Smart Structures and Systems, *Vol. 29, No. 6 (2022) 791-797*
https://doi.org/10.12989/sss.2022.29.6.791

Hierarchical fault propagation of command and control system

Tingyu Zhang[1,2a], Hong-Zhong Huang*[1,2], Yifan Li[1,2b], Sizhe Huang[3] and Yahua Li[3]

[1]*School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, China*
[2]*Center for System Reliability and Safety, University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, China*
[3]*Wuhan Digital Engineering Research Institute, Wuhan, 430070, China*

(*Received October 31, 2021, Revised January 25, 2022, Accepted March 29, 2022*)

Abstract. A complex system is comprised of numerous entities containing physical components, devices and hardware, events or phenomena, and subsystems, there are intricate interactions among these entities. To reasonably identify the critical fault propagation paths, a system fault propagation model is essential based on the system failure mechanism and failure data. To establish an appropriate mathematical model for the complex system, these entities and their complicated relations must be represented objectively and reasonably based on the structure. Taking a command and control system as an example, this paper proposes a hierarchical fault propagation analysis method, analyzes and determines the edge betweenness ranking model and the importance degree of each sub-system.

Keywords: command and control system; edge betweenness; hierarchical fault propagation

1. Introduction

Fault propagation phenomenon is ubiquitous in real systems, such as large-scale power failures, traffic jams and internet breakdowns, which are closely related to component unit fault propagation. A correct decision of critical fault propagation path identification further influences the complete analysis of the system fault propagation mechanism and the acquisition of the system fault propagation model in advance. For the construction of fault propagation models, research on Petri nets, Bayesian nets, and graph theory individually provides the construction of fault models from different perspectives.

Murata (1989). made an introductory discussion on the high-level network in Petri net and its application in a logic program. Bai (2021). proposed a fault analysis framework based on improved Petri net and analyzes an engineering example. Zuleta *et al*. (2007) avoided identifying a few SOE signals or events with the large time difference between them by refining attribute functions. Jensen. (2007). introduced the most probable explanation for the observed evidence, detecting conflicts in the evidence entered into the network, determining optimal strategies, analyzing for relevance, and performing sensitivity analysis. Li *et al*. (2020) proposed a new reliability assessment method for the systems suffering from CCF in a dynamic environment. Li *et al*. (2019) proposed a method to incorporate fuzzy probability and Bayesian network (BN) into multi-state systems (MSSs) with CCFs. In graph theory, Nakano and Nakanishi (1974) proposed an algorithm for constructing a square matrix reachable matrix by using a diagnostic matrix. Kolda *et al*. (2014) compared BTER to other scalable models and show that it gives a better fit to real data and provided idealized degree distributions and clustering coefficient profiles that can be tuned for user specifications. Tarifa and Nicolás (1997) presented a new model-based approach procedure for fault diagnosis in conventional chemical processes.

DEMATEL (Decision Making Trial and Evaluation Laboratory) was proposed in 1971 by the United States National Laboratories. It is used to solve complex and difficult real-world problems by graph theory and matrices. By analyzing the logical relationships between factors in a system, DEMATEL constructs a direct influence matrix to obtain the influence of factors. The influence includes the degree of being influenced, the reason degree, and the centrality degree. This technique is particularly effective for uncertain relationships systems.

Lee *et al*. (2013) proved that raising the initial relation matrix to the power of infinity may not converge to zero and hence total influence may not converge. Seyed-Hosseini and Safaei (2006) proposed methodology can cover some of the inherent shortcomings of the conventional Risk Priority Number (RPN) method and like. Wu and Lee (2007) proposed an effective method combining fuzzy logic and DEMATEL to segment required competencies for better promoting the competency development of global managers. Fang *et al*. (2009) reconsidered this method from the perspective of a complex network and proposed an improved networked DEMATEL method to analyze national economic data from four aspects. Lin and Tzeng (2009) compare various industrial

---

*Corresponding author, Professor
E-mail: hzhuang@uestc.edu.cn
[a]Ph.D. Student
E-mail: zhangtingyu@std.uestc.edu.cn
[b]Ph.D. Student
E-mail: liyifan@std.uestc.edu.cn

ISSN: 1738-1584 (Print), 1738-1991 (Online)

Table 1 The composition of command and control system

Subsystem	Code	Function
Command& Dispatch	*C*	The functions of broadcast, live broadcast and group scheduling, recording all scheduling passwords and callbacks in real-time
Data Storage	*D*	Analyze, classify, record and store the data in the large-capacity storage equipment
Network Transmission	*N*	To ensure the safety and accuracy of the data.
Unify-Time	*T*	Be responsible for the timing signal of all subsystems, and set up the punctuality mode of different servers
Status Display	*S*	Display terminals, integrated display equipment and supporting control equipment allocated to each subsystem
Information Processing	*I*	Complete the project's downlink data and send uplink instructions, and includes data records, data distribution and data management
Computer Management and UPS	*U*	Ensures the stability of information transmission and provides a reliable environment for system equipment
Technical Support	*G*	Command and control emergency, including fault judgment and emergency alarm, system monitoring management, network timing and other functions
Decision Monitoring	*M*	Monitoring of the decision command, compared with results to optimize decision

clusters using the DEMATEL technique to establish industrial structures.

