), \\ \bar{h}_{jl}(k) &= \frac{w_{jl}(k)}{\sum_{j=1}^{J_l} w_{jl}(k)}, \\ w_{jl}(k) &= \prod_{\mu=1}^{p} M_{j\mu\, l}(x_{\mu\, l}(k)), \end{aligned} \tag{4}$$

in which $M_{j\mu\, l}(x_{\mu\, l}(k))$ is the grade of membership of $x_{\mu\, l}(k)$ in $M_{j\mu\, l}$. Substituting Eq. (4) into Eq. (5), we have the lth isolated closed-loop subsystem

$$\begin{aligned} X_l(k+1) &= \sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k) H_{ijl} X_l(k), \\ H_{ijl} &= A_{il} - B_{il}F_{jl} \end{aligned} \tag{5}$$

According the above IF-THEN rules, the interconnected N_l is described

$$X_l(k+1) = \sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k) H_{ijl} X_l(k) + \sum_{\substack{n=1\\ n\neq l}}^{L} C_{nl} X_n(k)$$

The relation to the model and controller design is not presented as follows

$$\begin{aligned} X_l(k+1) &= \sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k) H_{ijl} X_l(k) \\ &+ [\Re_l(X_l(k)) - \sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k) H_{ijl} X_l(k)] \end{aligned} \tag{6}$$

where

$$\begin{aligned} & H_{ijl} = A_{il} - B_{il}K_{jl}, \\ & \Re_l(X_l(k)) \equiv f_l(X_l(k), U_l(k)), \\ & e_l(k) = \left[\Re_l(X_l(k)) - \sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k) H_{ijl} X_l(k) \right] \\ & \sum_{k=0}^{N} \boldsymbol{x}(k)^T \boldsymbol{Q} \boldsymbol{x}(k) \le \gamma^2 \sum_{k=0}^{N} \boldsymbol{b}(k)^T \boldsymbol{b}(k) \qquad \forall N \end{aligned} \tag{6}$$

where $\boldsymbol{Q}$ is a positive definite weighting matrix.

Eq. (7) considers the initial condition.

$$\begin{aligned} & \sum_{k=0}^{N} \boldsymbol{x}(k)^T \boldsymbol{Q} \boldsymbol{x}(k) \\ & < \boldsymbol{x}(0)^T \boldsymbol{P} \boldsymbol{x}(0) + \gamma^2 \sum_{k=0}^{N} \boldsymbol{b}(k)^T \boldsymbol{b}(k) \qquad \forall N \end{aligned} \tag{7}$$

where $\boldsymbol{P}$ are some definite positive matrices.

In the accompanying, a solidness standard is proposed to ensure the asymptotic framework N. Preceding examination of asymptotic steadiness, the following criterion is obtained.

Theorem 1: The neural network (NN) large-scale system $\boldsymbol{N}$ is asymptotically stable, once there exist positive constants κ and the feedback gains F_{jl}, $l = 1, 2, \cdots, L$ are selected to satisfy

$$\begin{aligned} & \hat{\lambda}_{ijl} = \lambda_M(Q_{ijl}) + \alpha_{ijl}, \\ & \text{for} \quad i = u \le r_l;\ j = v \le J_l; \\ & \hat{\lambda}_{ijvl} = \lambda_M(Q_{ijvl}) + 2\alpha_{ijl}, \\ & \text{for} \quad i = u \le r_l;\ j < v \le J_l; \\ & \hat{\lambda}_{ijul} = \lambda_M(Q_{ijul}) + 2\alpha_{ijl}, \\ & \text{for} \quad i < u \le r_l;\ j = v \le J_l; \\ & \hat{\lambda}_{ijuvl} = \lambda_M(Q_{ijuvl}) + 4\alpha_{ijl}, \\ & \text{for} \quad i < u \le r_l;\ j < v \le J_l; \end{aligned} \tag{8}$$

where $\bar{Q}_{ijl} = \sum_{n=1}^{L} \kappa H_{ijl}^T P_l C_{nl} C_{nl}^T P_l H_{ijl}$,

$$\tilde{\lambda}_M(\bar{Q}_{ijl}) = \lambda_M(\bar{Q}_{ijl}) + \sum_{n=1}^{L} \kappa^{-1}\frac{L-1}{L},$$

$$\eta_l = \sum_{n\neq l}^{L} (L-1)\lambda_M(P_n)\|C_{l\,n}\|^2 ,$$

$\alpha_{ijl} = \tilde{\lambda}_M(\bar{Q}_{ijl}) + \eta_l, \quad Q_{ijl} = H_{ijl}^T P_l H_{ijl} - P_l,$
$Q_{ijvl} = H_{ijl}^T P_l H_{ivl} + H_{ivl}^T P_l H_{ijl} - 2P_l,$
$Q_{ijul} = H_{ijl}^T P_l H_{ujl} + H_{ujl}^T P_l H_{ijl} - 2P_l,$
$Q_{ijuvl} = H_{ijl}^T P_l H_{uvl} + H_{uvl}^T P_l H_{ijl} + H_{ivl}^T P_l H_{ujl} + H_{ujl}^T P_l H_{ivl} - 4P_l,$ $P_l = P_l^T > 0$ and $\lambda_M(.)$ denote the maximum eigenvalues.

Proof: See Appendix 1.

