

Hierarchical structure parameters in three dimensional turbulence: She-Leveque model

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Abstract. Hierarchical structure parameters, proposed in She-Leveque model, are investigated for velocity components obtained from different flow types over a large range of Reynolds numbers $255 < Re_\lambda < 720$. The values of intermittency parameter β , with respect to a fixed velocity component, are observed nearly same for all four types of turbulence. The parameter γ , for streamwise velocity components is nearly the same but significantly different for vertical components in different flows. It is also observed that for both parameters, an obvious relation between the longitudinal and transverse components $\beta_T < \beta_L$ (and $\gamma_T < \gamma_L$) always holds. However, the difference between β_L and β_T is found very small in all types of turbulent flows, we studied here. It is evidenced that at low Reynolds numbers, the deviations from K41 scaling are mainly due to the most intense structures and slightly because of more heterogeneous hierarchy of fluctuation structures. However, at higher Reynolds numbers the deviations seem as a consequence of the most intense structures only. Over all, the study suggests that the hierarchy parameter β may be consider as a universal constant.

Keywords: hierarchical structure parameters; intermittency; structure functions

1. Introduction

Turbulence is a ubiquitous phenomena where the velocity fluctuates strongly with a hierarchical process in which energy transfer from large to small scales (Frisch and Kolmogorov 1995). One of the main components of statistical theory of turbulence is p th-order velocity structure function that provides basic knowledge for the analysis of the scaling properties of fully developed turbulence (Harris *et al.* 1977). A central piece of information on the small scale motion in turbulence lies in the p th-order structure function that is defined as: where the velocity difference $\delta u_r = u(x+r) - u(x)$, yields velocity of eddies of size r , $\langle \bullet \rangle$ denotes ensemble average done over x and p is order of structure function. Structure functions are called longitudinal structure functions $S_L^{(p)}$ and transverse structure functions $S_T^{(p)}$ when flow r is parallel and perpendicular to u respectively.

$$S^{(p)}(r) = \langle |\delta u_r|^p \rangle, \quad (1)$$

According to the Kolmogorov theory for homogenous

and isotropic turbulence (referred to as K41 theory hereafter) statistical properties of turbulence are scale invariant within the inertial range and scaling exponent increases linearly with order of structure functions, i.e., $S^{(p)}(r) \propto r^{p/3}$ (Praskovskiy *et al.* 1993) Studies on various kinds of turbulence revealed the fact that due to intermittent behavior of energy dissipation, scaling of structure functions significantly deviates from K41 scaling and such deviation is known as ‘anomalous scaling’ denoted by $\zeta^{(p)}$ (Anselmetti *et al.* 1984, Fukayama *et al.* 2000, Shen and Warhaft 2002, Jiang *et al.* 2006, Benzi *et al.* 2010), (McComb, 2014, Iyer *et al.* 2017, Iyer *et al.* 2020) Different theoretical and phenomenological models are proposed to formulate this anomalous scaling (Kolmogorov 1962, Frisch *et al.* 1978, Roberto Benzi *et al.* 1984). One of the most accepted descriptions of anomalous scaling is provided by Hierarchical Structure model (referred to as HS model hereafter) proposed by She and Leceque (1994). Several studies including experimental (Chavarria *et al.* 1995, Saw *et al.* 2017) and numerical models (Fisher *et al.* 2008, Berera and Ho 2018) has established the validity of HS model. Recently some researcher used different methods for nonlinear modeling (Tohidi *et al.* 2018, Arefi and Zenkour 2017, Arani *et al.* 2018, Krommer *et al.* 2016, Yeh 2016) and for other structures (Boussoula *et al.* 2020, AlSaleh and Fuggini 2020, Lee *et al.* 2019, Zahrai and

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Kakouei 2019, Poplawski *et al.* 2019).

The longitudinal and transverse components of hierarchy parameters β and γ in different flow types measured at various Reynolds numbers is found. The hierarchy parameter β measures the degree of hierarchical similarity whereas γ determines the singularity of the most intermittent structures of a turbulent flow. A smaller value of β indicates that flow is more heterogeneous while a smaller γ implies that the most intermittent structures are more singular. It is noted that that β -values for longitudinal components are nearly same and close to the theoretical value. The transverse components of β , in different flows, are also close to each other and to the corresponding values of β_L , which leads to the possibility that β is universal constant. For same flows, almost same longitudinal but significantly different transverse values of γ are observed which show the relation of γ with non-universal aspects of flows.

The objective of this work is to calculate the hierarchy parameters with both kinds, experimental and Direct Numerical Simulation (DNS), of datasets measured at variety of Reynolds numbers. The paper is organized as follows. In Section 2, we present an overview of Hierarchical Structure model especially, we explain basic parameters β and γ of this model. In Section 3, we give a brief description of experimental and DNS datasets being used in this study. The results obtained from the datasets with discussion on hierarchical structure parameters followed by the probability density functions and structure functions are given in Section 4. Finally, concise concluding remarks are listed in Section 5.