ISM (Interpretative Structural Modeling Method) is an analysis method and proposed for related issues of complex socio-economic systems in 1973. Its basic principle is to build the system structure model based on graph theory and matrix tools. Clarify the hierarchical relationship between the elements and the elements in the system. ISM is often used in the qualitative and quantitative phases of complex systems combined with structural analysis. Sajid *et al.* (2017) presented an ISM approach that was used to identify relationships among risk factors while a Bayesian Network (BN) approach was employed to define the strength of dependence and conduct a risk analysis. Mandal and Deshmukh (1994) developed an interpretive structural model to show the inter-relationship of different criteria and their levels of importance in the vendor selection process.

In the hierarchical division and decision-making of complex systems, if DEMATEL and ISM are used together, expert knowledge can be integrated systematically to make decisions. Song *et al.* (2016) proposed a rough weighted DEMATEL which has merit in flexibly manipulating the vagueness and ambiguity involved in risk analysis. Zhou *et al.* (2006) proposed a new method for constructing and adjusting complex system hierarchies through integrated DEMATEL and ISM models. Its calculation is simple, and the results are easy to observe, which makes this combined method more convenient when the target model is slightly adjusted. Shen *et al.* (2014) combined both methods and adopted a multi-level hierarchical structure model to obtain key subsystems, and intuitively presented the logical relationship between multiple fault subsystems and their mutual influence.

When combined with DEMATEL and ISM, the above methods can effectively represent the importance ranking of objects of products with mature design, but most existing methods rely on expert knowledge and experience or fault data without considering the inherent topological characteristics of the system. Once the lack of sample data and expert experience due to product updates are not applicable, their role will be very limited, and the final results will deviate from the actual situation to a large extent. Therefore, to make the hierarchical division of complex systems more independent and objective, a hierarchical system fault sequencing method based on DEMATEL, and ISM is proposed. Based on module sequencing using these two methods, the transmissibility based on edge betweenness is further introduced to characterize the influence degree of fault propagation and diffusion in the command and control system. Section 2 builds the direct impact matrix of the fault based on the structural composition of the accusation system, and conduct normalization processing. Considering the direct and indirect fault integration effects among each subsystem. Section 3 constructs the overall impact matrix, and fault delamination of complex systems is obtained by removing the influence of overstep and self-influence and using a directional edge connection between hierarchical subsystems. In Section 4, transmissibility analysis and propagation comparison are derived by the results of the previous three sections, and the important Propagation path is finally obtained.

2. A fault propagation model for the CCS

In this section, the fault propagation model is established based on fault data of the command & control system, and the modules are sorted based on their importance.

2.1 An Initial fault model

Different from the common physical space combat system, the present command and control system (CCS) designed with complex electronic information is used to solve conflicts and defects between network space nodes and their attributes, including computer hardware components, application software, database, network link connection, port characteristics, and communication protocols. Levis *et al.* (2015) describe the information architecture in structured analysis and explain the architectural view of the framework and related products. Considering the close and intricate relationship between

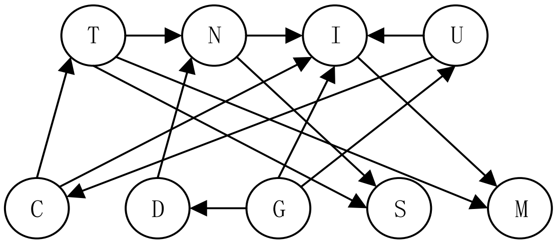

Fig. 1 Command & control system fault propagation

Table 2 System failure statistics in 1000 hours

Failure Frequency	The fault system	Caused by
4	*N*	*C*
3	*I*	*C*
3	*N*	*D*
5	*S*	*N*
3	*I*	*N*
1	*S*	*T*
2	*D*	*T*
2	*M*	*I*
2	*C*	*U*
3	*I*	*U*
4	*D*	*G*
1	*I*	*G*
2	*U*	*G*

each sub-system, many methods describe nodes and their attributes in the network space by using topological diagrams with weights, to realize the positioning of fault information in the network space. In addition, the importance evaluation indicators of the main influencing factors of the system also include the size of the components, the number of component connections, and the minimum distance between nodes.

