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Developing efficient model updating approaches for different structural complexity - an ensemble learning and uncertainty quantifications

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Abstract. Model uncertainty is a key factor that could influence the accuracy and reliability of numerical model-based analysis. It is necessary to acquire an appropriate updating approach which could search and determine the realistic model parameter values from measurements. In this paper, the Bayesian model updating theory combined with the transitional Markov chain Monte Carlo (TMCMC) method and K-means cluster analysis is utilized in the updating of the structural model parameters. Kriging and polynomial chaos expansion (PCE) are employed to generate surrogate models to reduce the computational burden in TMCMC. The selected updating approaches are applied to three structural examples with different complexity, including a two-storey frame, a ten-storey frame, and the national stadium model. These models stand for the low-dimensional linear model, the high-dimensional linear model, and the nonlinear model, respectively. The performances of updating in these three models are assessed in terms of the prediction uncertainty, numerical efforts, and prior information. This study also investigates the updating scenarios using the analytical approach and surrogate models. The uncertainty quantification in the Bayesian approach is further discussed to verify the validity and accuracy of the surrogate models. Finally, the advantages and limitations of the surrogate model-based updating approaches are discussed for different structural complexity. The possibility of utilizing the boosting algorithm as an ensemble learning method for improving the surrogate models is also presented.

Keywords: kriging; Markov chain Monte Carlo; model updating; polynomial chaos expansion; structural complexity

1. Introduction

In structural analysis, the numerical model is usually used to analyze and predict the structural response. However, discrepancies between model predictions and real measurements inevitably occur due to uncertainties from the limited knowledge of uncertain material properties or the simplification in modeling (Mottershead and Friswell 1993, Liang *et al*. 2019, Zhang *et al*. 2019). To reduce these uncertainties in structural numerical analysis, model updating techniques have been commonly adopted to determine the most plausible structural model for an instrumented structural system (Ching *et al*. 2006). Model updating, also referred to as model inversion or model calibration, has been widely used for structural performance assessment and response prediction (El-Borgi *et al*. 2005, Chung *et al*. 2012, Ni *et al*. 2012, Yu and Chung 2012, Friswell and Mottershead 2013, Zhang *et al*. 2020b). Model updating integrates measurements with a numerical model to update the model and to better describe the structural properties (Ewins 2009).

Model updating techniques can be generally classified into matrix updating methods and parameter updating methods. The matrix updating methods are usually referring to the updating of stiffness or mass matrices (Yang *et al*. 2016). However, matrix updating methods may violate structural connectivity so the updated parameters do not always have physical meanings, that is, they can hardly be related to the changes in the parameters of the model (Hou and Xia 2020). On the other hand, the updated objects of parameter updating methods are physical parameters. In the parameter updating approach, model parameters are updated to minimize an objective function which is the misfit between model-predicted data and measured data (Kanev *et al*. 2007, Ren and Chen 2010). In this sense, the parameter updating method is essentially an optimization problem. In this way, model updating methods reduce the uncertainty of the model and make the structural response predicted by the model close to reality as much as possible.

Based on whether uncertainty is considered or not, model updating techniques can also be classified into deterministic model updating method and stochastic model updating method. In the process of the deterministic model updating method, the uncertainties of parameters are not considered (Simoen *et al*. 2015). Parameters are treated as unknown constants, fixed within the specific condition of experimental trials (Zhang *et al*. 2019). Therefore, the

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updated results can only reproduce the experimental results under specific experimental circumstances. It is difficult to give the generalized updated structural response values. Moreover, the uncertainties associated with the results of model updating are not explicitly considered in deterministic methods. In the stochastic model updating methods, the probabilistic theory is applied to treat the uncertainty and utilized to estimate the statistical distributions of the parameters, hence a universal model updated response can arise (Feng *et al*. 2020).

The Bayesian model updating approach is among the stochastic model updating methods. It has the advantage of handling uncertainties while taking account of prior information (Jaynes 2004, Sivia and Skilling 2006, Zhang *et al*. 2018). In the Bayesian updating framework, the posterior distribution of updating physical parameters is expressed as a product of the prior distribution and the likelihood function. A suitable likelihood function is constructed to involve the available information in the measured data. For some complex structures, however, the explicit expression of the likelihood function may be difficult or even impossible since the relationships between physical parameters and structural measured output variables (also termed as "features" in the following context) are usually not direct (Feng *et al*. 2020). In addition, for complex large-scale structures, the computational cost in Bayesian model updating tends to become unaffordable or even impractical. Thus, certain surrogate models, like Kriging (Zhou and Lu 2020) and PCE (Jacquelin *et al*. 2016, He *et al*. 2021) have been applied to model updating as alternatives to the very sophisticated numerical models.

In the pioneering works, Collins *et al*. (1974) had first proposed the Bayesian model parameter identification method. Beck and Katafygiotis (1998) built a more comprehensive and rigorous framework for Bayesian model updating and defined the concept of system identifications. Recently, numerous studies of Bayesian model updating have been completed on both numerical examples and real-world applications (Ntotsios *et al*. 2009, Behmanesh and Moaveni 2015, Kuok and Yuen 2016, Lam *et al*. 2018, Ma *et al*. 2019, Zhang *et al*. 2020a).

Although the Bayesian model updating framework quantifies the uncertainty of updating results, the obtained uncertainties are sensitive to the complexity of the structure, reflected in the parameter dimensions and the input-output relationships. In the presence of such complex models, the updating result is often unrealistic. The high dimension and nonlinearities not only pose a great challenge to the Bayesian model correction process, but also make the application of the surrogate model to the likelihood function difficult.

As is known, many model updating techniques assume that structural behaviors remain in the linear stage after the damage has occurred (Song *et al*. 2018). Several successful applications of linear structural model updating can be found in the literature (Teughels and Roeck 2005, Moaveni *et al*. 2008). However, the assumption of linear dynamic models is progressively violated with an increasing level of excitation. For structures subjected to large excitations such as seismic loads, structural responses may exhibit high-order nonlinearities (Prawin and Rao 2018, Hou and Xia 2020). Therefore, nonlinear models should be considered for structural response prediction and structural assessment. Unfortunately, current literature on real-world applications of nonlinear model updating is still limited. Asgarieh *et al*. (2014) performed nonlinear FE model updating of a large-scale three-story RC frame. Hemez and Doebling (2001) emphasized the importance and necessity of considering nonlinearity in the modeling when the measured response is nonlinear. They illustrated that inaccurate results can be obtained by fitting linear models to measure nonlinear data. For nonlinear systems, the relationship between the updating physical parameters and the features is also often nonlinear, posing a great challenge for the application of surrogate models in likelihood functions. Thus, considering nonlinearity in the models seems to be necessary for robust and accurate identification of nonlinear structural systems.

Furthermore, with the increase of complexity of the structural model, more structural parameter updatings are considered (Mashayekhi and Santini-Bell 2019). However, for high dimensional problems, a direct calculation of the posterior distribution is sometimes difficult or even impractical. Most existing analytical methods such as surrogate models are not efficient, which is often referred to as the curse of dimensionality. In addition, with the increase of parameter dimensions, the model updating problem may become unidentifiable. The analytical form of the posterior PDF may not be available. Even with linear models, the model updating problem may be potentially ill-posed, i.e., the problem is not globally identifiable. The problem becomes even more challenging when only some of the degrees of freedom (DOF) of the model are measured. In this case, to perform an efficient Bayesian model updating analysis, a robust sampling algorithm is required.

Markov chain Monte Carlo (MCMC) sampling method is one of the successfully applied sampling approaches for Bayesian model updating approach. It provides an alternative way to estimate the posterior PDF of uncertain parameters by drawing random samples from the target distributions (Katafygiotis *et al*. 2000). MCMC can provide a full characterization of the posterior uncertainty, even when the model class is not globally identifiable (Yuen 2010). It allows direct sampling from the posterior distribution without the need to solve the potentially high dimensional integral in the Bayesian formulation (Straub and Papaioannou 2015). Beck and Au (2002) proposed Adaptive Metropolis-Hastings (AMH) algorithm with higher sampling efficiency, but it is inefficient in high-dimensional problems due to kernel density estimation. For handling high-dimensional model updating problems, Ching *et al*. (2006) had utilized Gibbs sampling method (Geman and Geman 1984). Ching and Chen (2007) also proposed the improved Transitional Markov Chain Monte Carlo (TMCMC) algorithm, suitable for sampling in high-dimensional situations. Because of its excellent performance in complex distributions, TMCMC has been employed in some recent research work (Muto and Beck 2008, Rocchetta *et al*. 2018).