The Evolutionary Bat Algorithm (EBA) has been proposed to rely on the complex and ambiguous echolocation framework used by bats in the normal world. Unlike other group knowledge calculations, the main purpose of EBA is to have only one parameter, called the mean value. This parameter must be resolved before the calculation can be used for troubleshooting. The choice of diverse media determines the degree of seemingly surprising progress in the development process. In this review, we choose air as the medium because it is the primary medium of existence for common habitats inhabited by bats. ABE tasks can be summarized in four steps: Getting started: Distribute fake operators across the table area by randomly assigning addresses. Development: Wrong dealer behavior. Generate an irregular number and see if it is greater than the fixed pulse output rate. If the result is positive, the wrong expert walks irregularly.

$$x_i^t = x_i^{t-1} + D,$$

where x_i^t illustrates, and the coordinate of the i-th artificial agent at the t-th iteration, x_i^{t-1} means D is the moving distance that the artificial agent goes in this iteration, and the coordinate of the i-th artificial agent at the last iteration.

$$D = \gamma \cdot \Delta T$$

where $\Delta T \in [-1,\ 1]$ is a random number. $\gamma = 0.17$ is used in our experiment because the chosen medium is air, and γ is a constant corresponding to the medium chosen in the experiment.

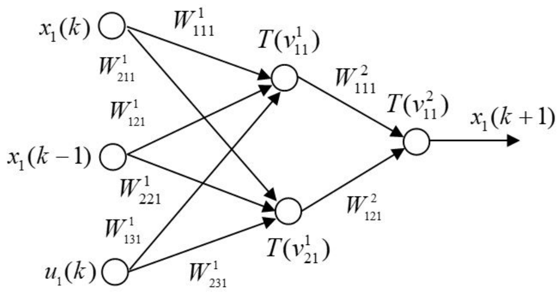

Fig. 2 The state of subsystem 1

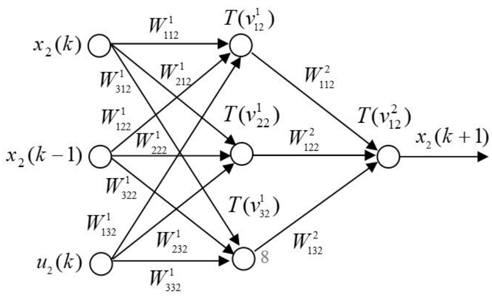

Fig. 3 The second isolated NN subsystem

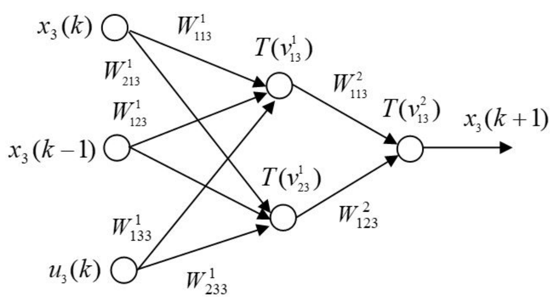

Fig. 4 The third isolated NN subsystem

$$x_i^{t_R} = \beta(x_{\text{best}} - x_i^t),\ \ \beta \in [0,\ \ 1]$$

where x_{best} indicates the coordinate of the near best solution found so far throughout all artificial agents; β is a random number; and $x_i^{t_R}$ represents the new coordinates of the artificial agent after the operation of the random walk process.

4. Example

Consider an NN multiple system described as follows. The first isolated NN subsystem is shown in Fig. 2. The algebraic developments of the example are shown in Appendix 2.

If you want to use EBA to solve the best problem, you must first design your training function. Another feature of system stability problems is that the solutions detected by the algorithm are as follows. The training function is based on the stability criteria derived from the LMI condition by the Lyapunov exponential method, as described above. Check the solution using the AND operation of the adaptive function to generate the binary classification result of the found solution. Therefore, the following parameters to satisfy the Theorem are obtained and also the H infinity

$$\sum_{k=0}^{N} \boldsymbol{x}(k)^T \boldsymbol{Q}\boldsymbol{x}(k) < \boldsymbol{x}(0)^T \boldsymbol{P}\boldsymbol{x}(0) + \gamma^2 \sum_{k=0}^{N} \boldsymbol{b}(k)^T \boldsymbol{b}(k)$$

has been checked satisfied.