Since that, only paying attention to the characteristics of the node itself and ignoring the overall composition makes the final analysis result produce unexpected differences, and the remedy to this problem is to combine key points in the structural model and representative statistics. Based on the above considerations, research on the fault propagation in CCS combines ISM and DEMATEL and is given here. Table 1 shows the composition of a typical CCS its specific functions.

Based on the functions performed, the CCS fault propagation model is shown in Fig. 1, where the arrow represents the direction of the fault transfer.

2.2 Direct influence matrix

The subsystem fault impact matrix $P=\left[p_{ij}\right]_{n\times n}$ is first constructed based on the CCS fault propagation directed graph, and the element is expressed as

$$P_{ij}=\begin{cases} b & \text{the number of failures of system } j \text{ caused by system } i \\ 0 & \text{no fault correlation between system } i \text{ and system } j \end{cases} \tag{1}$$

where b is the failure frequency between subsystems. In the experiment conducted by Wuhan Digital Engineering Research Institute, the traceability failures of each subsystem under different task modes under 1000 hours were statistically sorted out, including mutual support of each subsystem under different combat tasks, as shown in Table 2.

Based on the propagation model, the direct impact matrix is expressed as follows

$$P=\begin{array}{c} C \\ D \\ N \\ T \\ S \\ I \\ U \\ G \\ M \end{array}\begin{bmatrix} 0 & 0 & 4 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2.3 Subsystem reachability matrix

Then, the direct impact matrix is normalized to a matrix T as

$$T=\left[t_{ij}\right]=\frac{P}{\left(\max\limits_{1\le i\le n}\sum_{j=1}^{n}p_{ij}\right)} \tag{2}$$

where $\max\limits_{1\le i\le n}\sum_{j=1}^{n}p_{ij}$ is the maximum row sum of the matrix P.

Further, the comprehensive accumulation of direct and indirect faults is considered to construct the comprehensive influence matrix using $Q=[q_{ij}]$, i.e.

$$Q=\lim_{k\to\infty}\left(T+T^2+\cdots+T^k\right)=T\left(I-T\right)^{-1} \tag{3}$$

where I is the unit matrix; q_{ij} is the value of the comprehensive failure influence of system v_i on system v_j; $q_{ij}\neq 0$ denotes fault propagation relationship between systems based on the comprehensive failure impact matrix T, the unit matrix I is used to construct the overall influence matrix as $H=[h_{ij}]=Q+I$.

The fault reachability matrix is given by $A=[a_{ij}]_{n\times n}$

$$a_{ij}=\begin{cases} 1, & h_{ij}>\lambda \\ 0, & h_{ij}\le\lambda \end{cases}\quad\left(i,j=1,2,3,\cdots,n\right) \tag{4}$$

where λ is the settled threshold, and λ is set to 0.01 in this paper since the system is smaller. From the ISM method, the reachable set R_i of system V_i is established. The value of 1 is set to all columns in row i of matrix A. The antecedent set S_i is established for the system which is set corresponding to all rows of 1 in column i. Where S_i and R_i are expressed as follows

$$R_i=\left\{v_j\middle|v_j\in V,a_{ij}=1\right\},\left(i=1,2,\cdots,n\right) \tag{5}$$

$$S_i=\left\{v_j\middle|v_j\in V,a_{ji}=1\right\},\left(i=1,2,\cdots,n\right) \tag{6}$$

Table 3 Importance ranking of subsystems

Layer	L1	L2	L3	L4	L5
Subsystem	S,M	T,I	U,C,N	D	G

If $C_i=R_i \cap S_i=R_i$ is tenable, system v_i belongs to the termination system set and is located in the topmost L1 layer, usually being regarded as a fault input. Then, the iteration repeats until all systems are deleted and the decomposition of the reachability matrix divides the node hierarchy. The obtained results are shown in Table 3.

3. Estimation of directed edge load properties based on betweenness

In this section, the model structure index is extracted, and the model characteristic parameters affecting the fault propagation process are evaluated to prepare for the qualitative analysis of the transmissibility of each subsystem and each propagation path.

3.1 Skeleton matrix extraction

The framework matrix is extracted, and the layered fault propagation directed graph model of the charging system is constructed according to the order of dividing. The structure matrix A' is formed as follows,

$$A' = \begin{array}{c} S \\ M \\ T \\ I \\ N \\ \mathrm{U} \\ C \\ D \\ G \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

In the DEMATLE method, the fault propagation relationship (including its duality) across layers or between the same layer is not allowed, the leaping element “1” and the identity matrix need to be removed from the structure matrix, which is represented between nodes U and C in the third layer of the system in this paper. Therefore, the final skeleton matrix is,

$$A'' = \begin{array}{c} S \\ M \\ T \\ I \\ N \\ U \\ C \\ D \\ G \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

According to the above-mentioned steps, the system fault propagation graph becomes the directed graph with propagation from bottom to top, as shown in Fig. 2. Since the failure relationship in the same layer is eliminated, it is necessary to introduce a virtual node D* in their next level to simulate the propagation process. In the fault propagation directed graph model, the virtual node does not exist. Its effect only adjusts the hierarchical structure of the corresponding component and could be used in the calculation of final path identification and fault extraction, while the actual system component node does not have a corresponding indicator parameter.