In general, structural complexity gives a significant challenge in the application of Bayesian model updating. Although the MCMC-based Bayesian model updating method can efficiently evaluate the posterior marginal PDFs of the uncertain parameters for the target structural systems without calculating high-dimension numerical integration, high dimension remains a major dilemma for many corresponding analytical methods including surrogate models. Besides, nonlinearities, which represent not only material nonlinearity but also the nonlinear relationships between updating parameters and measured features, also create obstacles for Bayesian model updating. Hence, it is of interest to determine the applicability of Bayesian model updating method and surrogate models in these complex situations.

This paper applies a Bayesian model updating framework under three structural examples with different complexity, including a two-storey frame, a ten-storey frame, and the national stadium model. The three models stand for the low-dimensional linear model, the high-dimensional linear model, and the nonlinear model, respectively. In this paper, the high-dimensional problem refers to the dimensionality of updating parameters, and nonlinearity includes two meanings, nonlinearity in the structural properties and nonlinearity in the relationship between the updating physical parameters and the measured features. By investigating the examples, this paper also shows the advantages and limitations of two surrogate models, i.e., the Kriging predictor and PCE. The results provide suggestions on how to select surrogate models in model updating of complex structures. Furthermore, in order to improve the performance of surrogate models, this paper also proposes an ensemble learning method by integrating the Bootstrap Aggregating (Bagging) and the Adaboost algorithms.

The remainder of this paper is organized as follows. Section 2 presents a Bayesian structural model updating framework integrating the TMCMC algorithm, model validation, and K-means cluster analysis (MacQueen 1967, Lloyd 1982). Section 3 investigates three examples with different structural complexity. In Section 3, an extension study is also made to use the ensemble learning method to improve the surrogate model-based Bayesian model updating approaches. Finally, Section 4 summarizes the research findings.

2. Methodology

2.1 Bayesian model updating theorem

Inferences about uncertain parameters in Bayesian model updating analysis are based on posterior distribution. By the Bayes' formula, the posterior PDF of parameter θ is given as $p(\theta|D,M)$, which includes the model class M and measured data D

$$p(\theta|D,M) = \frac{p(D|\theta,M)p(\theta|M)}{p(D|M)} \quad (1)$$

where $p(\theta|M)$ is prior distribution, representing the prior knowledge about the updated parameters based on experience and historical data. $p(D|M)$ is the normalizing constant, also called evidence, which makes the probability volume under the posterior PDF equal to unity. The direct integral of $p(D|M)$ is not feasible in the general case. It usually needs the help of an advanced sampling technique such as MCMC to approximate the density function. $p(D|\theta, M)$ is the likelihood function, representing the probability of obtaining measured data D based on an instance of the model parameter θ. The likelihood function is constructed according to a class of probabilistic and physical models of the problem. For normal random uncertainties, the likelihood function can be expressed as

$$p(D|\theta,M) = exp\left(-\frac{J_g(\theta)}{2\sigma_\varepsilon^2}\right) \quad (2)$$

where σ_ε^2 is a measure of the size of the prediction error and $J_g(\theta)$ is a goodness-of-fit function normalized by the prediction error variance. $J_g(\theta)$ can be expressed as

$$J_g(\theta) = \sum_{i=1}^{m} \frac{\widehat{\Omega}^{(i)^2} - \Omega^{(i)}(\theta)^2}{\widehat{\Omega}^{(i)^2}} \quad (3)$$

where $\widehat{\Omega}^{(i)}$ and $\Omega^{(i)}(\theta)$ are the measured features and the model predictive features. m denotes the parameter dimensions. $p(\theta|D,M)$ is the posterior distribution, on which inferences about parameters are based.

The posterior PDF in Eq. (1) completely describes the plausibility of the model parameters θ, but its topology may usually be very complicated, especially in the high-dimensional case (Yuen 2010). Based on the model class M and the data D, model parameters θ can be classified into three categories (Katafygiotis and Lam 2002), namely globally identifiable, locally identifiable, and unidentifiable, depending on whether the set of maximum likelihood estimates is a singleton, finite, or uncountable continuum respectively, in the parameter space (Cheung and Beck 2009). For dynamic examples, parameter identifiability depends on the number of observed degrees of freedom (Yuen 2010). A high complex structure may require an efficient optimization algorithm to help in finding the global optimal solution.

2.2 Sampling

As observed in Eq. (1), a direct calculation of the posterior distribution is sometimes cumbersome and not possible. Sampling is an efficient way to approximate the posterior distribution. Monte Carlo simulation (MCS) is regarded as a suitable means for stochastic analysis because of its high accuracy and simple principle. But the huge amount of calculation is an obstruction for its implementation (Schueller *et al*. 2009). Furthermore, the important region of the posterior PDF is usually concentrated in a very small subset of the parameter space, resulting in a failure to compute structural response and parameters.

Therefore, more advanced sampling techniques like Markov Chain Monte Carlo sampling (MCMC) are developed and applied. The MCMC method estimates the posterior distribution of the parameters of interest through random sampling in the probability space. In MCMC, samples of θ are drawn from the desired posterior distribution based on Metropolis-Hastings (MH) algorithm. The basic idea of MH is to construct a Markov chain in the data sampling with a transfer matrix Q so that its stationary distribution is exactly the target distribution (Metropolis *et al*. 1953, Hastings 1970). Along the Markov chain, we get a transfer sequence of $\{\theta_0, \theta_1, \theta_2,..., \theta_m, \theta_{m+1},..., \theta_n\}$ and if the Markov chain has converged at the m-th step. The samples $\{\theta_m, \theta_{m+1},..., \theta_n\}$ can be deemed as representations of θ following the posterior distribution function $p(\theta|D,M)$. Although the Markov chain samples are dependent, they can still be used for statistical averaging as if they were independent.

The detailed MCMC approach is conducted as follows. First, an initial dataset of θ_0 is sampled from the prior distribution $p(\theta|D,M)$. Then a candidate dataset θ^* is sampled from a proposal distribution $q(\theta^*|\theta)$ conditioned on the initial value θ_0. The proposal distribution is usually unknown and chosen as a symmetric distribution like normal distribution (Ravenzwaaij *et al*. 2018). To determine if the candidate state θ^* is the next state of the Markov chain, an acceptance rate, related to the predetermined proposal distribution, is calculated by

$$\alpha = min\left\{\frac{p(\theta^*)q(\theta^*|\theta)}{p(\theta)q(\theta|\theta^*)}, 1\right\} \tag{4}$$

The candidate state θ^* is accepted and set as the next state of the Markov chain $\theta_{k+1}=\theta^*$ with probability α, or it is rejected and set $\theta_{k+1} = \theta_k$ with probability 1-α. The process is repeated until N Markov chain samples have been simulated. Since the stationary distribution of the Markov chain is not affected by the initial distribution, the MH algorithm is efficient even the prior distribution $p(\theta|M)$ differs substantially from the posterior distribution $p(\theta|D,M)$.

In the case of high dimensional integration, the acceptance rate of the MH algorithm is usually small which makes the candidate state θ^* been rejected too often. The sampling efficiency is closely related to the selection of the proposal distribution q. To tackle this issue, Beck and Au (2002) proposed the Adaptive Metropolis-Hastings (AMH) algorithm, in which the multiple-level idea was applied to link the gap between the prior PDF and the target PDF. The AMH algorithm has a higher sampling efficiency, and its proposal distribution is not fixed.

The proposal distributions are constructed as a sequence of PDFs $\{p_0, p_1,...,p_m\}$ converging to the posterior distribution $p(\theta|D,M)$. p_0 is taken as the prior distribution to simulate sample $\{\theta_1^{(1)},\theta_2^{(1)},...,\theta_N^{(1)}\}$, and p_1 is constructed as a weighted sum of N Gaussian PDFs centered among the samples, also called kernel sampling density functions (Ang *et al*. 1992).

$$p_1(\theta) = \sum_{i=1}^{N} w_i \phi(\theta|\theta_i^{(1)}, \Sigma_i) \tag{5}$$

where ϕ is the multi-dimensional Gaussian PDF with mean $\theta_i^{(1)}$ and covariance matrix Σ_i, and w_i is the corresponding probability weight. Σ_i of the proposal distribution is updated according to the samples $\{\theta_i^{(1)}\}$ generated by the Markov chain to adaptively approximate the target distribution. Then p_{k+1} is constructed using the samples $\{\theta_1^{(k+1)}, \theta_2^{(k+1)},..., \theta_N^{(k+1)}\}$ simulated by the proposal distributions p_k $(k = 1, 2, \dots, m)$ of the last state as in Eq. (5).