2. Materials and methods

2.1 Hierarchical structure model

HS model was proposed by She and Leveque in 1994 that can explain small-scale fluctuations in the inertial range using hierarchy of moment ratios (She and Leveque 1994). The hierarchy $F^{(p)}(r)$ is defined with the velocity structure functions $S^{(p)}(r) = \langle |\delta u_r|^p \rangle$ as

$$F^{(p)}(r) = \frac{S^{(p+1)}(r)}{S^{(p)}(r)}, \quad p = 0, 1, 2, \dots \quad (2)$$

which has the dimension of Δu_r and called p th-order HS function. The amplitude of the fluctuations is also equal to $F^{(p)}$ that increases monotonically with p (Jiang *et al.* 2006) and therefore characterizes fluctuation structures of increasing intensities from the mean $F^{(0)}(r) = \langle |\Delta u_r| \rangle$ to the most intermittent structure $F^{(\infty)}(r) = \lim_{p \rightarrow \infty} F^{(p)}(r)$. The hierarchy covers both: a range of scale and a range of intensities. The most intense structures $F^{(\infty)}(r)$ has a special role in definition of the dynamical state of turbulence, and all other fluctuation structures follow the hierarchical relation

$$\frac{F^{(p+1)}(r)}{F^{(\infty)}(r)} = A^{(p)} \left(\frac{F^{(p)}(r)}{F^{(\infty)}(r)} \right)^\beta, \quad (3)$$

where $0 \leq \beta \leq 1$ is a parameter and $A^{(p)}$ is independent of

r . This assumption implies a general formula which can determine whole set of scaling exponents $\zeta^{(p)}$ in fully developed turbulence

$$\zeta^{(p)} = \gamma p + C(1 - \beta^p) \quad (4)$$

where γ represents the amplitude of the most intermittent structure and C is its co-dimension. For further development of HS model, the hypothetical quantity $F^{(\infty)}(r)$ is eliminated, from Eq. (3), yields scaling relation

$$\frac{F^{(p+1)}(r)}{F^{(2)}(r)} = \frac{A^{(p)}}{A^{(1)}} \left(\frac{F^{(p)}(r)}{F^{(1)}(r)} \right)^\beta \quad (5)$$

By employing hierarchical symmetry assumptions of Eq. (3), She *et al.* developed methods called “ β -test” and “ γ -test” to measure β and γ (Liu *et al.* 2004). The theoretical values of β , γ and C equal to $(\frac{2}{3})^{1/3}$, $\frac{1}{9}$ and 2 respectively. The HS model incorporates the energy cascade as a log-Poisson process and can be construct as a limiting case of random β -model (Dubrulle 1994). In HS model, β measures the similarity between the two successive HS functions $F^{(p)}$ and $F^{(p+1)}$ and measure of degree of singularity of the most intermittent structure is called γ . The parameter β provides quantitative description of the degree of order (disorder) and homogeneity (heterogeneity) of chaotic flow (Liu *et al.* 2004). A β -value of unity implies an ordered and homogenous flow with uniformly distributed structures where $F^{(p)}$ and $F^{(p+1)}$ are identical. A smaller β is a manifestation of a flow containing mixture of order and disorder and ultimately more heterogeneous, i.e., flow having different HS functions. The parameter γ determines how singular the most intermittent structure is. A smaller γ -value implies that the most intermittent structures are more singular. The values of β and γ , for K41 scaling model are found 1 and 1/3 respectively (She and Zhang 2009, Baroud *et al.* 2003).

While working with different experimental, numerical datasets and GOY shell model, Liu and She focused on longitudinal components of β and γ and found respective values of $\beta = 0.874, 0.852$ and 0.874 whereas respective γ -values are $0.045, 0.09$ and 0.125 . They showed that γ is sensitive to the flow type and also varies with the statistical sample size (Poff and Zimmerman 2010). From Couette-Taylor and free jet turbulence, the measured γ -value, for longitudinal components, was found ~ 0.10 (She *et al.* 2001, Frisch and Kolmogorov 1995). There is a lack of consensus among the turbulence community about β , measured from longitudinal and transverse velocity components (Grossmann *et al.* 1997, Bortave 1997). These studies also showed that different β -values of β for longitudinal and transverse velocity components (She and Zhang 2009). These studies also showed different β -values for different orders of structure functions and hence concluded a non-universal nature of β . On the other hand, Jiang *et al.* conducted experiments in a turbulent mixing layer and observed approximately same β -values for both velocity components and pointed out that β can be considered a universal constant (Jiang *et al.* 2006). These conflicts provide grounds for further investigations of universality of HS parameter β , and here, we have collected