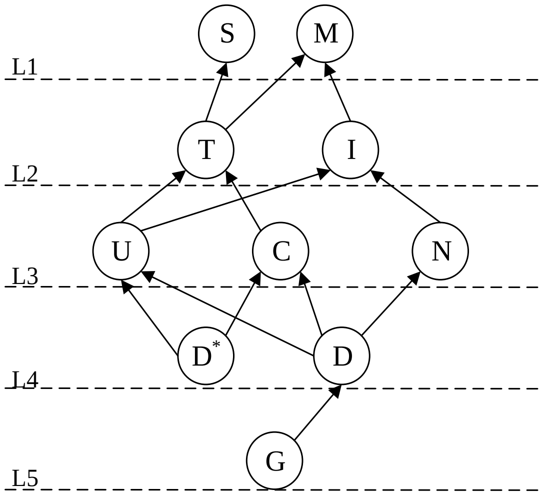

Fig. 2 Initial hierarchical fault propagation of the subsystem

Table 4 Common node importance characterization methods

Type	Definition & Function	Pros & Cons
Degree	The number of edges adjacent to the nodes; the sum of the in and out degrees	The network structure is not considered, while is easy to calculate
Approach degree	Reciprocal of the sum of the shortest distances from one node to all other nodes; Reflects the centrality of the node in the network	Considers the network topology, but only applied to sectional network topologies
Betweenness	The proportion of the number of nodes passing through the shortest path in the network; Characterizes the role of nodes in connected networks	Considers the global network relationship while the shortest path calculation process is complicated

3.2 Typical importance evaluation index

A network model is a simplified abstract representation of the fault propagation relationship of complex systems. Nodes in the network have different inequality characteristics due to their different physical locations, i.e., each node has a different importance to the whole network or other nodes. The importance reflects the impact of each node on the fault propagation and provides theoretical support for identifying the critical fault propagation path. The common node importance characterization methods and their advantages are listed in Table 4.

In addition to that, Li *et al.* (2004) developed an approach based on the shortest distance by quantifying the