By updating the sampled dataset, the proposal distribution will gradually approximate the target distribution and thus obtain the posterior distribution. AMH algorithm is applicable to multimode, peak, and flat PDFs because of its adaptive nature. However, this approach is still inefficient for high-dimensional problems, since a dramatically large number of samples are required to construct a proposal PDF which can generate samples with reasonably high acceptance probability.

In this paper, the TMCMC method is used in the sampling. Like AMH, TMCMC uses a sequence of intermediate PDFs to converge to the posterior PDF (Ching and Chen 2007). It is quite useful in drawing samples from complicated PDFs. Nevertheless, kernel density estimation is not necessary for TMCMC. Instead, a resampling approach is employed to handle high-dimensional PDFs more efficiently. The estimation of the evidence $p(D|M)$, which is crucial for Bayesian model class selection, is also allowed in TMCMC. The sequence of intermediate PDFs in TMCMC is

$$p_j(\theta) \propto p(\theta|M)p(D|M,\theta)^{p_j} \tag{6}$$

where p_j denotes the iteration stage, starting from $p_0 = 0$ in the first iteration and progressively until p_n reaches one in the last iteration. p_j is adaptively computed from the samples in the previous step. TMCMC algorithm generates new samples from high intermediary likelihood values, allowing for sampling from very complex posterior distributions.

Before obtaining p_{j+1}, the plausibility weights need to be computed by

$$\begin{aligned} w(\theta_{j,k}) &= \frac{p(\theta_{j,k}|M)p(D|M,\theta_{j,k})^{p_{j+1}}}{p(\theta_{j,k}|M)p(D|M,\theta_{j,k})^{p_j}} \\ &= p(\theta_{j,k}|M)p(D|M,\theta_{j,k})^{p_{j+1}-p_j} \\ k &= 1,2,\dots,N_j \end{aligned} \tag{7}$$

The plausibility weight $w(\theta_{j,k})$ here indicates the distance between $p_j(\theta)$ and $p_{j+1}(\theta)$. It reflects the iteration speed of the intermediate PDFs converging to the target distribution. A common assumption of the coefficient of variation (C.O.V) of this value is the unity which is adopted in this study (Ching and Chen 2007).

The acceptance rate is calculated by normalizing the weights by

$$\theta_{j,k} = \theta_{j,l} \text{ with the probabillity } \frac{w(\theta_{j,k})}{\sum_{l=1}^{N_j} w(\theta_{j,l})} \quad k = 1,2,\dots,N_j \tag{8}$$

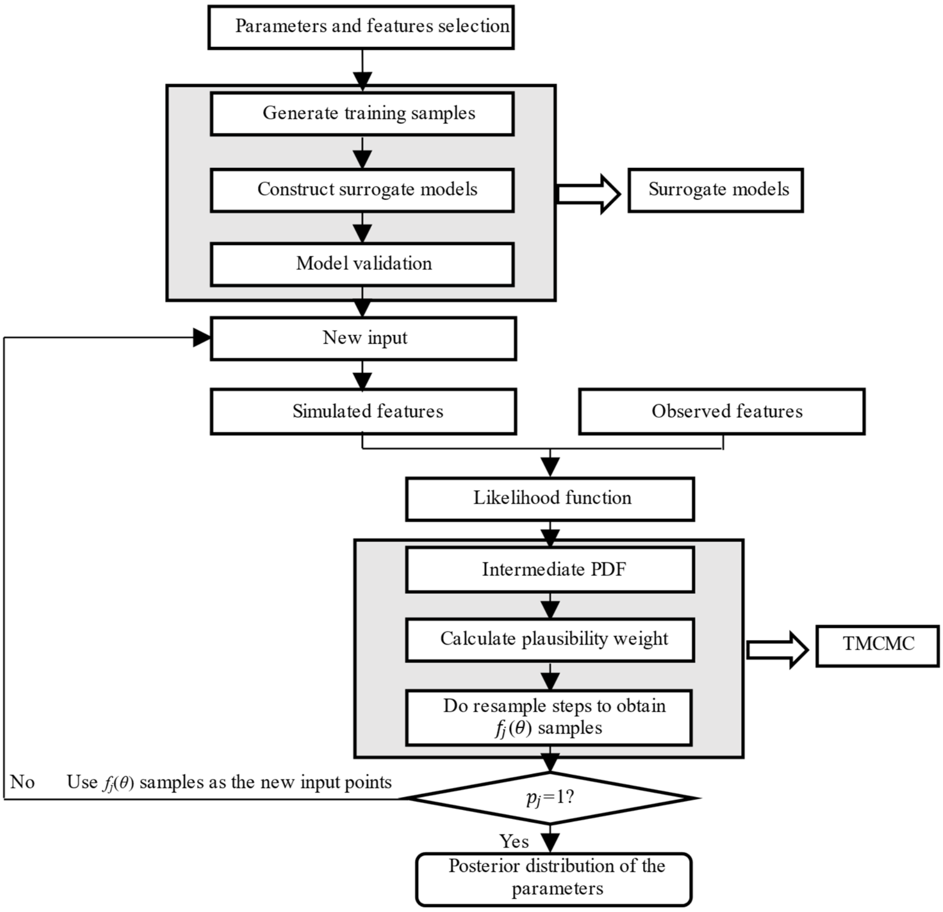


Fig. 1 Flow chart of the Bayesian updating framework

In this way, the intermediate PDFs are automatically selected to approach the posterior distribution

2.3 Surrogate model

The likelihood function must include the mechanical models to relate the observation to the model parameters θ. However, such a mapping does not always have an explicit analytical expression due to the complex circumstances in many cases. A common method is to build a surrogate model, transforming the engineering problem into a black box problem, to directly model the input-output relationship without caring about the specific internal physical process. In addition, computation efficiency is another important issue for Bayesian model updating. The computational cost of the surrogate model is significantly smaller than that of the original numerical model, especially for large complex structures.

The first step to construct a surrogate model, reflecting the relationship between input parameters θ and features y, is to find a set of data points that could best represent the relationship. The selection of the datasets directly affects the accuracy and calculation speed of the surrogate models. The process of generating the training dataset for surrogate models is known as the design of the experiment (DoE). This paper adopts a sparse grid method to generate the training dataset. The $2n + 1$ by n sparse matrix $\boldsymbol{\xi}$ is

$$\boldsymbol{\xi} = \begin{bmatrix} 0 & \cdots & 0 \\ & & -1 \\ & \iddots & \\ -1 & & \\ & & 1 \\ & \iddots & \\ 1 & & \end{bmatrix} \tag{9}$$

The matrix $\boldsymbol{\xi}$ is transformed so that the range of its elements coincides with the range of the training set. For each dimension, the origin and two other points are used to estimate the coefficients of a second-order polynomial. Therefore, only $2N+1$ (N is the parameter dimension) points are needed in all. Subsequently, based on the experimental samples generated by the sparse grid method, a surrogate model is established to reduce the computational burden of model updating.

After a surrogate model is built, the prediction accuracy can be evaluated by comparing the predictions and measurement. Validation metrics are the mathematical tools to measure the difference between system response quantities obtained from the surrogate models and experimental measurements (Deng *et al*. 2017). In this paper, three validation metrics including Root Mean Square Error (RMSE), Mahalanobis distance (MD), and Bhattacharyya distance (BD) are employed. These validation metrics reflect the fitting error of the model prediction feature and the measurement feature from different perspectives. Let an N-dimensional vector y_n denote the model predicted features, and an N-dimensional vector y_e denote the measured features.