Table 1 Some important parameters of four datasets, i.e., nearly isotropic decaying channel flow at four measurement locations, jet flow, numerically simulated homogenous shear turbulence (denoted by DNS1), and homogenous isotropic turbulence (denoted by DNS2)

	$x/M = 20$	$x/M = 30$	$x/M = 40$	$x/M = 48$	Jet	DNS1	DNS2
ϵ	22.8	9.13	4.72	3.41		1.080	0.88
η	0.11	0.14	0.16	0.18	0.174	0.0051	0.0026
λ	5.84	7.13	8.25	8.78	5.86	0.160	0.1023
Re_λ	716	676	650	626	293	255	400
Grid points	–	–	–	–	–	2048 ³	2048 ³

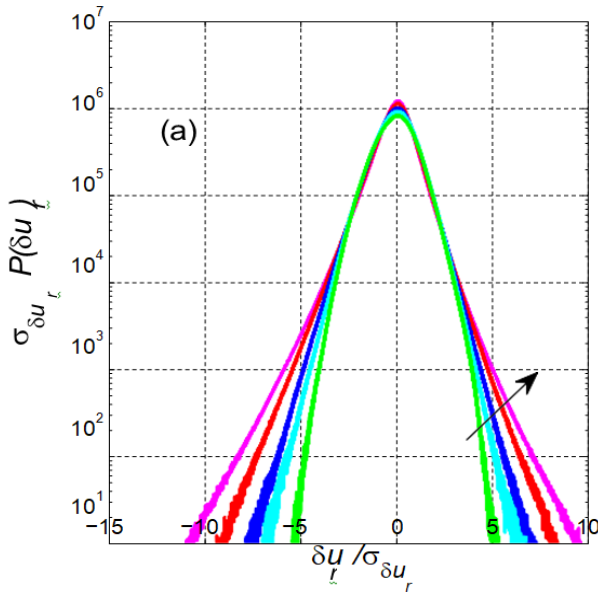


Fig. 1(a) (Color online) Normalized probability density functions measured with DNS dataset of homogenous shear turbulence for velocity increments at various separation scales ranging from dissipation to integral scales. The arrows indicate that PDFs are expanding outwards with decreasing scales

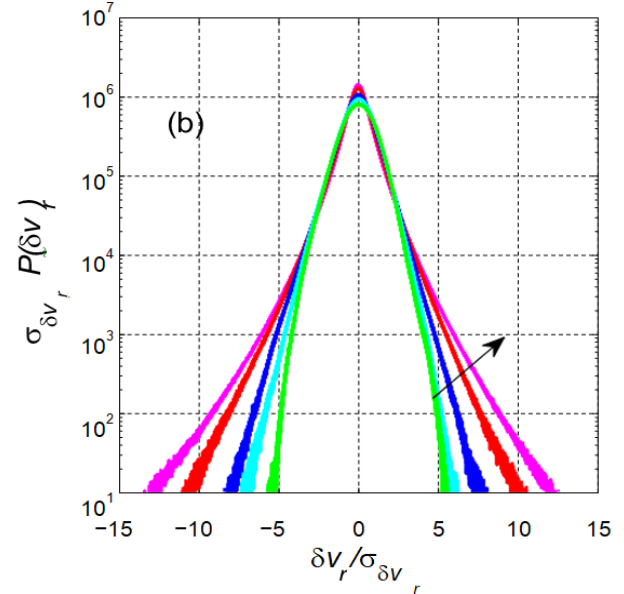


Fig. 1(b) (Color online) Normalized probability density functions measured with DNS dataset of homogenous shear turbulence for transverse velocity increments at various separation scales ranging from dissipation to integral scales. The arrows indicate that PDFs are expanding outwards with decreasing scales

some more evidences that may be helpful to settle this issue.

3. Datasets

In this study, we use four different kinds of datasets including two experimental and two DNS, measured at variety of Reynolds numbers. The first experimental dataset is taken from nearly isotropic decaying channel flow measured at four downstream locations, i.e., $x/M = 20, 30, 40$ and 48 where M is the mesh size. To achieve high Reynolds numbers, an active-grid technique is used at the entrance of the wind tunnel. The Taylor's micro-scale based Reynolds numbers Re_λ at downstream locations, are approximately 720, 680, 650 and 620 respectively. The mean velocities $\langle u \rangle$ at these four locations ranges from $12.0 - 10.8 \text{ ms}^{-1}$. From first to last location, the turbulent intensities decrease continuously and slowly, i.e., $15.4 - 10.0\%$ whereas anisotropy ratios increase from $1.13 - 1.16$. The sampling frequency, f_s , and measurement noise, f_N , are 40 kHz and 20 kHz respectively. At these measurement