$$\begin{aligned}
&W_{113}^1 = -0.5,\ W_{213}^1 = 0.25,\ W_{123}^1 = 1,\\
&W_{223}^1 = 0.2,\ W_{133}^1 = -0.5,\ W_{233}^1 = 0.75,\\
&W_{113}^2 = 0.5,\ W_{123}^2 = -1,\ W_{112}^1 = 0.5,\\
&W_{212}^1 = 0.5,\ W_{312}^1 = 0.25,\ W_{122}^1 = 0.4,\\
&W_{132}^1 = 0.25,\ W_{232}^1 = 0.8,\ W_{332}^1 = -0.25,\\
&W_{222}^1 = 0.35,\ W_{322}^1 = 0.5,\ W_{112}^2 = 0.25,\\
&W_{122}^2 = -0.75,\ W_{132}^2 = 1,\ W_{111}^1 = 1,\\
&W_{211}^1 = -1,\ W_{121}^1 = -0.5,\ W_{221}^1 = -0.6,\\
&W_{131}^1 = 0.3,\ W_{231}^1 = -0.4,\ W_{111}^2 = 0.75,\\
&W_{121}^2 = 1, \text{ and}\\
&A_{111} = A_{112} = A_{121} = A_{122} = A_{211} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},\\
&A_{212} = \begin{bmatrix} -0.4444 & -0.2667 \\ 1 & 0 \end{bmatrix},\\
&A_{221} = \begin{bmatrix} 0.3333 & -0.1667 \\ 1 & 0 \end{bmatrix},\\
&A_{222} = \begin{bmatrix} -0.1111 & -0.4333 \\ 1 & 0 \end{bmatrix},\\
&B_{111} = B_{112} = B_{121} = B_{122} = B_{211} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\\
&B_{212} = \begin{bmatrix} -0.1778 \\ 0 \end{bmatrix},\ B_{221} = \begin{bmatrix} 0.1000 \\ 0 \end{bmatrix},\\
&B_{222} = \begin{bmatrix} -0.0778 \\ 0 \end{bmatrix},\\
&\bar{A}_{11} = A_{111} = A_{112} = A_{121} = A_{122} = A_{211},\\
&\bar{A}_{21} = A_{212},\ \bar{A}_{31} = A_{221},\ \bar{A}_{41} = A_{222},\\
&\bar{B}_{11} = B_{111} = B_{112} = B_{121} = B_{122} = B_{211},
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\bar{B}_{21} = B_{212},\ \bar{B}_{31} = B_{221},\ \bar{B}_{41} = B_{222},\\
&\bar{A}_{12} = A_{1pst} = A_{2111} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad p,s,t = 1,2,\\
&\bar{A}_{22} = A_{2112} = \begin{bmatrix} 0.1276 & 0.2551 \\ 1 & 0 \end{bmatrix},\\
&\bar{A}_{32} = A_{2121} = \begin{bmatrix} -0.1913 & -0.1339 \\ 1 & 0 \end{bmatrix},\\
&\bar{A}_{42} = A_{2122} = \begin{bmatrix} -0.0638 & 0.1212 \\ 1 & 0 \end{bmatrix},\\
&\bar{A}_{52} = A_{2211} = \begin{bmatrix} 0.0638 & 0.0510 \\ 1 & 0 \end{bmatrix},\\
&\bar{A}_{62} = A_{2212} = \begin{bmatrix} 0.1913 & 0.3061 \\ 1 & 0 \end{bmatrix},\\
&\bar{A}_{72} = A_{2221} = \begin{bmatrix} -0.1276 & -0.0829 \\ 1 & 0 \end{bmatrix},\\
&\bar{A}_{82} = A_{2222} = \begin{bmatrix} 0 & 0.1722 \\ 1 & 0 \end{bmatrix},\\
&\bar{B}_{12} = B_{1pst} = B_{2111} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad p,s,t = 1,2,\\
&\bar{B}_{22} = B_{2112} = \begin{bmatrix} -0.1276 \\ 0 \end{bmatrix},\\
&\bar{B}_{32} = B_{2121} = \begin{bmatrix} -0.3061 \\ 0 \end{bmatrix},\\
&\bar{B}_{42} = B_{2122} = \begin{bmatrix} -0.4337 \\ 0 \end{bmatrix},\\
&\bar{B}_{52} = B_{2211} = \begin{bmatrix} 0.0319 \\ 0 \end{bmatrix},\\
&\bar{B}_{62} = B_{2212} = \begin{bmatrix} -0.0957 \\ 0 \end{bmatrix},\\
&\bar{B}_{72} = B_{2221} = \begin{bmatrix} -0.2742 \\ 0 \end{bmatrix},\\
&\bar{B}_{82} = B_{2222} = \begin{bmatrix} -0.4018 \\ 0 \end{bmatrix},\\
&\bar{A}_{13} = A_{111} = A_{112} = A_{121} = A_{122} = A_{211} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},\\
&\bar{A}_{23} = A_{212} = \begin{bmatrix} -0.1736 & -0.1389 \\ 1 & 0 \end{bmatrix},\\
&\bar{A}_{33} = A_{221} = \begin{bmatrix} -0.1736 & 0.3472 \\ 1 & 0 \end{bmatrix},\\
&\bar{A}_{43} = A_{222} = \begin{bmatrix} -0.3472 & 0.2083 \\ 1 & 0 \end{bmatrix},\\
&\bar{B}_{13} = B_{111} = B_{112} = B_{121} = B_{122} = B_{211} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\\
&\bar{B}_{23} = B_{212} = \begin{bmatrix} -0.5208 \\ 0 \end{bmatrix},\\
&\bar{B}_{33} = B_{221} = \begin{bmatrix} -0.1736 \\ 0 \end{bmatrix},\\
&\bar{B}_{43} = B_{222} = \begin{bmatrix} -0.6944 \\ 0 \end{bmatrix},\\
&C_{21} = \begin{bmatrix} 0.13 & -0.12 \\ 0 & 0 \end{bmatrix},\\
&C_{31} = \begin{bmatrix} -0.12 & -0.1 \\ 0 & 0 \end{bmatrix},\\
&C_{12} = \begin{bmatrix} 0.1 & -0.15 \\ 0 & 0 \end{bmatrix},\\
&C_{32} = \begin{bmatrix} 0.12 & 0.1 \\ 0 & 0 \end{bmatrix},\\
&C_{13} = \begin{bmatrix} 0.16 & -0.13 \\ 0 & 0 \end{bmatrix},\\
&C_{23} = \begin{bmatrix} -0.15 & 0.12 \\ 0 & 0 \end{bmatrix}.
\end{aligned} \tag{9}$$