RMSE represents the square root of the second sample moment of the differences between y_n and y_e, which is given as following

$$RMSE = \sqrt{\frac{1}{N}(y_n - y_e)^T(y_n - y_e)} \tag{10}$$

Mahalanobis distance is the weighted distance considering the covariance of data samples (Mahalanobis 1936). The correlation between variables is considered in the form of the covariance matrix as following

$$d_M = [(y_n - y_e)^T C^{-1}(y_n - y_e)]^{\frac{1}{2}} \tag{11}$$

where C^{-1} is the inverse of the covariance matrix C of y_n and y_e. Bhattacharyya distance considers the probability characteristics of two samples (Bhattacharyya 1943). It is expressed as the negative logarithm of the Bhattacharyya coefficient (BC), which quantifies the probability overlap between the sampling distribution and the experimental measurement distribution as following

$$d_B = -\,ln[BC(p_n, p_e)] = ln\left[\int_y \sqrt{p_n(y)p_e(y)}\right] \tag{12}$$

where p_n and p_e are PDFs of the simulated sample and measured sample respectively.

Then the surrogate model is used to predict the features $y^{'}$ according to the new input data $\theta^{'}$. The process is the same thing as taking the conditional probability

$$y = p(y'|\theta', \theta, y) \tag{13}$$

In this paper, two surrogate models, including Kriging and PCE model, are utilized in model updating. Kriging model could be expressed as the summation of a parametric model $F(\beta,x)$ and non-parametric random process $z(x)$ (Rasmussen 2003)

$$y(x) = F(\beta, x) + z(x) \tag{14}$$

where $F(\beta,x)$ represents the regression model which is composed of polynomial functions; $z(x)$ is a standard Gaussian stationary random process reflecting the noise, enabling the Kriging model to show the non-linear state of the structure. With the introduction of $z(x)$, the Kriging model overcomes the shortcoming of the polynomial response surface method in the nonlinear problem. The covariance function of $z(x)$ can be expressed as a kernel function

$$Cov[z(x_i), z(x_j)] = \sigma^2 R(\theta, x_i, x_j) \tag{15}$$

where σ^2 denotes the process variance, x_i and x_j are the components of x, R is an admissible correlation function with hyper-parameter θ, greatly affecting the fitting accuracy of the Kriging model (Liang *et al.* 2011). In the process of iterative optimization, new samples are added by considering the distribution of the initial samples. In such a way, the Kriging model can be well established.

In PCE model, the model feature is treated as a random process (Wiener 1938)

$$\hat{s}(x,t|\theta) = \alpha_0(x,t) + \sum_{i_1=1}^{\infty} \alpha_{i_1}(x,t)\,\Gamma_1(\xi_{i_1}(\theta)) + \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i_1} \alpha_{i_1 i_2}(x,t)\Gamma_2(\xi_{i_1}(\theta), \xi_{i_2}(\theta)) + \cdots \tag{16}$$

where $a_0(x,t)$ and $a_{i_1 i_2 \ldots i_j}(x,t)$ are the PCE coefficients, and $\Gamma_j(\xi_{i_1}, \xi_{i_2}, \ldots, \xi_{i_j})$ are a set of polynomial chaos with respect to the independent random variables ξ_{i_1}, ξ_{i_2}, …, ξ_{i_j} (Wiener 1938). PCE offers an efficient way of including nonlinear effects in stochastic analysis (Ghanem and Spanos 2003). It can be truncated by finite random dimensions N and degrees M as

$$\hat{s}(x,t|\xi(\theta)) = \sum_{i=1}^{Q} \alpha_i(x,t)\,\Psi_i(\xi(\theta)) \tag{17}$$

where $Q = (N+M)!/(N!M!)$, $\alpha_i(x,t)$ are the PCE coefficients, and $\Psi_i(\xi(\theta))$ are the orthogonal basis function.

2.4 Ensemble learning

In order to reduce the generalization errors in Kriging and PCE model, the ensemble learning technique is utilized to strategically combine the two surrogate models to improve the performance. Two commonly used ensemble learning algorithms are Bootstrap AGGregatING (Bagging) and Adaptive Boosting (Adaboost).

Bagging is a parallel ensemble learning algorithm (Breiman 1996), which generates a series of randomly sampled subsets D_i from the entire training set D for the model training. The bootstrap method can resample the datasets with replacement, and therefore some training points can be repeated in various training subsets. The models trained by D_i are then combined by averaging the output variables. In this way, Bagging can reduce model variance and help to avoid overfitting. On the contrary, however, the fitting degree of the training set will be worse, that is, the bias of the model will be larger.

Unlike Bagging, Adaboost is a serial ensemble learning algorithm which mainly aims to reduce bias (Schapire 1990). In AdaBoost, the sample weight of the training points is iteratively adjusted to make subsequent models focus on the training points with large errors. In this way, previous points with larger errors are more likely to appear in the next training sample. The initial sample weight is set as uniform, so that all training points have an equal probability to be drawn.

Surrogate models are strong learners with little bias, so there is no need to directly utilize Adaboost algorithm to improve the fitting accuracy on the training set. However, the fitting accuracy is greatly influenced by DoE. In this paper, an ensemble learning algorithm suitable for surrogate models is presented by combining Bagging and Adaboost. The complete algorithm can be summarized as follows:

(1) Select a training dataset D containing m points and initialize $w_{0,i} = 1/m$. Determine the number of surrogate models needed in the ensemble learning process p. Repeat the following 2-4 for j = 1, 2,…, p.
(2) Generate n training subsets and train n surrogate models $S_1^{(j)}, S_2^{(j)}, \dots, S_n^{(j)}$ using bootstrap sampling. The average is taken as a new surrogate model $S^{(j)}$.
(3) Calculate the predictive error $e_{j,i}$ (i = 1, 2,…, nm) and the model weight:

$$a_j = \frac{1}{2} ln \frac{1 - \sum_1^{nm} e_{j,i}/nm}{\sum_1^{nm} e_{j,i}/nm} \tag{18}$$

(4) Update the sample weight:

$$w_{j,i} = \frac{w_{j-1,i}}{Z_j} a_j^{1-e_{j,i}} \tag{19}$$

(5) At the end of the algorithm, the final model can be expressed as a linear combination $\sum_{j=1}^{p} S^{(j)}$.

Based on the above-mentioned techniques, the Bayesian model updating framework in this paper is illustrated in Fig. 1. First, the updated parameters and measured features are determined. Then a certain number of training points are used to generate surrogate models. Validation metrics are further utilized to evaluate the fitting accuracy of models. The features obtained by surrogate models are compared with the measured features, generating the likelihood function. Then the TMCMC iterative procedure is applied to estimate the posterior distribution and the evidence. Finally, the model parameter can be updated by identifying the maximums in the posteriors. The performance of the developed framework in different complex structures will be investigated in the following section.

3. Case study

3.1 Low-dimensional linear model: two-storey frame

Considering a two-storey frame modeled through a linear shear building model with two degrees of freedom (DOF). The initial storey masses including the mass contributions from the columns are all taken as m = 11170 *kg*. The initial inter-storey stiffness values are all modeled as K_0 = 46.08 *MN*/*m*. Damping is not considered in the analysis.

In the process of Bayesian model updating, the inter-storey stiffness of each floor is considered to be updating parameter. Instead of using directly the absolute value of each updating parameter, a dimensionless scaling damage factor (also termed as "stiffness parameter" in the following context) is defined as the ratio of the updated inter-storey stiffness to its initial condition ($\theta_i = K_i/K_0$, i = 1, 2). The

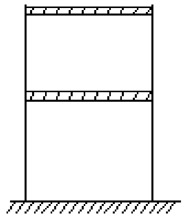


Fig. 2 Two-storey frame

origin value of θ is $[0.7, 0.8]^T$. The parameters are assumed to be independent and identically distributed following uniform distribution ($\theta \sim U[0,3]$) due to the lack of prior knowledge. During the parameter calibration, the first two frequencies are applied as structural features. The stiffness matrix is given by Eq. (20), and the modal frequencies are the eigenvalues of the stiffness matrix given in Eq. (21)

$$K = K_0 \begin{bmatrix} \theta_1 + \theta_2 & -\theta_2 \\ -\theta_2 & \theta_2 \end{bmatrix} \tag{20}$$

$$|K - \omega^2 M| = 0 \tag{21}$$

In this example, the terms $\Omega^{(i)}(\theta)$ and $\widehat{\Omega}^{(i)}$ in Eq. (3) are measured frequencies and modal frequencies. The actual modal frequencies are calculated according to Eq. (21). The observations of the first two frequencies are simulated from Gaussian distributions with the mean value of f = [5.5, 14.9] on the assumption of the origin value of θ. And the C.O.V for each variable is assigned as 0.01 to account for uncertainties. The number of samples N is set 200.