locations, nearly two decades of inertial ranges are found where power-law behaviors are observed for both velocity components with scaling exponents close to $-5/3$. More details about the dataset are available in the article of Kang *et al* (Kang *et al.* 2003). Another experimental dataset is obtained from fully developed jet flow measured at $Re_\lambda = 293$. In this flow, jet exit velocity and the Kolmogorov scale were 40 m/s and 0.174 mm respectively (Tong 2001). The direct numerical simulation is emerging as a powerful tool in study of turbulence for last three decades. Beside experimental datasets, we use two DNS datasets measured for different flow environments and Reynolds numbers. The first numerical simulation, denoted by DNS1 here, is performed for homogenous shear turbulence with an imposed velocity gradient in a cubic box of grid points 2048^3 . The DNS1 dataset is measured with mean shear $d\langle u_1 \rangle / dx_2 = 1.5$ and at $Re_\lambda = 255$ (Wang 2010). The second numerical dataset, we denote it by DNS2, is simulated for homogenous isotropic turbulence with numerical resolutions of 2048^3 grid points and $Re_\lambda \approx 400$. For DNS2, The eddy turnover time at the integral scale and the mean energy dissipation rate are 2.2 and 0.88 . Further

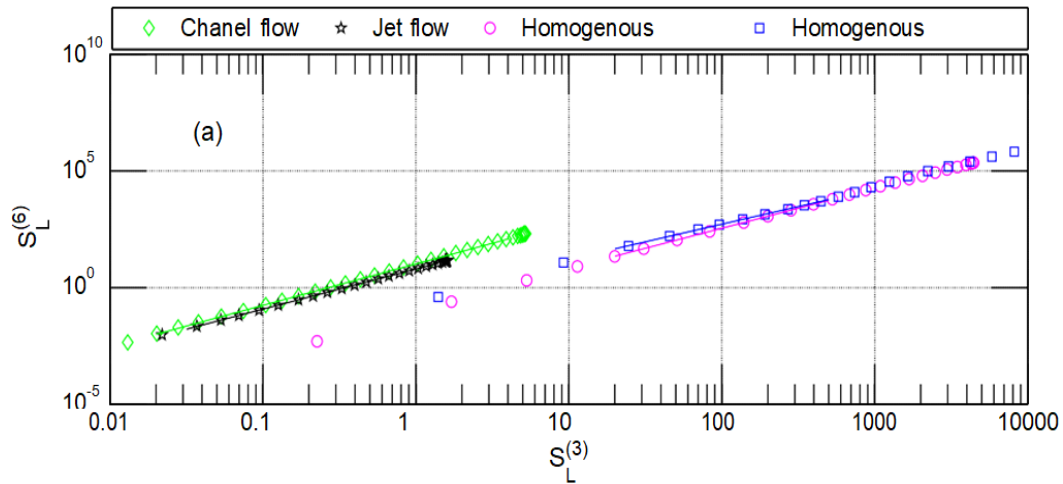


Fig. 2(a) (Color online) Sixth-order of longitudinal structure functions measured with ESS using datasets of nearly isotropic channel flow (\diamond), jet flow (\star), homogenous shear (\circ) and homogenous isotropic turbulence (\square) at $Re_\lambda = 630, 293, 255$ and 400 respectively. Solid lines of corresponding colors represent the inertial-ranges

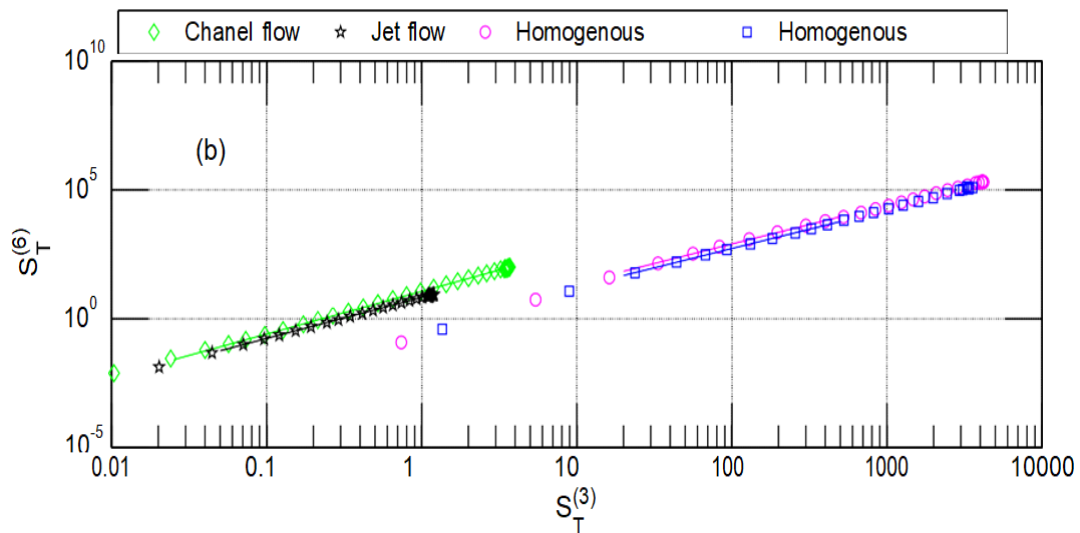


Fig. 2(b) (Color online) Sixth-order of transverse structure functions measured with ESS using datasets of nearly isotropic channel flow (\diamond), jet flow (\star), homogenous shear (\circ) and homogenous isotropic turbulence (\square) at $Re_\lambda = 630, 293, 255$ and 400 respectively. Solid lines of corresponding colors represent the inertial-ranges

