Smart Structures and Systems, *Vol. 27, No. 5 (2021) 729-743*
DOI: https://doi.org/10.12989/sss.2021.27.5.729

Vibration analysis of FG cylindrical shell: Evaluation of Ritz-polynomial mixed with ring terms

Mohamed A. Khadimallah*[1,2], Muzamal Hussain [3], Muhammad Nawaz Naeem [3], Amjad Qazaq [1], Abdulaziz Alqahtani [1] and Abdelouahed Tounsi [4,5]

[1] *Prince Sattam Bin Abdulaziz University, College of Engineering, Civil Engineering Department, Al-Kharj, 16273, Saudi Arabia*
[2] *Laboratory of Systems and Applied Mechanics, Polytechnic School of Tunisia, University of Carthage, Tunis, Tunisia*
[3] *Department of Mathematics, Govt. College University Faisalabad, 38000, Faisalabad, Pakistan*
[4] *YFL (Yonsei Frontier Lab), Yonsei University, Seoul, Korea*
[5] *Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia*

(*Received June 11, 2020, Revised December 30, 2020, Accepted January 2, 2021*)

Abstract. Here the Rayleigh - Ritz method has been applied to derive the shell vibration frequency equation. This equation has been formed as an eigenvalue problem form. MATLAB software package has been utilized for extracting shell frequency spectra. Nature of materials used for construction of cylindrical shells also has visible impact on shell vibration characteristics. For isotropic materials, the physical properties are same everywhere, the laminated and functionally graded materials vary from point to point. Here the shell material has been taken as functionally graded material. Moreover, the impact of ring supports around the shell circumferential has been examined for the various positions along the shell axial length. These shells are stiffened by rings in the tangential direction. These ring supports are located at various positions along the axial direction round the shell circumferential direction. These variations have been plotted against the locations of ring supports for three values of exponents of volume fraction law. For three conditions, frequency variations show different behavior with these values of exponent law. The influence of the positions of ring supports for simply supported end conditions is very visible. The frequency first increases and gain maximum value in the midway of the shell length and then lowers down. The comparisons of frequencies have been made for efficiency and robustness for the present numerical procedure.

Keywords: Lagrangian functional; ring supports; volume fraction; FGM

1. Introduction

More than one type of materials is used to structure the functionally graded materials (FGMs) by some material manufacturing law and their physical properties vary from one surface to the other surface. In these surfaces, one has highly heat resistance property while other may preserve great dynamical perseverance and differs mechanically and physically in regular manner from one surface to other surface, making them of dual physical appearance. All these materials have changeable outer and inner sides and their physical properties greatly differ from each other (Suresh and Mortensen 1997, Koizumi 1997).

These materials are organized by various techniques and their applications are seen in dynamical elements such as plates, beams and shells. A large use of shell structures in practical applications makes their theoretical analysis an important field of structural dynamics. Since a shell problem is a physical one, so their vibrational behaviors are distorted by variations of physical and material parameters. To elude any complications which may risk a physical system their analytical investigation is done. Loy and Lam (1997) investigated the shell vibrations with ring supports that restricted the motion of cylindrical shells (CSs) in the transverse direction. This influence was inducted by the polynomial functions.

Xiang *et al*. (2002) formed some closed form solution functions for studying vibrations of cylindrical shells. The mid-way ring supports were clamped around the shells. Mehar *et al*. (2017a, b, c, d) studied the frequency response of FG CNT and reinforced CNT using the simple deformation theory, finite element modeling, Mori-Tanaka scheme. They investigated a new frequency phenomenon with the combination of Lagrange strain, Green-Lagrange, for double curved and curved panel of FG and reinforced FG CNT. The characteristics of sandwich and grades CNT was found with labeling the temperature environ. The thermo elastic frequency of single shallow panel was determined using Mori-Tanaka formulations. The research of these authors has opened a new frequency spectrum for other material researchers. Wang *et al*. (1997) scrutinized the vibrations of ring-stiffened CSs using Ritz polynomial functions. Materials of both shells and rings were of isotropic nature. These shells were stiffened with isotropic rings having three types of locations on the shell outer surface. To increase the stiffness of CSs was stabilized by