Based on the Bayesian method and TMCMC, the scatter plots for the two uncertain parameters are plotted in Fig. 3. In the same figure, a confidence ellipse for the samples is also illustrated. The confidence ellipse represents a contour that allows the visualization of the confidence interval, given as the region that contains 95% of the parameter samples. It is seen that the TMCMC algorithm has found the high probability areas after a few iterations (or burn-in period) and the posterior distribution has two local maxima: [0.73,0.84] and [1.6,0.38], of which the former is very close to the true value [0.7,0.8]. The updated frequencies are detailed in Table 1.

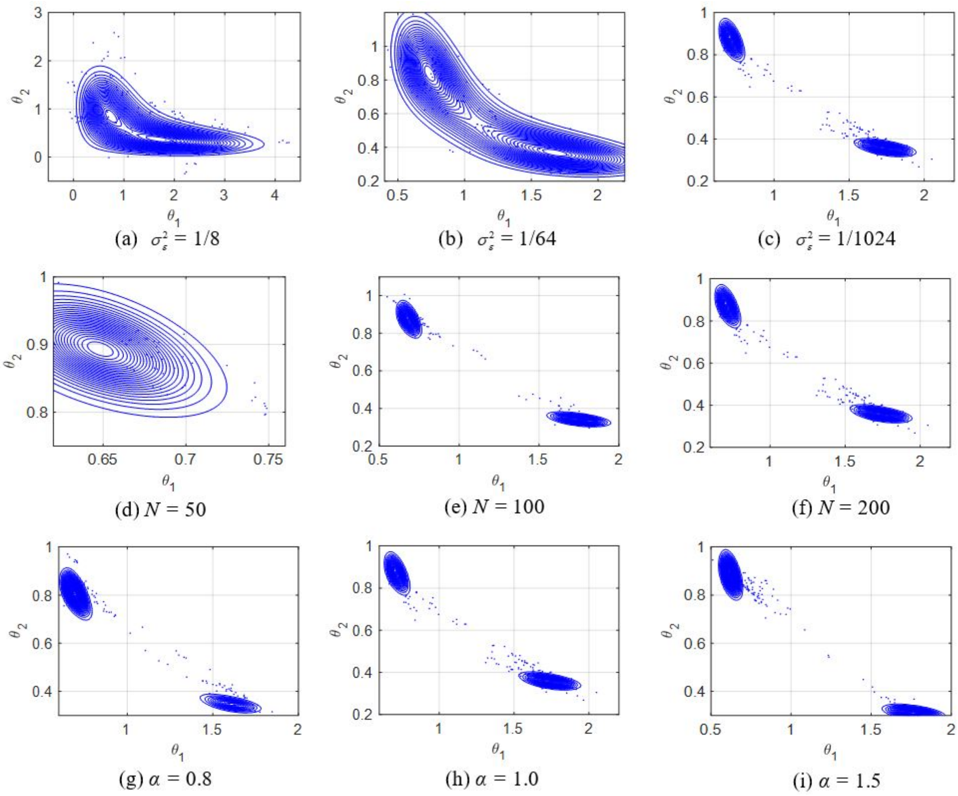
To test the robust of the Bayesian updating framework, a sensitivity analysis is performed to investigate the influences of the predictive error σ_ε^2, the sample size N, and the form of the prior distribution. For the prior distribution, a lognormal distribution with a variance having fluctuation is tested. The fluctuation factor is α, which is defined as the ratio of the variance to the initial variance. The initial value of σ_ε^2, N and α are 1/1024, 200, and 1.0, respectively. The posterior distribution for using different values of the above

Table 1 Caption

	f_1 (Hz)	f_2 (Hz)
Updated value	5.62	14.69
Origin value	5.50	14.90
Error (%)	2.14	-1.43

(a) Prior PDF (b) Posterior PDF

Fig. 3 Prior and posterior PDF



(a) $\sigma_\varepsilon^2 = 1/8$ (b) $\sigma_\varepsilon^2 = 1/64$ (c) $\sigma_\varepsilon^2 = 1/1024$

(d) $N = 50$ (e) $N = 100$ (f) $N = 200$

(g) $\alpha = 0.8$ (h) $\alpha = 1.0$ (i) $\alpha = 1.5$

Fig. 4 The contour line of posterior distribution with changes in prediction error, sample size, and the form of prior distribution

above factors in the model updating is shown in Fig. 4. It can be observed that the change in three factors does not affect the posterior mean but the variance. The sample of posterior distribution may be incomplete when the prior variance is large or the sample size is small. On the other hand, if N is too large, the change in the prior distribution has little effect on the posterior PDF, even though the samples of the first few iterations are very divergent.

3.2 High-dimensional linear model: ten-storey frame

A more complex ten-storey frame is been analyzed herein. The updated parameters are still the stiffness parameters θ. A description of the parameters is presented in Table 2. The measured frequencies are simulated by providing origin values for parameters. The uncertainties associated with the measured values are assumed to have a C.O.V of 0.01. The number of samples N is set 3000.

In this case, both analytical method and surrogate models have been used in the model updating. Kriging model and PCE model are both utilized as candidate surrogate models with the same training set containing 21 points. To construct a robust Kriging model, the zero-order, one-order, and two-order polynomial functions are selected to formulate the regression model, and the correlation function R in Eq. (15) is selected as the Gaussian function, exponential function, and linear function, respectively. After comparing the Mean Square Error (MSE) of the modal frequencies of the above nine Kriging models, this paper utilizes the model with the smallest MSE, i.e., the model with the second-order polynomial function and Gaussian function.

In order to evaluate the feasibility of the two surrogate

Table 2 Description of updated parameters in ten-storey frame

Parameters	Prior distribution	Interval	Origin value
θ_1	Uniform	[0,3]	1.5
θ_2-θ_9	Uniform	[0,3]	1.0

Table 3 Fitting errors of surrogate models

	RMSE	MD	BD
Kriging	0.1045	0.0120	0.0111
PCE	0.2054	0.0133	0.1716

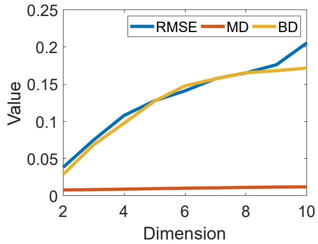


Fig. 5 Validation metrics of PCE model under different parameter dimensions

models, a set of test points independent of the training points are generated and used as input variables for model validation. The set of test points contains 50 points. Three validation metrics including Root Mean Square Error (RMSE), Mahalanobis distance (MD) and Bhattacharyya distance (BD) are employed and detailed in Table 3.

It shows from Table 3 that the Kriging model can generate a more precise outcome for the ten-storey frame than the PCE model indicated by all the validation metrics. The effect of parameter dimension on PCE model was subsequently investigated. It is worth noting that the predictive accuracy of the PCE model decreases with the increase of parameter dimensions as shown in Fig. 5.

The two surrogate models and analytical model are all utilized and compared in the Bayesian model updating. In this example, there are ten combinations of values of uncertain parameters corresponding to the measured features (at this moment, the parameters are locally identifiable). For such a high dimension case, it is difficult to visualize the posterior distribution, so K-means cluster analysis is performed to eliminate the local optimal solution.

The updated parameters before and after clustering are presented in Table 4. Fig. 6 shows the scatter plot after clustering. Before clustering, the updated results from the analytical model and PCE model are not very satisfactory. For each updated parameter except θ_3, the error when using the Kriging model is smaller than that when employing the PCE model, and the differences are especially evident for θ_1 and θ_8. The reason for the slightly large gap in θ_1 and θ_8 may be that degree of aggregation of parameters obtained by the two models differs in high-dimensional space, which can also be seen in Fig. 6. After clustering, the local optimal solutions are eliminated. Therefore, there is a slightly large difference between θ_1 and θ_8 before clustering but a small difference after clustering. The reason for the good performance of the Kriging model might be the infill criteria, which helps the Kriging model find the global optimal solution. After clustering, however, the errors dramatically decrease to less than 10%, indicating clustering significantly improves the predicting accuracy of surrogate models. The results of updated frequencies before clustering are given in Table 5. It is seen the errors are all within 5%.