details about this dataset can be found in (Bec *et al.* 2009). Some other parameters of these datasets are provided in Table 1. The validity of HS model, especially the universality of β , in various flows implies that different turbulent environments can be differentiated through the property of the most intermittent fluctuations.

4. Results and discussion

4.1 Probability density functions and structure functions

To observe how the distributions of two velocity fields change with scale r see for example (Liu 2017, Friedrich 2020) we measure probability density functions at various separation scales with experimental and DNS datasets and these measurements for longitudinal and transverse velocity

increments: δu_r and δv_r of homogenous shear turbulence are shown in Fig. 1 (similar figures are obtained for other datasets). We see that for larger r both PDFs are close to Gaussian distribution and for smaller r the tails of the PDFs become longer. Longitudinal scaling exponents are measured within the inertial ranges $[0.02, 4]$, $[0.06, 1.5]$, $[20, 500]$ and $[20, 500]$ whereas inertial ranges for transverse scaling exponents are $[0.06, 3.5]$, $[0.07, 1.0]$, $[20, 500]$ and $[20, 500]$ for channel flow, jet flow, homogenous shear and homogenous isotropic turbulence respectively. Among these four datasets, the first two are obtained experimentally whereas the last two are measured with DNS techniques.

In this work, we measure scaling exponents of longitudinal and transverse structure functions using Extended Self-Similarity (ESS) method, a property of turbulence in which scaling of p th-order velocity structure functions versus the third-order structure function can be

Table 2 Scaling exponents of longitudinal and transverse structure functions measured with ESS from streamwise fourth location of nearly isotropic channel flow, jet flow, numerically simulated homogenous shear (DNS1), homogenous isotropic (DNS2) flows, Benzi et al. respectively. In the last column, $\zeta_L^{(p)}$ obtained from The S-L model are also given

	$x/M = 48$		Jet flow		DNS1		DNS2		Benzi		S-L
	$Re_\lambda = 630$		$Re_\lambda = 293$		$Re_\lambda = 255$		$Re_\lambda = 400$		$Re_\lambda = 600$		model
p	$\zeta_L^{(p)}$	$\zeta_T^{(p)}$	$\zeta_L^{(p)}$	$\zeta_T^{(p)}$	$\zeta_L^{(p)}$	$\zeta_T^{(p)}$	$\zeta_L^{(p)}$	$\zeta_T^{(p)}$	$\zeta_L^{(p)}$	$\zeta_T^{(p)}$	$\zeta_L^{(p)}$
1	0.36	0.38	0.37	0.39	0.37	0.38	0.37	0.40	–	–	0.36
	0.69	0.71	0.70	0.72	0.70	0.72	0.70	0.73	0.71	0.71	0.70
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	–	–	1.00
	1.28	1.25	1.27	1.23	1.26	1.24	1.28	1.21	1.29	1.27	1.28
	1.54	1.47	1.51	1.43	1.50	1.44	1.55	1.37	–	–	1.54
	1.78	1.66	1.72	1.59	1.71	1.62	1.81	1.49	1.78	1.68	1.78
	1.99	1.82	1.90	1.73	1.91	1.77	2.07	1.59	–	–	2.001
	2.19	1.97	2.01	1.84	2.11	1.91	2.32	1.67	2.18	1.92	2.21

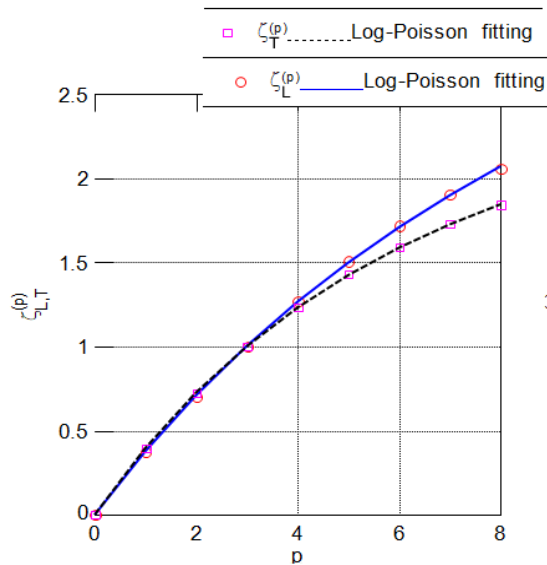


Fig. 3(a) (Color online) Longitudinal and transverse hierarchical structure parameters β and γ measured by log-Poisson fittings using corresponding scaling exponents $\zeta_{L,T}^{(p)}$ of experimentally measured Jet and