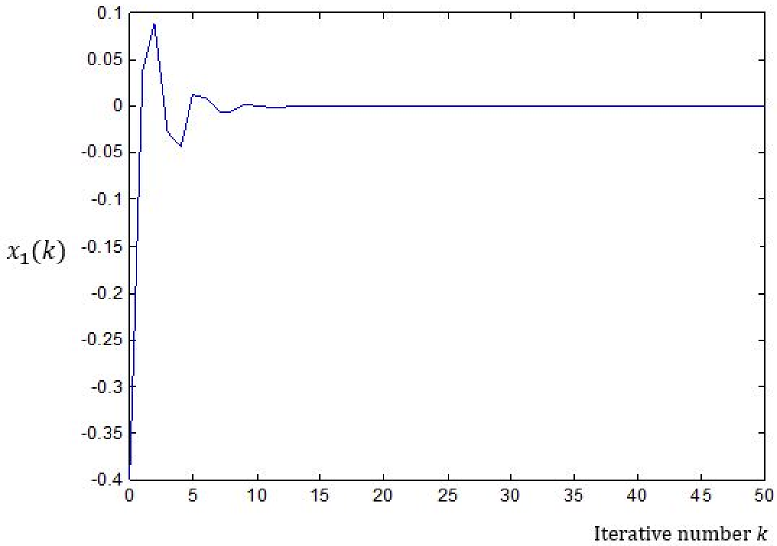

Fig. 5 The state $x_1(k)$ of subsystem 1

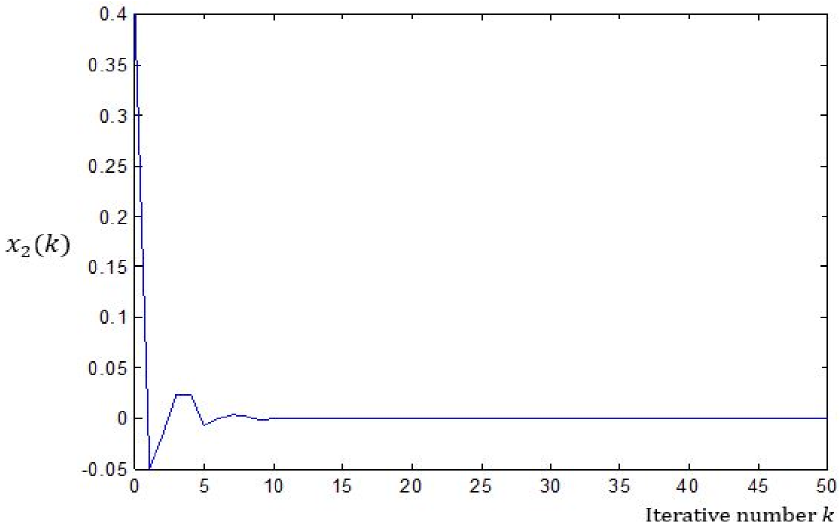

Fig. 6 The state $x_2(k)$ of subsystem 2

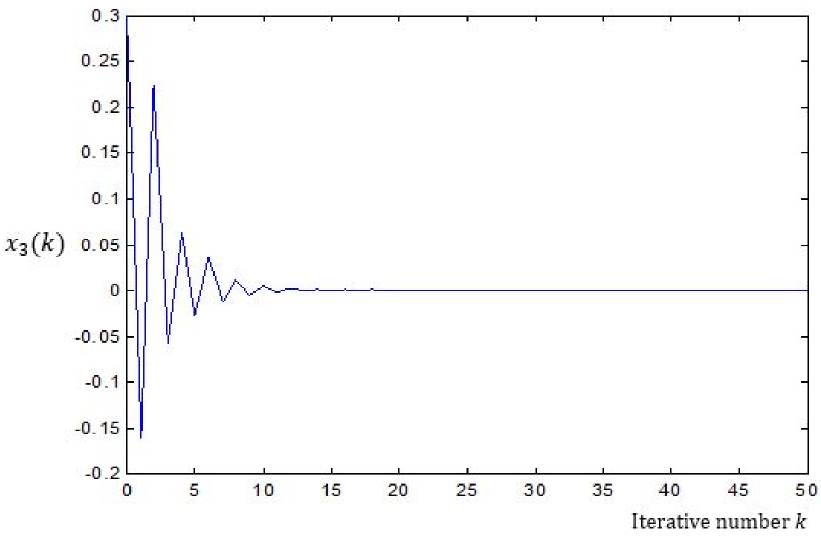

Fig. 7 The state $x_3(k)$ of subsystem 3

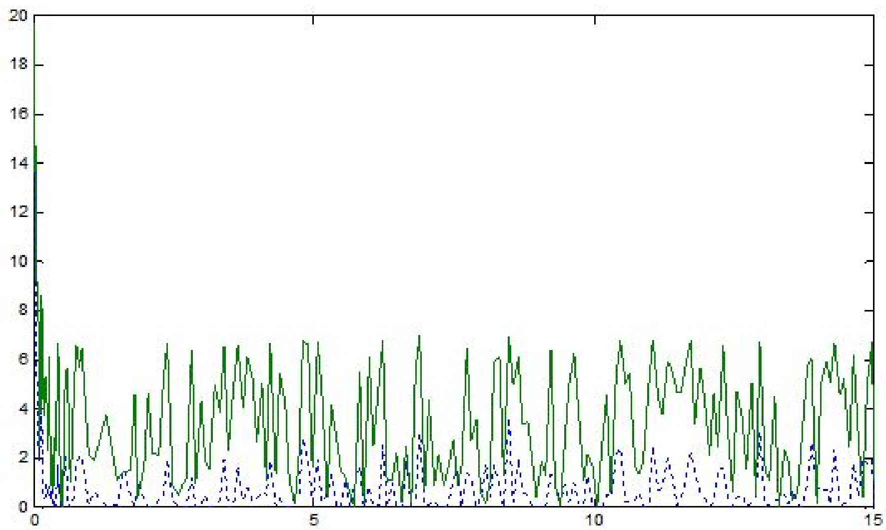

Fig. 8 The validation of controlled system with decentralized control of NNDI

Therefore, the NNDI system Eqs. (B13), (B21) and (B26) can be asymptotically controlled with the following controller design.