Table 5 Statistic of the shortest path

Arbitrary two nodes	Shortest path	Arbitrary two nodes	Shortest path
$v_G \longrightarrow v_D$	$v_G e_{G\to D} v_D$	$v_C \longrightarrow v_I$	$v_C e_{C\to I} v_I$
$v_G \longrightarrow v_N$	$v_G e_{G\to D} v_D e_{D\to N} v_N$	$v_C \longrightarrow v_I$	$v_C e_{C\to T} v_T$
$v_G \longrightarrow v_I$	$v_G e_{G\to D} v_D e_{D\to N} v_N e_{N\to I} v_I$ $v_G e_{G\to D} v_D e_{D\to C} v_C e_{C\to I} v_I$ $v_G e_{G\to D} v_D e_{D\to U} v_U e_{U\to I} v_I$	$v_C \longrightarrow v_M$	$v_C e_{C\to I} v_I e_{I\to M} v_M$ $v_C e_{C\to T} v_T e_{T\to M} v_M$
$v_G \longrightarrow v_M$	$v_G e_{G\to D} v_D e_{D\to N} v_N e_{N\to I} v_I e_{I\to M} v_M$ $v_G e_{G\to D} v_D e_{D\to C} v_C e_{C\to I} v_I e_{I\to M} v_M$ $v_G e_{G\to D} v_D e_{D\to C} v_C e_{C\to T} v_T e_{T\to M} v_M$ $v_G e_{G\to D} v_D e_{D\to U} v_U e_{U\to T} v_T e_{T\to M} v_M$ $v_G e_{G\to D} v_D e_{D\to U} v_U e_{U\to I} v_I e_{I\to M} v_M$	$v_D \longrightarrow v_M$	$v_D e_{D\to N} v_N e_{N\to I} v_I e_{I\to M} v_M$ $v_D e_{D\to C} v_C e_{C\to I} v_I e_{I\to M} v_M$ $v_D e_{D\to C} v_C e_{C\to T} v_T e_{T\to M} v_M$ $v_D e_{D\to U} v_U e_{U\to T} v_T e_{T\to M} v_M$ $v_D e_{D\to U} v_U e_{U\to I} v_I e_{I\to M} v_M$
$v_G \longrightarrow v_T$	$v_G e_{G\to D} v_D e_{D\to C} v_C e_{C\to T} v_T$ $v_G e_{G\to D} v_D e_{D\to U} v_U e_{U\to T} v_T$	$v_G \longrightarrow v_S$	$v_G e_{G\to D} v_D e_{D\to C} v_C e_{C\to T} v_T e_{T\to S} v_S$ $v_G e_{G\to D} v_D e_{D\to U} v_U e_{U\to T} v_T e_{T\to S} v_S$
$v_C \longrightarrow v_U$	$v_C e_{C\to U} v_U$	$v_U \longrightarrow v_C$	$v_U e_{U\to C} v_C$
$v_G \longrightarrow v_C$	$v_G e_{G\to D} v_D e_{D\to C} v_C$	$v_U \longrightarrow v_I$	$v_U e_{U\to T} v_T$
$v_G \longrightarrow v_U$	$v_G e_{G\to D} v_D e_{D\to U} v_U$	$v_U \longrightarrow v_I$	$v_U e_{U\to I} v_I$
$v_D \longrightarrow v_T$	$v_D e_{D\to U} v_U e_{U\to T} v_T$ $v_D e_{D\to C} v_C e_{C\to T} v_T$	$v_U \longrightarrow v_M$	$v_U e_{U\to I} v_I e_{I\to M} v_M$ $v_U e_{U\to T} v_T e_{T\to M} v_M$
$v_D \longrightarrow v_U$	$v_D e_{D\to U} v_U$	$v_U \longrightarrow v_S$	$v_U e_{U\to T} v_T e_{T\to S} v_S$
$v_D \longrightarrow v_C$	$v_D e_{D\to C} v_C$	$v_T \longrightarrow v_S$	$v_T e_{T\to S} v_S$
$v_D \longrightarrow v_N$	$v_D e_{D\to N} v_N$	$v_T \longrightarrow v_M$	$v_T e_{T\to M} v_M$
$v_C \longrightarrow v_S$	$v_C e_{C\to T} v_T e_{T\to S} v_S$	$v_N \longrightarrow v_M$	$v_N e_{N\to I} v_I e_{I\to M} v_M$
$v_D \longrightarrow v_I$	$v_D e_{D\to N} v_N e_{N\to I} v_I$ $v_D e_{D\to C} v_C e_{C\to I} v_I$ $v_D e_{D\to U} v_U e_{U\to I} v_I$	$v_N \longrightarrow v_I$	$v_N e_{N\to I} v_I$
		$v_D \longrightarrow v_S$	$v_D e_{D\to C} v_C e_{C\to T} v_T e_{T\to S} v_S$ $v_D e_{D\to U} v_U e_{U\to T} v_T e_{T\to S} v_S$

damage to network connectivity. Dangalchev (2006) introduced residual closeness to evaluate the average closeness centrality after the removal of nodes and measure the vulnerability of the network. Considering the dynamic nature of network nodes when they increase or decrease, Restrepo *et al.* (2006) believe that the importance of nodes can also be measured by the change of the maximum eigenvalue of the network adjacency matrix. Tan *et al.* (2006) proposed a point that if the entire network becomes more agglomerate after contracting a node and its neighbours into a new node, this node is considered to be more important. Betweenness centrality was first proposed by Bavelas (1948), which uses the number of the shortest path and the total number of paths it is in to represent the importance of the path and was further promoted and improved by Shimbel (1953) and Shaw (1954) in node's potential power in controlling the information flow in a network. Freeman (1977) generalized the concept of in-between graph theory and extended it to connected and unconnected networks, forming the now common definition of betweenness the higher the number of times that a node or edge acts as an "intermediary", the greater its mediation centrality will be.

In this paper, an important evaluation method combining betweenness and the number of adjacent edges is adopted. Evaluation of the importance of network nodes can be achieved by extracting structural indicators of the fault propagation model.

Table 6 Edge betweenness rank

Directed edge	$L(e_{i\to j})$	Rank	Directed edge	$L(e_{i\to j})$	Rank
$e_{G\to D}$	7	1	$e_{U\to T}$	2.4	6
$e_{D\to C}$	4.5	2	$e_{N\to I}$	2.1	7
$e_{D\to U}$	4.5	2	$e_{D\to N}$	2.1	7
$e_{T\to S}$	4	3	$e_{I\to M}$	1.8	8
$e_{C\to T}$	3.4	4	$e_{U\to I}$	1.6	9
$e_{T\to M}$	2.7	5			

The edge betweenness $L(e)$ is the proportion of edges that pass all shortest paths in the directed graph pattern. $D = (V, E)$ $(V = \{v_1, v_2, \ldots, v_n\}, E = \{e_{i\to j}\}(1 \le i, j \le n))$; and the directed edge $L(e_{i\to j})$ connecting node v_i to node v_j is expressed as follows.

$$L(e_{i\to j}) = \sum_{\substack{v_i, v_j, v_o, v_k \in V; e_{i\to j} \in E \\ (o,k) \neq (i,j)}} \frac{N_{ok}\left(e_{i\to j}\right)}{N_{ok}} \tag{7}$$

where N_{ok} is the number of the shortest paths between nodes v_o and v_k. $N_{ok}(e_{i\to j})$ is the number of the shortest paths between nodes v_o and v_j through the edge $e_{i\to j}$.