obtained even for low Reynolds number flows (Benzi *et al.* 1993). The ESS plots of sixth-order longitudinal and transverse functions are shown in Fig. 2 for all four datasets, being considered here. The ESS property is clearly observed for both velocity components with slightly shorter scaling ranges for transverse structure functions measured from experimental datasets. For channel flow, jet flow, homogenous shear and homogenous isotropic turbulence, the longitudinal scaling exponents are measured within the scaling ranges $[0.02,4]$, $[0.06,1.5]$, $[20,500]$ and $[20,500]$ of $S_L^{(3)}$ respectively whereas transverse scaling exponents respective scaling ranges are $[0.06,3.5]$, $[0.07,1.0]$, $[20,500]$ and $[20,500]$ of $S_T^{(3)}$.

The relative longitudinal and transverse scaling exponents measured by least-square fittings of the ESS plots of fourth measurement location of channel, jet, DNS1 and DNS2 datasets are provided in Table II. The scaling exponents measured at first three locations in channel flow

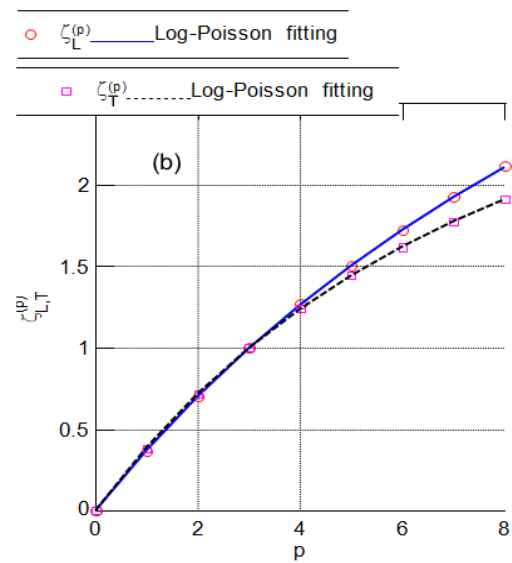


Fig. 3(b) (Color online) Longitudinal and transverse hierarchical structure parameters β and γ measured by log-Poisson fittings using corresponding scaling exponents $\zeta_{L,T}^{(p)}$ of numerically simulated homogenous shear turbulence at $Re_\lambda = 293$ and 255 respectively

are nearly similar to those obtained at fourth location which are not given because they require a lot of space in the Table. In Table II, recently measured scaling exponents by Benzi *et al.* using numerical simulation of homogenous and isotropic turbulence with resolutions of 1856^3 at $Re_\lambda \approx 600$ and theoretical predictions from S-L model are also given for comparison (Benzi *et al.* 2010, She and Leveque 1994). An obvious observation confirming previous studies, for all flows here, is that $\zeta_T^{(p)}$'s are showing more deviations from K41 scaling than $\zeta_L^{(p)}$'s when $p > 3$. Moreover, we observe that $\zeta_L^{(p)}$'s measured from all four locations of channel flow, DNS1 and DNS2 are consistent with the $\zeta_L^{(p)}$ obtained by Benzi *et al.* and theoretical predictions of S-L model, however, higher-order longitudinal scaling exponents estimated from jet flow are slightly smaller than $\zeta_L^{(p)}$'s of all other flows. A possible reason that the flow is

Table 3 Hierarchical structure parameters β and γ measured for longitudinal and transverse velocity components with experimental and DNS datasets. In rows two to five, HS parameters are calculated from nearly isotropic experimental dataset measured at downstream four locations, i.e., $x/M = 20, 30, 40$ and 48 for $Re_\lambda \approx 720, 680, 650$ and 630 respectively. The HS parameters are measured from jet flow at $Re_\lambda = 293$ and given in the sixth row whereas in the last two rows these parameters are obtained from numerically simulated datasets of homogenous shear and homogenous isotropic turbulence at $Re_\lambda = 400$ and 255 respectively

	β_L	β_T	γ_L	γ_T
$x/M = 20$	0.87 ± 0.004	0.84 ± 0.004	0.10 ± 0.004	0.06 ± 0.003
$x/M = 30$	0.86 ± 0.007	0.84 ± 0.004	0.10 ± 0.007	0.06 ± 0.003
$x/M = 40$	0.86 ± 0.006	0.84 ± 0.002	0.10 ± 0.007	0.06 ± 0.002
$x/M = 48$	0.87 ± 0.004	0.84 ± 0.003	0.11 ± 0.004	0.06 ± 0.002
Jet flow	0.85 ± 0.005	0.82 ± 0.002	0.08 ± 0.005	0.03 ± 0.002
DNS1	0.86 ± 0.003	0.83 ± 0.002	0.09 ± 0.003	0.04 ± 0.002
DNS2	0.87 ± 0.005	0.79 ± 0.003	0.12 ± 0.004	-0.001 ± 0.002