The marginal PDFs for the updated parameters, constructed from the Markov chain samples, are shown in Fig. 7. The maximum value of the posterior distribution of each parameter is plotted. For most parameters, the maximum value is closer to the true value than the mean value listed in Table 4, which might indicate that the maximum value of the posterior distribution is more suitable to estimate the parameters than the posterior mean. It is worth noting that the spread of variables is smaller when the PCE model is employed. This indicates that the PCE-based model updating method yields less parameter uncertainty. Fig. 8 displays the updated values of f_1-f_{10} versus the iteration number in different methods. The result

Table 4 Updated parameters before and after clustering (% errors in percentage)

Variables	Analytic solution		Kriging		PCE	
	Before clustering	After clustering	Before clustering	After clustering	Before clustering	After clustering
θ_1	1.92 (28.0)	1.60 (6.7)	1.69 (12.9)	1.48 (2.4)	1.97 (38.3)	1.55 (3.3)
θ_2	1.14 (14.0)	0.94 (-5.6)	0.96 (-3.9)	0.93 (6.7)	1.09 (8.6)	1.07 (6.8)
θ_3	1.01 (0.9)	1.12 (12.3)	1.00 (0.1)	0.98 (2.1)	0.97 (-2.6)	0.91 (-8.8)
θ_4	0.89 (-11.1)	0.92 (-7.7)	1.01 (0.5)	1.05 (4.1)	1.06 (-5.9)	0.97 (-3.3)
θ_5	1.04 (4.0)	1.01 (1.0)	1.05 (4.9)	1.04 (4.4)	1.06 (6.4)	0.97 (-2.5)
θ_6	1.00 (0.2)	0.99 (-0.8)	1.06 (6.3)	1.01 (1.1)	1.15 (15.4)	0.89 (-9.5)
θ_7	0.98 (-2.5)	0.99 (-0.8)	1.02 (2.2)	1.03 (2.6)	1.11 (10.5)	0.93 (-6.6)
θ_8	1.03 (3.4)	0.94 (-6.4)	1.06 (6.1)	1.05 (4.8)	1.22 (22.5)	0.90 (-9.6)
θ_9	1.00 (-0.2)	0.91 (-9.0)	1.06 (6.0)	1.08 (7.5)	1.03 (3.0)	1.06 (5.9)
θ_{10}	0.96 (-3.7)	0.96 (-3.7)	1.24 (23.7)	1.07 (6.8)	1.25 (24.5)	1.11 (11.3)

(a) Kriging: θ_1-θ_2 (b) PCE: θ_1-θ_2

Fig. 6 Scatter plot of posterior data samples after clustering

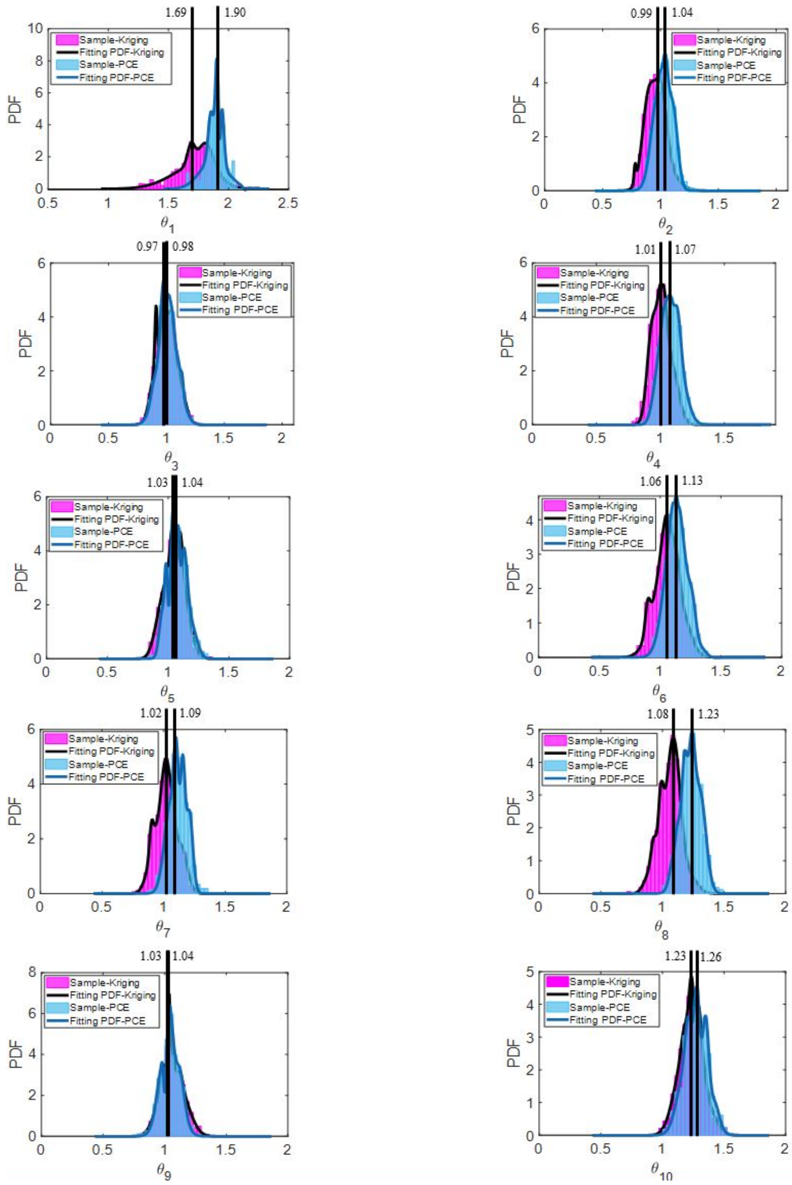


Fig. 7 The contour line of posterior distribution with changes in prediction error, sample size, and the form of prior distribution

plotted in Fig. 8 shows that the Kriging model is more violent and susceptible than the other methods in the initial steps. However, all three methods converge reasonably fast and at almost the same iteration step. This means that the convergence speed is acceptable no matter which method is applied.

As illustrated in Table 5, the predictive accuracy of frequencies is higher than that of updated parameters, and the accuracy of the analytical method is higher than that of the surrogate models. For all three methods, the first six modes are almost the same. Nevertheless, for the seventh to tenth order modes, the errors of the surrogate models are

Table 5 Updated structural frequencies before clustering

Frequency		f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
Measurements (Hz)		1.58	4.70	7.70	10.51	13.07	15.31	17.17	18.63	19.66	20.25
Analytical solution	Value (Hz)	1.57	4.69	7.78	10.57	13.39	15.72	17.23	18.92	19.77	20.12
	Error (%)	-0.2	-0.1	1.1	0.6	2.5	2.7	0.3	1.6	0.6	-0.7
Kriging	Value (Hz)	1.60	4.85	7.71	10.67	13.35	16.08	17.71	19.15	20.44	20.69
	Error (%)	1.5	3.3	0.2	1.5	2.2	5.0	3.1	2.8	4.0	2.1
PCE	Value (Hz)	1.62	4.73	7.84	10.76	13.36	15.76	17.71	19.27	20.30	21.49
	Error (%)	2.8	0.7	1.9	2.3	2.2	3.0	3.2	3.5	3.3	6.1