The layered system fault propagation model is calculated by the proposed method, and the statistics of the shortest fault path are obtained in Table 5, and results are shown in Table 6.

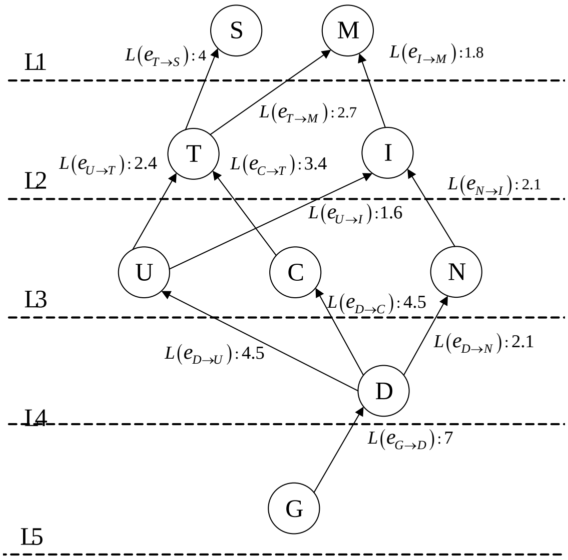

Fig. 3 Fault propagation with edge betweenness

4. Transmissibility analysis and propagation comparison of subsystems

In particular, the virtual node is deleted here. Reasons in: (1) The system scale is small and there are few fault samples, so the equivalent virtual node analysis will produce large errors; (2) The presence of virtual nodes at the initial level will further expand the error in (1); (3) Deleting the virtual node D^* will make the risk assessment of node D higher, but this is far less than the error caused by retaining the virtual node. In summary, the final fault propagation with edge betweenness is shown in Fig. 3.

Compared with Fig. 1, the output and input levels of nodes in the hierarchical model have changed a lot, which is caused by the difference between the recorded fault information and the initial system topology model. In other words, the failure propagation probabilities of different sides in the initial model are different. Therefore, to explore their respective importance, often there is a need for fitting each system reliability function. Considering that it is difficult to establish an accurate fit when each subsystem lacks data, there are situations where the failure rate changes after the system task are replaced. Here is a method to evaluate the importance degree of each node by combining the inherent propagation model with hierarchical propagation. The transmissibility of node X is defined as follows

$$H=\left(\|\boldsymbol{\eta}\|_2\right)^n\left[I\sum L(\mathrm{e}_{\to X})+O\sum L(\mathrm{e}_{X\to})\right] \tag{8}$$

where $\boldsymbol{\eta}=[\eta_1,\eta_2,\dots,\eta_1]$ is fault propagation vector which stands for the diffusion property of fault propagation from high level to the low level, and $\eta_i=\frac{\text{Number of nodes in } L_i}{\text{Number of nodes in } L_{i+1}}$, $i<L_{\max}$. n stands for node X in n-th level; $I=\frac{\text{Input degree after hierarchy}}{\text{Node original input degree}}$, $O=\frac{\text{Output degree after hierarchy}}{\text{Node original output degree}}$, $\sum L(e_{\to X})$ and $\sum L(e_{X\to})$ respectively stand for sum edge betweenness to X and sum edge betweenness from X.

Table 7 Node transmissibility rank

Node	Transmissibility	Node	Transmissibility
G	1033.77	T	183.87
D	5277.90	I	41.77
U	329.06	S	15.22
C	240.03	M	6.77
N	81.30		

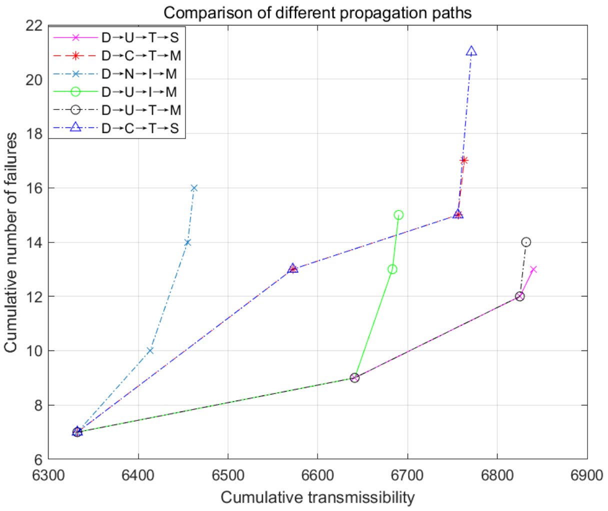

Fig. 4 Cumulative failures of each path except the start node

Herein, the effect of topological structure has been taken into account in the calculation of the number of edge betweenness. That is to say, in the process of determining ηi, only need to pay attention to the level of each node, but not their out and in degree. Since edge betweenness is calculated with sufficient out degree and in degree, it doesn't have to be considered repeatedly.