*Corresponding author, Ph.D., Assistant Professor,
E-mail: am.khadimallah@gmail.com;
mohamedamine.khadimallah@fsgf.rnu.tn

http://www.techno-press.org/?journal=sss&subpage=7
ISSN: 1738-1584 (Print), 1738-1991 (Online)

ring-stiffeners. Isotopic materials are the constituents of these rings. A large use of shell structures in practical applications makes their theoretical analysis an important field of structural dynamics. Since a shell problem is a physical one, so their vibrational behaviors are distorted by variations of physical and material parameters. To elude any complications which may risk a physical system their analytical investigation was done. Mehar and Panda (2016a, b, 2018a) computed the vibration behavior, bending and dynamic response of FG reinforced CNT using shear deformation theory and finite element method. For the sake of generality, the mathematical model was presented with the mixture of Green Lagrange method. The convergence of these methodologies has been checked for the variety of results. The composite plates with different graded was investigated with isotropic and core phase. Sharma *et al.* (1998) determined frequencies of composite cylindrical shells containing fluid. They estimated the axial modal deformations by trigonometric functions. Sharma (1974) analyzed vibration frequencies circular cylinder with using the Rayleigh - Ritz formulation and made comparisons of his results with some experimental ones. Chung *et al.* (1981) investigated the vibrations of fluid-filled CSs and presented an analysis of experimental and analytical investigation. Mehar and Panda (2018b) investigated the curved shell and CNT vibration with thermal environment using higher order deformation theory. This CNT was mixed with different configurations of the layers. The results have been verified with the earlier investigations. Sewall and Naumann (1968) considered the vibration analysis of CSs based on analytical and experimental methods. The shells were strengthened with longitudinal stiffeners. Mehar and Panda (2018c) investigated numerically the deflection behavior of carbon nanotube-reinforced composite plate is using the finite-element method and the result accuracy is established via three-point experimental bending test data. Alazzawy and Jweeg (2010) conducted the vibration solution for laminated simply supported closed cylindrical shells. This solution is obtained using General Third Shell Theory (GTT). Also, the critical in-plane fatigue load is studied and the required equilibrium equations are developed, the effects of tension or compression in-plane load on the natural frequencies are discussed also.

Jiang and Olson (1994) recommended the characteristics of analysis of stiffened shell using finite element method to diminish large computational efforts which are required in the conventional finite element analysis. Mehar and Panda (2018d) developed a general mathematical model for the evaluation of the theoretical flexural responses of the functionally graded carbon nanotube- reinforced composite doubly curved shell panel using higher-order shear deformation theory with thermal load. Najafizadeh and Isvandzibaei (2007) applied ring supports to CSs for vibration analysis of along the tangential direction and founded their research on angular deformation theory of higher order. The angular deformation was used for shell equations and determined the effects of constituent volume fractions and shell configurations on the shell vibrations. FG material parameters were changed step by step. Bisen *et al*. (2018) and Mehar and Panda (2019) studied the structural response of reinforced material and FG-CNT using the numerical and experimental properties. The results are verified with the open existing literature. The computer software MATLAB was used for the frequency results. The higher order finite element and higher order mid-plane kinematics. The mixture rule was defined for the different materials. Power-law, sigmoid and exponential distribution of the individual constituents through the thickness direction.

Shah *et al*. (2009) and Sofiyev and Avcar (2010) studied stability of CSs based on Rayleigh - Ritz and Galerkin technique using elastic foundations. The structures of cylindrical shell are tackled under the exponential law and axial load. Mehar *et al*. (2018a) evaluated the frequency behavior of nanolpate structure using FEM including the nonlocal theory of elasticity. Computer generated results are created by using the software first time robustly to check the vibration of nanoplate. The efficiency was checked by comparing the results of available data. Jweeg and Alazzawy (2007) investigated the transient solutions for laminated simply supported closed cylindrical shells subjected to a uniform dynamic pressure at the outer surface of the cylinder. These solutions are obtained by using General Third Shell Theory (GTT). Rectangular pulse, triangular pulse, sinusoidal ulse and (ramp-constant) load-time varying functions are studied and the required equilibrium equations are developed. Naeem *et al*. (2013) conducted the vibrational behavior of submerged FG-CSs. The problem of submerged cylindrical shells was frequently met where fluid envelopes a structure. Ramteke *et al*. (2020a) obtained the finite element solutions of static deflection and stress value for the functionally graded structure considering variable grading patterns (power-law, sigmoid and exponential) including the porosity effect. The unknown values are obtained computationally via a customized computer code with the help of cubic-order displacement functions considering the varied distribution of porosity (even and uneven) through the panel thickness.