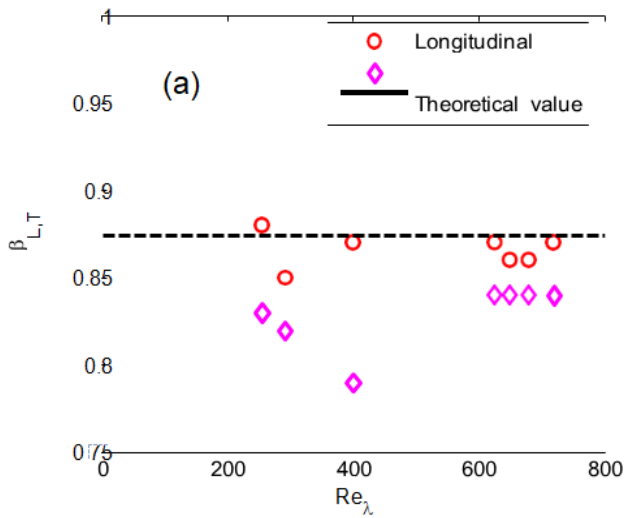


Fig. 4(a) (Color online) Longitudinal and transverse hierarchical parameters measured for β at different Reynolds numbers: $255 < Re_\lambda < 720$ from nearly isotropic channel flow, jet flow, homogenous shear and homogenous isotropic turbulence. The black dashed lines are used for theoretical representation of HS parameters β and γ having values 0.874 and $1/9$ respectively.

measured at relatively low Reynolds numbers due to which not only its longitudinal but also transverse scaling exponents are observed smaller. From the remaining transverse scaling exponents, $\zeta_T^{(p)}$'s from channel flow and DNS1 are showing very good agreement with the results of Benzi *et al.*, however, higher-order $\zeta_T^{(p)}$ measured from DNS2 are significantly smaller than all other corresponding results of Table II which needs further investigations to understand it.

4.2 Hierarchical structure parameters

We measure hierarchical parameters β and γ by applying Log-Poisson fitting on corresponding scaling exponents of four datasets. The HS parameters β characterize the hierarchical similarity and γ provides the degree of singularity of the most intense fluctuation structures which is equal to h_{\min} , the smallest singularity

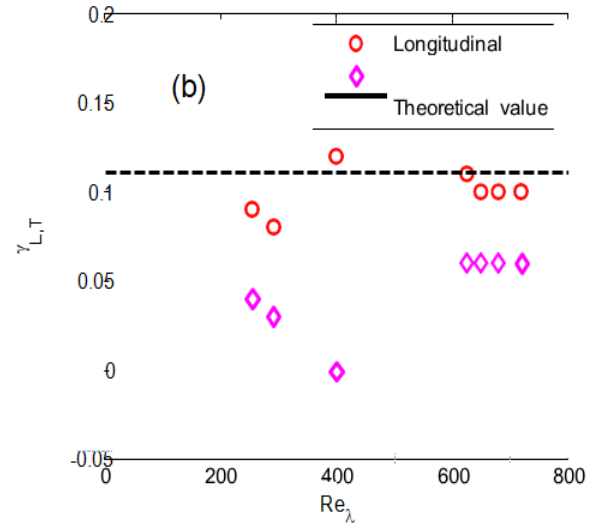


Fig. 4(b) (Color online) Longitudinal and transverse hierarchical parameters measured for γ at different Reynolds numbers: $255 < Re_\lambda < 720$ from nearly isotropic channel flow, jet flow, homogenous shear and homogenous isotropic turbulence. The black dashed lines are used for theoretical representation of HS parameters β and γ having values 0.874 and $1/9$ respectively

index in the framework of multifractality. According to HS model, the parameter β is believed more intrinsic which depends on mechanism of cascade and the universality of the self-scaling of the dissipation fluctuation is implied by the universality of β . From the theory, it can be found that $F^{(\infty)}(r) \sim r^\gamma$ which implies that γ is a property of the high-order moments, i.e., $p \rightarrow \infty$ where turbulence properties can be explored from tail of the PDF. The measurement of HS parameters for longitudinal and transverse components of experimentally measured jet flow and numerically simulated homogenous shear turbulence is shown in Fig. 3. In Table III, we give measured values of hierarchical structure parameters β and γ for longitudinal and transverse velocity components of different flow types. The values of hierarchy parameters are obtained by best fits to the scaling exponents ζ^p within 95% confidence intervals. From Table III, measurements of β show that the longitudinal values measured from experimental and DNS dataset are nearly