Therefore, the quantitative impact of nodes should only be recorded in I and O, while H, represent transmissibility, is a comprehensive evaluation parameter that integrates the essential structure and the failure statistical results. It can be obtained at a lower cost and assists in the evaluation of reliability. According to Eq. (8), the transmissibility of each node is shown in Table 7.

According to the statistical failure records in Table 2 and the hierarchy of Fig. 3, the transmissibility and the number of faults is accumulated along different paths. Since the first step of all paths is to propagate from G to D, the first process of all paths is uniformly omitted. Finally, the comparison of the six main propagation paths is shown in Fig. 4.

Obviously, path $D\to N\to I\to M$ fails the fastest in the lowest transmissibility growth. In other words, it has the advantage in topological structure but has produced worse results. That means, the reliability of node N and node I are lower, so that they are more likely to pass the fault to the next node on a large scale. In contrast, the U node in the same layer is more reliable that when its transmissibility increased by 329, only 2 more failures were caused. And it is undeniable that path $D\to C\to T\to S$ spreads the most faults (21) and the nodes it contains also have higher transmissibility, which makes it the main fault propagation path.

5. Conclusions

In this paper, the digraph model of fault propagation in command & control system is constructed by using digraph theory. Based on the matrix tool and integrated DEMATEL/ISM method, the system fault propagation directed graph model is constructed.

According to the established fault propagation model, the model index is extracted and used to evaluate the characteristic parameters of the model that affect fault propagation behaviour. It is an important part of the final system key fault propagation path identification principle from what we can infer the main propagation path is $G \rightarrow D \rightarrow C \rightarrow T \rightarrow S$. After further consideration of node importance, it is obvious that although node D is not the initial node, it is most likely the source of the failure, and the core path would be changed as $D \rightarrow U \rightarrow T \rightarrow S$. In future works, focus of attention will be the system fault propagation path identification method based on fault propagation strength and application to the reliability verification of the integrated function of the system.

Acknowledgments

This research was funded by the National Natural Science Foundation of China under contract No. 51775090.

References

Bai, S., Li, Y.F., Huang, H.Z., Yu, A.D. and Zeng, Y. (2021), "An improved petri net for fault analysis of an electronic system with hybrid fault of software and hardware", *Eng. Fail. Anal.*, **120**, 120105077. https://doi.org/10.1016/j.engfailanal.2020.105077.

Bavelas, A.A. (1948), "A mathematical model of group structures", *Human Organiz.*, **7**(3), 16-30. http://doi.org/10.17730/humo.7.3.f4033344851gl053.

Dangalchev, C. (2006), "Residual closeness in networks", *Mech. Appl.*, **365**(2), 556-564. https://doi.org/10.1016/j.physa.2005.12.020.

Fang, A.L., Gao, Q.S. and Zhang, S.Y. (2009), "Application of networked DEMATEL method in industry economic system analysis", *Math. Pract. Theory*, **39**(5), 78-83.

Freeman, L.C. (1977), "A set of measures of centrality based on betweenness", *Sociometry*, **40**(1), 35-41. http://doi.org/10.2307/3033543.

Jensen, F. (2007), "Bayesian networks and decision graphs", *Technom.*, **45**(2), 178-179. https://doi.org/10.1007/978-1-4757-3502-4.

Kolda, T., Pinar, A., Plantenga, T. and Seshadhri, C. (2014), "A scalable generative graph model with community structure", *SIAM J. Scientif. Comput.*, **36**(5), 424-452. http://doi.org/10.1137/130914218.

Lee, H.S., Tzeng, G.H., Yeih, W., Wang, Y.J. and Yang, S.C. (2013), "Revised DEMATEL: Resolving the infeasibility of DEMATEL", *Appl. Math. Model.*, **37**(10-11), 6746-6757. https://doi.org/10.1016/j.apm.2013.01.016.

Levis, A.H. and Wagenhals, L.W. (2015), "C4ISR architectures: I. Developing a process for C4ISR architecture design", *Syst. Eng.*, **3**(4), 225-247. https://doi.org/10.1002/1520-6858(2000)3:4<225::AID-SYS4>3.0.CO;2-%23.