If $x_1(k)$ is M_{11} Then $u_1(k) = -F_{11}X_1(k)$;
If $x_1(k)$ is M_{21} Then $u_1(k) = -F_{21}X_1(k)$,
If $x_2(k)$ is M_{12} Then $u_2(k) = -F_{12}X_2(k)$;
If $x_2(k)$ is M_{22} Then $u_2(k) = -F_{22}X_2(k)$
If $x_3(k)$ is M_{13} Then $u_3(k) = -F_{13}X_3(k)$;
If $x_3(k)$ is M_{23} Then $u_3(k) = -F_{23}X_3(k)$,

with the following optimal matrix and random membership functions

$$F_{11} = [0.75 \quad 0.5], \qquad F_{21} = [0.6 \quad 0.5],$$
$$F_{12} = [-0.5 \quad -0.3], \quad F_{22} = [0.2 \quad -0.25],$$
$$F_{13} = [0.3 \quad 0.2], \qquad F_{23} = [0.5 \quad 0.25],$$
$$P_1 = \begin{bmatrix} 76.5477 & -0.0361 \\ -0.0361 & 39.3816 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 73.6939 & 1.2549 \\ 1.2549 & 38.7814 \end{bmatrix},$$
$$P_3 = \begin{bmatrix} 65.0683 & 0.9759 \\ 0.9759 & 35.7440 \end{bmatrix},$$
$$\hat{\lambda}_{ijl} < 0, \quad \hat{\lambda}_{ijvl} < 0, \quad \hat{\lambda}_{ijul} < 0, \quad \hat{\lambda}_{ijuvl} < 0.$$

Simulation results of each subsystem are illustrated in Figs. 5-7 with random initial conditions. To ensure the validity of the system, the model testing along with an error analysis is used to demonstrate the advantage of the proposed NNDI control scheme. The validation of controlled systems with decentralized control via fuzzy model-based approach is depicted in Fig. 8.

5. Conclusions

This work proposes a sound foundation based on Lyapunov's immediate strategy for large-scale NN frameworks with conformation of asymptotic stability. On this resultant illustration and with a decentralized control plot, a large number of fuzzy controllers have been combined to balance out that NNDI framework. Finally, the proposed technique is demonstrated by a numerical simulation.

Acknowledgments

The authors are grateful for the research grants given to RY Wang from the Projects of Talents Recruitment of GDUPT (NO. 2019rc098) in Guangdong Province, Peoples R China No. 2019rc098 by ZY Chen as well as to the anonymous reviewers for constructive suggestions.

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Appendix A.
Proof of Theorem 1

Let the Lyapunov function for the neural network (NN) large-scale system ***N*** be defined as

$$V(k)=\sum_{l=1}^{L}X_l^T(k)P_lX_l(k) \tag{A1}$$

where $P_l=P_l^T>0$. We then evaluate the backward difference of $V(k)$ on the trajectories to get

$$\begin{aligned}\Delta V(k)&=V(k+1)-V(k)\\&=\sum_{l=1}^{L}[X_l^T(k+1)P_lX_l(k+1)-X_l^T(k)P_lX_l(k)]\\&=\sum_{l=1}^{L}\left\{\left[\sum_{i=1}^{r_l}\sum_{j=1}^{J_l}h_{il}(k)\bar{h}_{jl}(k)H_{ijl}X_l(k)+\varphi_l(k)]^T\right]P_l\left[\sum_{u=1}^{r_l}\sum_{v=1}^{J_l}h_{ul}(k)\bar{h}_{vl}(k)H_{uvl}X_l(k)+\varphi_l(k)\right]-X_l^T(k)P_lX_l(k)\right\}\\&=D_1+D_2+D_3+D_4+D_5+D_6,\end{aligned} \tag{A2}$$

where

$$D_1\equiv\sum_{l=1}^{L}\sum_{i=u=1}^{r_l}\sum_{j=v=1}^{J_l}h_{il}^2(k)\bar{h}_{jl}^2(k)X_l^T(k)(H_{ijl}^TP_lH_{ijl}-P_l)X_l(k)\le\sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l}h_{il}^2(k)\bar{h}_{jl}^2(k)X_l^T(k)\lambda_M(Q_{ijl})X_l(k) \tag{A3}$$

with $Q_{ijl}\equiv H_{ijl}^TP_lH_{ijl}-P_l$,

$$\begin{aligned}D_2&\equiv\sum_{l=1}^{L}\sum_{i=u=1}^{r_l}\sum_{j\neq v}^{J_l}h_{il}^2(k)\bar{h}_{jl}(k)\bar{h}_{vl}(k)X_l^T(k)(H_{ijl}^TP_lH_{ivl}-P_l)X_l(k)\\&\le\sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j<v}^{J_l}h_{il}^2(k)\bar{h}_{jl}(k)\bar{h}_{vl}(k)\,\lambda_M(Q_{ijvl})X_l^T(k)X_l(k)\end{aligned} \tag{A4}$$

with $Q_{ijvl}\equiv H_{ijl}^TP_lH_{ivl}+H_{ivl}^TP_lH_{ijl}-2P_l$,

$$\begin{aligned}D_3&\equiv\sum_{l=1}^{L}\sum_{i\neq u}^{r_l}\sum_{j=v=1}^{J_l}h_{il}(k)h_{ul}(k)\bar{h}_{jl}^2(k)X_l^T(k)(H_{ijl}^TP_lH_{ujl}-P_l)X_l(k)\\&\le\sum_{l=1}^{L}\sum_{i<u}^{r_l}\sum_{j=1}^{J_l}h_{il}(k)h_{ul}(k)\bar{h}_{jl}^2(k)\,\lambda_M(Q_{ijul})X_l^T(k)X_l(k)\end{aligned} \tag{A5}$$