similar. The measured values of β_L from the datasets obtained from channel and two DNS flows are more close to the theoretical value of β_L for fully developed turbulence: $(2/3)^{1/3} \approx 0.874$ than the $\beta_L = 0.85$ obtained from experimental dataset of jet flow. The smaller β_L -value in jet flow implies that the flow contains more heterogeneous structures than in other flows. In case of transverse components of β , two experimental and numerically simulated homogenous shear turbulence yield nearly similar values of β_T , i.e., around 0.84 however, a smaller $\beta_T = 0.79$ is measured from homogenous isotropic turbulence, possibly due to its relatively smaller $\zeta_T^{(p)}$ given in Table II.

Measured values of β_L and β_T are following similar trends as observed by Boratav and Grossmann *et al.*, i.e., $\beta_T < \beta_L$ but their measured values and difference are comparatively larger, e.g., β_L and β_T calculated by Grossmann *et al.* are 0.947 and 0.870 respectively (Grossmann *et al.* 1997). Our measurements show that β -values obtained from two velocity components of different flow types are nearly same which implies that β is almost a universal constant and show agreement with the observation of Jiang *et al.* (2006).

In Table III, we also give γ -values, measured from experimental and DNS datasets. We observe that γ_L 's measured from channel flow and DNS dataset of homogenous isotropic turbulence are very close to the theoretical value: $1/9$ and that measured by Liu and She from GOY shell model (Liu and She 2003). Relatively smaller values of $\gamma_L = 0.08$ and 0.09 are found from jet and homogenous shear turbulence respectively are very close to those obtained by Liu and She from DNS dataset of homogenous turbulence and experimental results of Jiang *et al.* (Jiang *et al.* 2006, Shih *et al.* 2005). These smaller γ_L -values indicate that the most intense fluctuations in jet and homogenous shear turbulence are comparatively more singular than in channel and homogenous isotropic flows. The transverse components of γ has significantly different values for each flow type, that is, $\gamma_T = 0.06, 0.03, 0.04$ and -0.001 for channel, jet, homogenous shear and homogenous isotropic flows respectively showing that homogenous isotropic turbulence contains comparatively more singular spanwise structures.

In Fig. 4, we show measured HS parameters for both velocity components of four flows as functions of Reynolds numbers. A smaller value of β_L is observed at $Re_\lambda = 293$, β_L found close to the theoretical values at higher Reynolds numbers and even at $Re_\lambda = 255$. In case of β_T , we find the lowest values 0.79, 0.82 and 0.83 at $Re_\lambda = 400, 293$ and 255 respectively whereas a fixed β_T value 0.84 is observed at higher Reynolds numbers. The longitudinal and transverse components of γ are following almost similar trends as observed for the corresponding components of β .

Jiang *et al.* argued that a larger deviation from K41 scaling is due to the most intense structures γ and does not necessarily imply a more heterogeneous hierarchy of fluctuation structures (Jiang *et al.* 2006). We observe that such deviation is mainly due to the most intense structures that yields smaller γ and slightly the more heterogeneous hierarchy of fluctuation structures that outputs smaller β

when Reynolds numbers are relatively low. The jet flow and homogenous isotropic turbulence are examples of this case. However, it seems that the deviation completely depend on the most intense structures and barely on the more heterogeneous hierarchy of fluctuations for high Reynolds numbers flow, as the case of channel flow at all four locations. Overall, the study prefers the possibility that β is universal constant which supports the claim of Prof. She Z. S. group.

5. Conclusions

In this study, we investigated two main parameters of HS model with two experimental and two DNS datasets at a variety of Reynolds numbers. The experimental datasets are measured from nearly isotropic decaying turbulence at four downstream locations and fully developed jet flow at Taylor-microscale Reynolds numbers 720–620 and 293 respectively while the DNS datasets are obtained from homogenous shear and homogenous isotropic turbulence at $Re_\lambda = 255$ and 400 respectively. It is found that intermittency parameter β , measured from longitudinal velocity components, has nearly same values for four different flows and are very close to the theoretical value. The values of β_T are also found very close to each other. On the other hand, longitudinal component of γ has almost same values which are clearly different from the values obtained from corresponding transverse components. For different flows, it is observed that values of γ_T are significantly different from each other. In all types of turbulent flows, it is also found that spanwise components of both parameters are smaller than the corresponding streamwise components. However, the difference between β_L and β_T is found very small though non-vanishing. We found that at low Reynolds numbers, the deviations from K41 scaling are primarily due to the γ and slightly because of β . However, at higher Reynolds numbers the deviations seem as consequence of γ only. On the basis of our findings, it may be concluded that β is a universal constant in all types of fully developed turbulent flows.

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