Li, P.X., Ren, Y.Q. and Xi, Y.M. (2004), "An importance measure of actors (set) within a network", *Syst. Eng.*, **22**(4), 13-20.

Li, Y.F., Huang, H.Z., Mi, J., Peng, W. and Han, X. (2022), "Reliability analysis of multi-state systems with common cause failures based on Bayesian network and fuzzy probability", *Ann. Oper. Res.*, **311**(1), 195-209. https://doi.org/10.1007/s10479-019-03247-6.

Li, Y.F., Liu, Y., Huang, T., Huang, H.Z. and Mi, J. (2020), "Reliability assessment for systems suffering common cause failure based on Bayesian networks and proportional hazards model", *Qual. Reliab. Eng. Int.*, **36**(7), 2509-2520. https://doi.org/10.1002/qre.2713.

Lin, C.L. and Tzeng, G.H. (2009), "A value-created system of science (technology) park by using DEMATEL", *Exp. Syst. Appl.*, **36**(6), 9683-9697. https://doi.org/10.1016/j.eswa.2008.11.040.

Mandal, A. and Deshmukh, S.G. (1994), "Vendor selection using interpretive structural modelling (ISM)", *Int. J. Oper. Prod. Manage.*, **14**(6), 52-59. http://doi.org/10.1108/01443579410062086.

Murata, T. (1989), "Petri nets: Properties, analysis and applications", *Proc. IEEE*, **77**, 541-580. http://doi.org/10.1109/5.24143.

Nakano, H. and Nakanishi, Y. (1974), "Graph representation and diagnosis for multiunit faults", *IEEE Trans. Reliab.*, **23**(5), 320-325. http://doi.org/10.1109/TR.1974.5215295.

Restrepo, J.G., Ott, E. and Hunt, B.R. (2006), "Characterizing the dynamical importance of network nodes and links", *Phys. Rev. Lett.*, **97**(9), 094102. https://doi.org/10.1103/physrevlett.97.094102.

Sajid, Z., Khan, F. and Zhang, Y. (2017), "Integration of interpretive structural modelling with Bayesian network for biodiesel performance analysis", *Renew. Energy*, **107**, 194-203. https://doi.org/10.1016/j.renene.2017.01.058.

Seyed-Hosseini, S.M. and Safaei, N. (2006), "Asgharpour, reprioritization of failures in a system failure mode and effects analysis by decision making trial and evaluation laboratory technique", *Reliab. Eng. Syst. Saf.*, **91**(8), 872-881, https://doi.org/10.1016/j.ress.2005.09.005.

Shaw, M.E. (1954), "Group structure and the behavior of individuals in small groups", *J. Psychol.*, **38**(1), 139-149. http://doi.org/10.1080/00223980.1954.9712925.

Shen, G.X., Sun, S.G. and Zhang, Y.Z. (2014), "System failure analysis based on DEMATEL-ISM and FMECA", *J. Central South Univ.*, **21**(12), 4518-4525. http://doi.org/10.1007/s11771-014-2456-8.

Shimbel, A. (1953), "Structural parameters of communication networks", *Bull. Math. Biophys.*, **15**(4), 501-507. http://doi.org/10.1007/BF02476438.

Song, W., Ming, X. and Liu, H.C. (2016), "Identifying critical risk factors of sustainable supply chain management: A rough strength-relation analysis method", *J. Clean. Prod.*, **143**, 100-115. https://doi.org/10.1016/j.jclepro.2016.12.145.

Tan, Y.J., Wu, J. and Deng, H.Z. (2006), "Evaluation method for node importance based on node contraction in complex networks", *Syst. Eng.*, **26**(11), 79-83.

Tarifa, E.E. and Nicolás, J. (1997), "Fault diagnosis, direct graphs, and fuzzy logic", *Comput. Chem. Eng.*, **21**, 649-654. https://doi.org/10.1016/S0098-1354(97)87576-X.

Wu, W.W. and Lee, Y.T. (2007), "Developing global managers competencies using the fuzzy DEMATEL method", *Exp. Syst. Appl.*, **32**(2), 499-507. https://doi.org/10.1016/j.eswa.2005.12.005.

Zhou, D.Q., Zhang, L. and Li, H.W. (2006), "A study of the system's hierarchical structure through integration of dematel and ISM", *Int. Conf. Mach. Learn. Cybernet. IEEE*, **77**(1), 1449-1453, https://doi.org/10.1109/ICMLC.2006.258757.

Zuleta, L., Madrigal, G.Z. and Carranza, D. (2007), "Hybrid system based on fuzzy inference and colored petri nets to identify electrical fault events in real time", *Electronics, Robotics and Automotive Mechanics Conference (CERMA 2007)*, September.