with $Q_{ijul}\equiv H_{ijl}^TP_lH_{ujl}+H_{ujl}^TP_lH_{ijl}-2P_l$,

$$\begin{aligned}D_4&\equiv\sum_{l=1}^{L}\sum_{i\neq u}^{r_l}\sum_{j\neq v}^{J_l}h_{il}(k)h_{ul}(k)\bar{h}_{jl}(k)\bar{h}_{vl}(k)X_l^T(k)(H_{ijl}^TP_lH_{uvl}-P_l)X_l(k)\\&\le\sum_{l=1}^{L}\sum_{i<u}^{r_l}\sum_{j<v}^{J_l}h_{il}(k)h_{ul}(k)\bar{h}_{jl}(k)\bar{h}_{vl}(k))\,\lambda_M(Q_{ijuvl}){X_l}^T(k)X_l(k)\end{aligned} \tag{A6}$$

with $Q_{ijuvl}\equiv H_{ijl}^TP_lH_{uvl}+H_{uvl}^TP_lH_{ijl}+H_{ivl}^TP_lH_{ujl}+H_{ujl}^TP_lH_{ivl}-4P_l$,

$$
\begin{aligned}
D_5 &\equiv \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k)X_l^T(k)H_{ijl}^T P_l\varphi_l(k) + \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k)\varphi_l^T(k)P_l H_{ijl}X_l(k) \\
&\le \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l}\sum_{n\neq l}^{L} h_{il}(k)\bar{h}_{jl}(k)\{\kappa X_l^T(k)H_{ijl}^T P_l C_{nl}C_{nl}^T P_l H_{ijl}X_l(k) + \kappa^{-1}X_n^T(k)X_n(k)\} \\
&\le \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k)(\lambda_M(\bar{Q}_{ijl}) + \sum_{n=1}^{L}\kappa^{-1}\left(\frac{L-1}{L}\right)X_l^T(k)X_l(k) = \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k)\tilde{\lambda}_M(\bar{Q}_{ijl})X_l^T(k)X_l(k)
\end{aligned}
\tag{A7}
$$

with $\bar{Q}_{ijl} = \sum_{n=1}^{L}\kappa H_{ijl}^T P_l C_{nl}C_{nl}^T P_l H_{ijl},\ \tilde{\lambda}_M(\bar{Q}_{ijl}) = \lambda_M(\bar{Q}_{ijl}) + \sum_{n=1}^{L}\kappa^{-1}\frac{L-1}{L}$.

$$
\begin{aligned}
D_6 \equiv \sum_{l=1}^{L}\varphi_l^T(k)\ P_l\varphi_l(k) &\le \sum_{l=1}^{L}[\,(L-1)\lambda_M(P_l)\|C_{1l}X_1(k)\|^2 + (L-1)\lambda_M(P_l)\|C_{2l}X_2(k)\|^2 + \cdots] \\
&\le \sum_{l=1}^{L}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)\bar{h}_{jl}(k)\eta_l\|X_l(k)\|^2
\end{aligned}
\tag{A8}
$$

with $\eta_l = \sum_{n\neq l}^{L}(L-1)\lambda_M(P_n)\|C_{l\,n}\|^2$.

Substituting Eqs. (A3)-(A8) into Eq. (A2) yields

$$
\begin{aligned}
\Delta V(k) \le \sum_{l=1}^{L}&\left\{\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}^2(k)\bar{h}_{jl}^2(k)\lambda_M(Q_{ijl}) + \sum_{i=1}^{r_l}\sum_{j<v}^{J_l} h_{il}^2(k)\bar{h}_{jl}(k)\bar{h}_{vl}(k)\,\lambda_M\left(Q_{ijvl}\right) + \sum_{i<u}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)h_{ul}(k)\bar{h}_{jl}^2(k))\,\lambda_M\left(Q_{ijul}\right)\right. \\
&\left. + \sum_{i<u}^{r_l}\sum_{j<v}^{J_l} h_{il}(k)h_{ul}(k)\bar{h}_{jl}(k)\,\bar{h}_{vl}(k)\lambda_M\left(Q_{ijuvl}\right) + \sum_{u=1}^{r_l}\sum_{v=1}^{J_l}\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{ul}(k)\bar{h}_{vl}(k)h_{il}(k)\bar{h}_{jl}(k)\alpha_{ijl}\right\}\|X_l(k)\|^2
\end{aligned}
$$