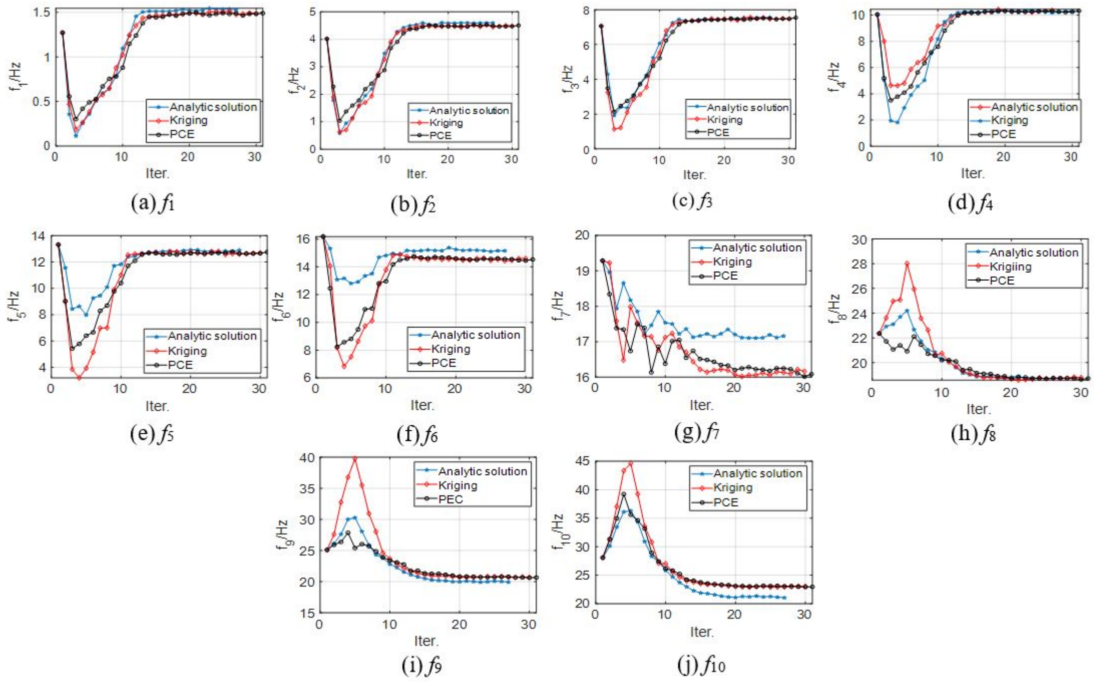


Fig. 8 Comparison of updated f_1-f_{10} in different methods

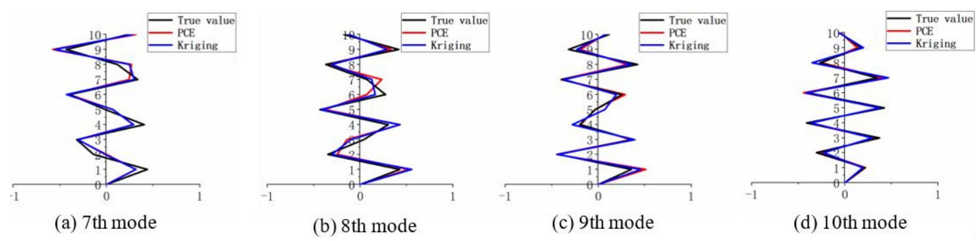


Fig. 9 Comparison of structure modes using updated model parameters from different surrogate models

slightly large as Fig. 9 shows. Considering that the application value of high-order modes in practical engineering is not significant, the updated results are quite ideal. It is feasible to use surrogate models to establish the relationship between the updated parameters and structural features. Moreover, surrogate models significantly simplify the establishment of the likelihood function, which is the most complicated step in the Bayesian model updating. The simplification is reflected in the shortened running time of the program. In the absence of a clear and quantitative understanding of the relationship between parameters and features, the analytical method is incapable to construct. However, generating a surrogate model only requires a set of input and output points. The advantage of the surrogate model over the analytical methods will be illustrated in the following model updating of the national stadium.

3.3 High-dimensional nonlinear model: national stadium

A much more complex nonlinear structure model, the national stadium of China, has been analyzed. Fig. 10

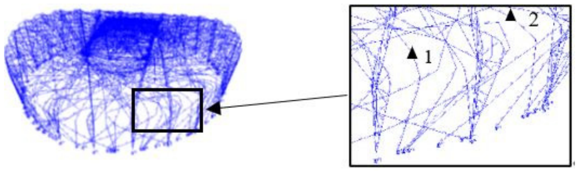


Fig. 10 National Stadium

shows the finite element model of the national stadium. The updated parameters are the scaled young's modulus of steel $\theta_i = E_i/10^{11}$ Pa, i = 1,2. The origin value of θ is $[1.75, 1.75]^T$. The parameters are assumed to be independent and identically distributed following uniform distributions. The first five modal frequencies f (Case A) and the extreme values of the horizontal velocity V of two nodes (Case B) are the investigated structural features. The features are assumed to obey Gaussian distributions with given mean values, on the assumption of the origin value of θ, with C.O.V of 0.01 to account for the uncertainties. The number of samples N is set 3000.

The Kriging model and PCE model are both utilized in this example analysis. The best candidate Kriging model with the minimum MSE is selected. As there are only two updated parameters, the number of design points of the two surrogate model is five. This means that the prior distribution will have a large impact on the correction results. Therefore, the mean values of the prior distribution are taken as five different values in this case to study the effect of the prior distribution on the posterior distribution when the sample size is small. A description of the parameters is presented in Table 6.

As Table 7 shows, the prior mean of θ has an obvious influence on updating accuracy. Table 8 illustrates the updated results of the frequencies at the prior mean equivalent to the origin value. It can be seen that the predictive accuracy of frequencies is higher than that of parameters. Fig. 11 shows the marginal posterior distribution of θ at the prior mean equivalent to the origin value. Fig. 12 illustrates the updated modes in case A. As is seen from Fig. 11, the maximum value of the posterior distribution is closer to the true value than the posterior mean. The spread of variables is smaller when the PCE model is employed. It indicates that the PCE-based model updating method yields less parameter uncertainty, which is consistent with the result obtained in the ten-storey frame.

From observations in Tables 7-8 and Fig. 11, the PCE model performs better than the Kriging model in the updating results of the parameters and the features. Another advantage of PCE is that the iteration steps are almost half of that in Kriging models. This shows that the PCE model is more applicable to low dimensional cases than Kriging

Table 6 Description of updated parameters in the national stadium

Parameters	Prior distribution	Interval	Origin value
θ_1	Uniform	$[\mu$-0.5, μ+0.5] $\mu \in \{1.05, 1.30, 1.50, 1.75, 2.00\}$	1.75
θ_2	Uniform	$[\mu$-0.5, μ-0.5] $\mu \in \{1.05, 1.30, 1.50, 1.75, 2.00\}$	1.75

Table 7 Parameters in initial and updated models (% errors in percentage)

Prior mean	1.05		1.30		1.50		1.75		2.00	
Parameters	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2	θ_1	θ_2
Frequency (Kriging)	1.85 (5.7)	1.80 (2.9)	1.89 (8.0)	1.85 (5.7)	1.85 (5.7)	1.81 (3.4)	1.83 (4.5)	1.82 (4.0)	1.98 (13.1)	1.94 (10.9)
Frequency (PCE)	1.87 (6.8)	1.69 (-3.5)	1.82 (4.1)	1.66 (-5.1)	1.85 (5.7)	1.67 (-4.6)	1.71 (-2.3)	1.74 (-0.6)	1.95 (11.4)	1.72 (-1.6)
Velocity (Kriging)	1.52 (-13.1)	1.48 (-15.3)	1.60 (-8.5)	1.56 (-10.9)	1.63 (-6.9)	1.59 (-9.1)	1.71 (-2.2)	1.69 (-3.4)	1.71 (-2.2)	1.73 (-1.2)
Velocity (PCE)	1.51 (-13.7)	1.51 (-13.7)	1.69 (-3.4)	1.61 (-8.0)	1.72 (-1.7)	1.66 (-5.1)	1.76 (0.6)	1.71 (-2.3)	1.80 (2.9)	1.86 (6.3)

Table 8 Updated features before cluster (% errors in percentage)

Features	Case A					Case B			
	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_4 (Hz)	f_5 (Hz)	$v_{max,1}$ (m/s)	$v_{min,1}$ (m/s)	$v_{max,2}$ (m/s)	$v_{min,2}$ (m/s)
Initial value	1.118	1.133	1.270	1.838	1.964	0.5101	-0.4826	0.7238	-0.5286
Kriging	1.109 (-0.82)	1.128 (-0.48)	1.271 (0.06)	1.823 (-0.82)	1.948 (-0.81)	0.5190 (1.74)	-0.4809 (-0.35)	0.7149 (-1.23)	-0.5210 (-1.44)
PCE	1.114 (-0.37)	1.128 (-0.44)	1.265 (-0.37)	1.830 (-0.44)	1.956 (-0.43)	0.5098 (-0.06)	-0.4851 (0.51)	0.7184 (-0.75)	-0.5283 (-0.06)