with $\alpha_{ijl} = \tilde{\lambda}_M\left(\bar{Q}_{ijl}\right) + \eta_l$

$$
\begin{aligned}
&= \sum_{l=1}^{L}\{\sum_{i=1}^{r_l}\sum_{j=1}^{J_l} h_{il}^2(k)\bar{h}_{jl}^2(k)\lambda_M(Q_{ijl}) + \sum_{i=1}^{r_l}\sum_{j<v}^{J_l} h_{il}^2(k)\bar{h}_{jl}(k)\bar{h}_{vl}(k)\,\lambda_M\left(Q_{ijvl}\right) \\
&\quad + \sum_{i<u}^{r_l}\sum_{j=1}^{J_l} h_{il}(k)h_{ul}(k)\bar{h}_{jl}^2(k))\,\lambda_M\left(Q_{ijul}\right) + \sum_{i<u}^{r_l}\sum_{j<v}^{J_l} h_{il}(k)h_{ul}(k)\bar{h}_{jl}(k)\,\bar{h}_{vl}(k)\lambda_M\left(Q_{ijuvl}\right) \\
&\quad + \sum_{i=u}^{r_l}\sum_{j=v}^{J_l} {h_{il}}^2(k)\,{\bar{h}_{jl}}^2(k)\alpha_{ijl} + \sum_{i=u}^{r_l}\sum_{j<v}^{J_l} {h_{il}}^2(k)\,\bar{h}_{jl}(k)\bar{h}_{vl}(k)2\alpha_{ijl} + \sum_{i<u}^{r_l}\sum_{j=v}^{J_l} h_{il}(k)h_{ul}(k)\bar{h}_{jl}^2(k)\,2\alpha_{ijl} \\
&\quad + \sum_{i<u}^{r_l}\sum_{j<v}^{J_l} h_{il}(k)h_{ul}(k)\bar{h}_{jl}(k)\,\bar{h}_{vl}(k)4\alpha_{ijl}\}\|X_l(k)\|^2 \\
&= \sum_{l=1}^{L}\left\{\sum_{j=1}^{J_l}\sum_{i=1}^{r_l} {\bar{h}_{jl}}^2(k){h_{il}}^2(k)\hat{\lambda}_{ijl} + \sum_{j<v}^{J_l}\sum_{i=1}^{r_l} \bar{h}_{jl}(k)\bar{h}_{vl}(k){h_{il}}^2(k)\,\hat{\lambda}_{ijvl} + \sum_{j=1}^{J_l}\sum_{i<u}^{r_l} {\bar{h}_{jl}}^2(k)h_{il}(k)h_{ul}(k)\hat{\lambda}_{ijul}\right. \\
&\quad \left. + \sum_{j<v}^{J_l}\sum_{i<u}^{r_l} \bar{h}_{jl}(k)\bar{h}_{vl}(k)h_{il}(k)h_{ul}(k)\hat{\lambda}_{ijuvl}\right\}\|X_l(k)\|^2
\end{aligned}
\tag{A9}
$$

Eventually, we have $\Delta V(k) < 0$ and the proof of theorem 1 is thereby completed.

Appendix B. Algebraic developments of the example

From this Figs. 2-4, we have

$$v_{e1}^1 = W_{e11}^1 x_1(k) + W_{e21}^1 x_1(k-1) W_{e31}^1 u_1(k) \quad e = 1, 2, \tag{B1}$$

$$v_{11}^2 = W_{111}^2 T(v_{11}^1) + W_{121}^2 T(v_{21}^1) \tag{B2}$$

$$x_1(k+1) = T(v_{11}^2), \tag{B3}$$

where

$$A_{ips} = \begin{bmatrix} g_{i1}\{g_{p1}W_{111}^2 W_{111}^1 + g_{s1}W_{121}^2 W_{211}^1\} & g_{i1}\{g_{p1}W_{111}^2 W_{121}^1 + g_{s1}W_{121}^2 W_{221}^1\} \\ 1 & 0 \end{bmatrix}, \tag{B14}$$

$$T(v_{e1}^1) = \frac{2}{1+\exp\left(-\frac{v_{e1}^1}{0.75}\right)} - 1, \quad e = 1, 2, \tag{B4}$$

$$T(v_{11}^2) = \frac{2}{1+\exp\left(-\frac{v_{11}^1}{0.75}\right)} - 1 \tag{B5}$$

According to Eqs. (B4)-(B5), the extreme value of derivative $T(v)$ can be obtai(ned as follows

$$g_{11} = \min_v \frac{dT(v)}{dv} = 0, \quad g_{21} = \max_v \frac{dT(v)}{dv} = \frac{2}{3} \tag{B6}$$

Further, based on the interpolation method, Eqs. (B4)-(B5) can be represented, respectively

$$T(v_{l1}^1) = \left(h_{l11}^1(k) g_{11} + h_{l21}^1(k) g_{21}\right) v_{l1}^1, \quad l = 1, 2, \tag{B7}$$

$$T(v_{11}^2) = \left(h_{111}^2(k) g_{11} + h_{121}^2(k) g_{21}\right) v_{11}^2. \tag{B8}$$

From Eqs. (B3) and (B8), we have

$$x_1(k+1) = (h_{111}^2(k) g_{11} + h_{121}^2(k) g_{21}) v_{11}^2 = \sum_{i=1}^{2} h_{1i1}^2(k) g_{i1} v_{11}^2. \tag{B9}$$

Substituting Eqs. (B2), (B6)-(B7) into Eq. (B9) yields

$$x_1(k+1) = \sum_{i=1}^{2} h_{1i1}^2(k) g_{i1} \sum_{p=1}^{2} W_{1p1}^2 T(v_{p1}^1) = \sum_{i=1}^{2} h_{1i1}^2(k) g_{i1} \sum_{p=1}^{2}\sum_{s=1}^{2} h_{1p1}^1(k) h_{2s1}^1(k) \{g_{p1} W_{111}^2 v_{11}^1 + g_{s1} W_{121}^2 v_{21}^1\}. \tag{B10}$$

By plugging Eq. (B1) into Eq. (B10), we obtain

$$x_1(k+1) = \sum_{i=1}^{2}\sum_{p=1}^{2}\sum_{s=1}^{2} h_{1i1}^2(k) h_{1p1}^1(k) h_{2s1}^1(k) \{g_{i1}[g_{p1}W_{111}^2 W_{111}^1 + g_{s1}W_{121}^2 W_{211}^1] x_1(k) + g_{i1}[g_{p1}W_{111}^2 W_{121}^1 + g_{s1}W_{121}^2 W_{221}^1] x_1(k-1) + g_{i1}[g_{p1}W_{111}^2 W_{131}^1 + g_{s1}W_{121}^2 W_{231}^1] u_1(k)\}. \tag{B11}$$