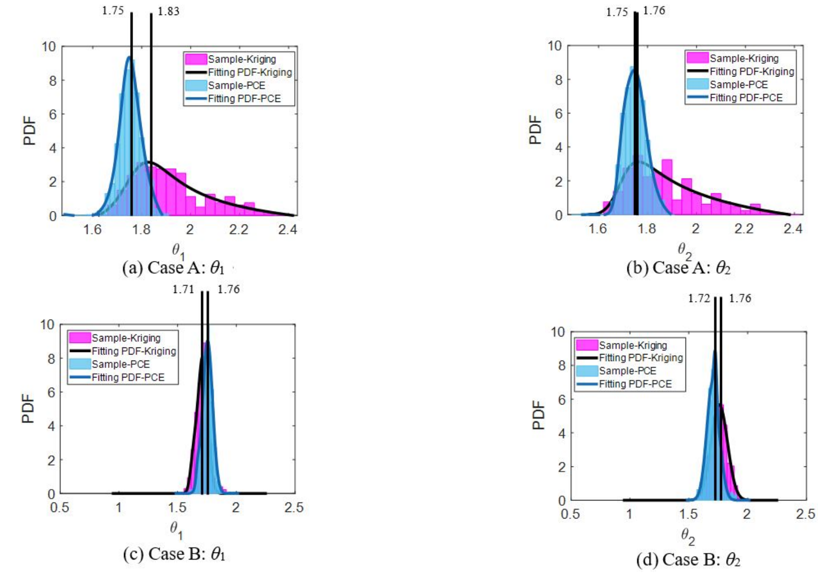


Fig. 11 Marginal posterior distributions of the updated parameters

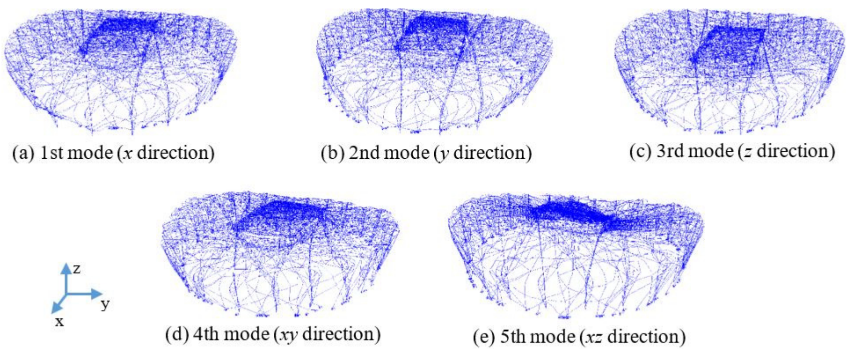


Fig. 12 Updated modes in case A

model, even though there is a nonlinear relationship between parameters and features.

One reason for having the errors is the model uncertainties due to a few training points. Another reason is that the frequencies of the national stadium are not very sensitive to the changes of updated parameters. Such small variations in measured features may cause a large difference in the parameters, resulting in relatively large errors in θ, especially when using the Kriging model.

From the above, when there are fewer sample points for constructing a surrogate model, the prior distribution has a great impact on the results. By introducing the clustering method, the updated results have strong robustness to the quantity of measured features. However, if we take the perspective of a frequentist, whose idea is to replace the whole population with a sample and derive the updated parameters backwards from the measured features, the error will be very large because the sample is too small. It reflects an advantage of the Bayesian model updating theorem over the deterministic updating methods.

3.4 Extension study: *Ensemble learning*

A final extension study is made to consider possible ways of improving the surrogate model based on Bayesian model updating approaches. The proposed ensemble learning method in Section 2.4 is applied to update the parameters in ten-storey frame and the national stadium. For the ten-storey frame, the model weights are estimated as a_1 = 0.3583, a_2 = 0.6417. The iteration steps of Kriging model, PCE model, and the ensemble model are all around 30. For the national stadium, the model weights are

Table 9 Comparison before and after ensemble in ten-storey frame (% errors in percentage)

Variables	Kriging	PCE	Ensemble model
θ_1	1.69 (12.9)	1.97 (38.3)	1.54 (2.9)
θ_2	0.96 (-3.9)	1.09 (8.6)	0.96 (-4.2)
θ_3	1.00 (0.1)	0.97 (-2.6)	0.95 (-4.6)
θ_4	1.01 (0.5)	1.06 (-5.9)	1.03 (3.4)
θ_5	1.05 (4.9)	1.06 (6.4)	1.01 (1.0)
θ_6	1.06 (6.3)	1.15 (15.4)	1.02 (2.3)
θ_7	1.02 (2.2)	1.11 (10.5)	0.97 (-3.2)
θ_8	1.06 (6.1)	1.22 (22.5)	0.97 (-2.9)
θ_9	1.06 (6.0)	1.03 (3.0)	1.03 (0.4)
θ_{10}	1.24 (23.7)	1.25 (24.5)	1.05 (4.8)

Table 10 Comparison before and after ensemble in national stadium (% errors in percentage)

Variables		θ_1	θ_2
Case A	Kriging	1.83 (4.5)	1.82 (4.0)
	PCE	1.71 (-2.3)	1.74 (-0.6)
	Ensemble model	1.75 (0.4)	1.74 (-0.6)
Case B	Kriging	1.71 (-2.2)	1.69 (-3.4)
	PCE	1.76 (0.6)	1.71 (-2.3)
	Ensemble model	1.75 (0.3)	1.75 (-0.1)

estimated as a_1= 0.4296, a_2= 0.5704 for case A and a_1 = 0.4670, a_2 = 0.5330 for case B. The iteration steps of PCE model and the ensemble model are both four while that of Kriging model is seven. From the model weights, it can also be seen that PCE is more suitable for low-dimensional problems while Kriging performs better in high-dimensional problems. Tables 9 and 10 compare the updated parameters in two surrogate models and the ensemble model for the ten-storey frame and the national stadium respectively. Figs. 13 and 14 show the box plots for a comparison of the distribution of some representative parameters using the three models. It can be seen that the ensemble learning technique greatly improves the predictive accuracy and inherits the fastest iteration speed.

4. Conclusions

This paper proposes a Bayesian structural model updating framework by integrating the Transitional Markov chain Monte Carlo (TMCMC) algorithm, model validation, and K-means cluster analysis. The proposed method was verified using three examples with different structural complexity. In the three examples, Kriging model and PCE model are investigated and compared to show their advantages and limitations.

- In the simple two-storey frame (low-dimensional linear model), it was found that the prediction error, sample size, and the form of prior distribution have little influence on the posterior mean of updated parameters but have a large impact on the variance. The analytical approach can be well applied with no difficulties.
- For the two complex cases (high-dimensional linear model and low-dimensional nonlinear model), the Kriging model and the PCE model are utilized to facilitate the updating calculations. It is shown that the updating accuracy of the PCE model is sensitive to the parameter dimensions. For a ten-story frame with high-dimensional parameters, the analytic method and the surrogate models are all applicable, but the errors of PCE are slightly larger. For the national stadium with low-dimensional parameters, the PCE model could guarantee higher updating accuracy, which shows the advantage of PCE model in dealing with nonlinear problems.
- To improve the performance of surrogate models, an ensemble learning algorithm suitable for the surrogate model is presented by combining Bagging and Adaboost. It shows such an ensemble learning of the surrogate models can greatly improve the model updating. This provides an opportunity to update parameters for more complex structures while computational efforts can be largely reduced.

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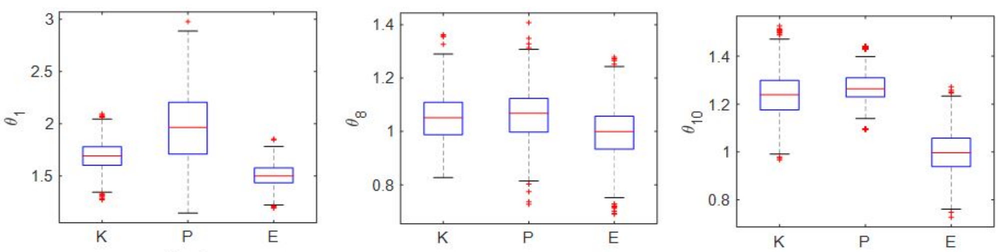

(a) θ_1 (b) θ_8 (c) θ_{10}

Fig. 13 Comparison of Kriging model, PCE model, and ensemble model in ten-storey frame

(a) θ_1 (b) θ_2

Fig. 14 Comparison of Kriging model, PCE model, and ensemble model in case A of national stadium

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