The NNDI matrix representation of Eq. (B11) is

$$X_1(k+1) = \sum_{i=1}^{2}\sum_{p=1}^{2}\sum_{s=1}^{2} h_{1i1}^2(k) h_{1p1}^1(k) h_{2s1}^1(k) \{A_{ips} X_1(k) + B_{ips} u_1(k)\} \tag{B12}$$

$$X_1(k+1) = \sum_{i=1}^{4} h_{i1}(k) \{\bar{A}_{i1} X_1(k) + \bar{B}_{i1} u_1(k)\} \tag{B13}$$

where

$$B_{ips} = \begin{bmatrix} g_{i1}\{g_{p1}W_{111}^2 W_{131}^1 + g_{s1}W_{121}^2 W_{231}^1\} \\ 0 \end{bmatrix}, \tag{B15}$$

$$X_1^T(k) = [x_1(k) \quad x_1(k-1)]. \tag{B16}$$

The second isolated NN subsystem is shown in Fig. 3. From this figure, we have

$$v_{e2}^1 = W_{l12}^1 x_2(k) + W_{e22}^1 x_2(k-1) + W_{e32}^1 u_2(k), \quad e = 1, 2, 3, \tag{B17}$$

$$v_{12}^2 = W_{112}^2 T(v_{12}^1) + W_{122}^2 T(v_{22}^1) + W_{132}^2 T(v_{32}^1), \tag{B18}$$

$$x_2(k+1) = T(v_{12}^2), \tag{B19}$$

where the transfer functions $T(v)$ are similar as (B4)-(B5) and thus $g_{11} = \min_v \frac{d\,T(v)}{dv} = 0$, $g_{21} = \max_v \frac{d\,T(v)}{dv} = \frac{5}{7}$.

Using the same procedure as that in subsystem1, we obtain NNDI in the following

$$X_2(k+1) = \textstyle\sum_{i=1}^{2}\sum_{p=1}^{2}\sum_{s=1}^{2}\sum_{t=1}^{2} h_{1i2}^2(k) h_{1p2}^1(k) h_{2s2}^1(k) h_{3t2}^1(k) \{A_{ipst} X_2(k) + B_{ipst} u_2(k)\}, \tag{B20}$$

$$X_2(k+1) = \sum_{i=1}^{8} h_{i2}(k) \{\bar{A}_{i2} X_2(k) + \bar{B}_{i2} u_2(k)\} \tag{B21}$$

where

$$A_{ipst}=\begin{bmatrix} g_{i2}\{g_{p2}W^2_{112}W^1_{112}+g_{s2}W^2_{122}W^1_{212}+g_{t2}W^2_{132}W^1_{312}\} & g_{i2}\{g_{p2}W^2_{112}W^1_{122}+g_{s2}W^2_{122}W^1_{222}+g_{t2}W^2_{132}W^1_{322}\} \\ 1 & 0 \end{bmatrix},$$
$$B_{ipst}=\begin{bmatrix} g_{i2}\{g_{p2}W^2_{112}W^1_{132}+g_{s2}W^2_{122}W^1_{232}+g_{t2}W^2_{132}W^1_{332}\} \\ 0 \end{bmatrix},$$
$$X_2{}^T(k)=[x_2(k) \quad x_2(k-1)].$$

The third isolated NN subsystem is shown in Fig. 4. From this figure, we have

$$v^1_{e3}=W^1_{e13}x_3(k)+W^1_{e23}x_3(k-1)+W^1_{e33}u_3(k),\quad e=1,2 \tag{B22}$$

$$v^2_{13}=W^2_{113}T(v^1_{13})+W^2_{123}T(v^1_{23}), \tag{B23}$$

$$x_3(k+1)=T(v^2_{13}), \tag{B24}$$

where the transfer functions $T(v)$ are similar as Eqs. (B4)-(B5) and thus $g_{11}=\min_v \frac{d\,T(v)}{dv}=0$, $g_{21}=\max_v \frac{d\,T(v)}{dv}=\frac{5}{6}$. Using the same procedure as that in subsystem1, we obtain NNDI below

$$X_3(k+1)=\sum_{i=1}^{2}\sum_{p=1}^{2}\sum_{s=1}^{2}h^2_{1i3}(k)h^1_{1p3}(k)h^1_{2s3}(k)\{A_{ips}X_3(k)+B_{ips}u_3(k)\} \tag{B25}$$

$$X_3(k+1)=\sum_{i=1}^{4}h_{i3}(k)\{\bar{A}_{i3}(k)X_3(k)+\bar{B}_{i3}\,u_3(k)\} \tag{B26}$$

where

$$A_{ips}=\begin{bmatrix} g_{i3}\{g_{p3}W^2_{113}W^1_{113}+g_{s3}W^2_{213}W^1_{213}\} & g_{i3}\{g_{p3}W^2_{113}W^1_{123}+g_{s3}W^2_{123}W^1_{223}\} \\ 1 & 0 \end{bmatrix},$$
$$B_{ips}=\begin{bmatrix} g_{i3}\{g_{p3}W^2_{113}W^1_{133}+g_{s3}W^2_{123}W^1_{233}\} \\ 0 \end{bmatrix},$$
$$X_3{}^T(k)=[x_3(k) \quad x_3(k-1)].$$