Ergin and Temarel (2002) did a vibration study of cylindrical shells. The shells lied in a horizontal direction and contained fluid and submerged in it. Goncalves and Batista (1987) gave an analytical investigation of submerged CSs with fluid. Ramteke *et al*. (2020b) studied the two directional graded structure has been developed using a commercial FE package ANSYS and the subsequent deflection responses. Additionally, the model includes the porosity within the graded structure considering even type of distribution pattern. The present model is derived using the basic steps available in the ANSYS platform through the batch input technique. Kareem and Majeed (2019) investigated the impact of transient response laminated shallow shells due to transverse foreign object analytically and experimentally. The analytical analysis is presented by using the new higher order shear deformation shell theory. Contact force and transverse deflection of laminated shell have been measured with a piezoelectric force transducer (load cell) and a piezoelectric bending transducer respectively. Amabili (1999) used Donnell's shallow-shell model with the quiescent, dense, inviscid and incompressible fluid. Also, the dense fluid is studied for the influence of both the internal and external side of the shell. In the external side of the shell, the fluid was considered as an unbounded domain in the radial direction, while internally, the shell was considered as filled completely. Mehar and Panda (2018e) reported the nonlinear finite solutions of the nonlinear flexural strength and stress behavior of nano sandwich graded structural shell panel under the combined thermomechanical loading. Wang and Lai (2000) examined a novel approach for the evaluation of eigen - frequencies of cylindrical shells. The numerical process adopted by them was alike the wave propagation approach (WPA). Zhang (2002) studied vibrations of CSs submerged in a fluid. It was seen that the fluid factor impressed vibration shell frequencies to a significant limit. Mehar *et al*. (2018b) studied the bending responses of nanotube-reinforced curved sandwich shell panel structure under the influence of the thermomechanical loading. Further, the temperature dependent material properties of the sandwich structure are assumed to evaluate the exact responses.

Alazzawy (2008) studied the static behavior (under torsion and bending loading) of machine tool column was studied in this paper. The effects of changing the cross sectional area of the column itself on the deformations (design parameters) was investigated. The adding of stiffeners and changing the stiffeners cross sectional area are also verified. The results show that using of stiffeners can produse a great reduction in deformation of the column structure under the stastic loading mentioned above. Ansari and Rouhi (2015) performed nonlocal model for the frequencies of multi-walled carbon nanotubes with small effects subject to various boundary conditions (BCs) using Rayleigh-Ritz technique. The governing equation was formulated based on Flügge's and nonlocal shell theory. Some new resonant frequencies were identified with the association of vibrational modes and circumferential modes into shell model. Sharma *et al*. (2019) studied the functionally graded material using sigmoid law distribution under hygrothermal effect. The Eigen frequencies are investigated in detail. Frequency spectra for aspect ratios have been depicted according to various edge conditions. Recently some researcher used different methods for nonlinear modeling (Tohidi *et al*. 2018, Arefi and Zenkour 2017, Ghobaei-Arani *et al*. 2018, Krommer *et al*. 2016, Yeh 2016) and for other structures (Boussoula *et al*. 2020, AlSaleh and Fuggini 2020, Lee *et al*. 2019, Zahrai and Kakouei 2019, Poplawski *et al*. 2019) and some other works as deformation theory (Mehar *et al*. 2016), finite element method (Mehar *et al*. 2018c), Green–Lagrange strain field ((Mehar *et al*. 2018c), Multiscale modeling approach (Mehar and Panda 2019, Mehar *et al*. 2019, higher order theory (Jweeg and Majeed 2020); finite element formulation (Dewangan *et al*. 2020a, b).

According to our knowledge, up to now little is known about the vibration analyses of ring supports and moreover, the influence of length-to-radius ratio, thickness-to-radius ratio and varying of exponents of volume fraction have not been investigated earlier for FG-CS based on RRM. These shells are stiffened with isotropic rings having three types of boundary conditions on the shell outer surface. These cylinders are stabilized by ring-stiffeners to increase the stiffness and strength. Isotopic materials are the constituents of these rings. A large use of shell structures in practical applications makes their theoretical analysis an important field of structural dynamics. Since a shell problem is a physical one, so their vibrational behaviors are distorted by variations of physical and material parameters. To elude any complications which may risk a physical system their analytical investigation is done.

2. Functionally graded material

The volume fraction V_f for CSs with power-law, the material property is expressed as (Chi and Chung 2006)

$$V_f = \left[\frac{z}{h} + \frac{1}{2}\right]^q \tag{1}$$

The power law exponent is denoted as q and h for thickness and z is the coordinate which varies from zero to infinity.

In practice a cylindrical shell is framed from a FGM which contains of two component materials these are denoted by Type-I and Type-II. If E_1 and E_2 as Young's moduli, ν_1 and ν_2 as Poisson's ratios, ρ_1 and ρ_2 mass densities respectively. Then effective material quantities: Young's modulus, E_{FGM}, Poisson's ratio, ν_{FGM} and mass density, ρ_{FGM} of the FGM are given as

$$\begin{aligned} E_{FGM} &= [E_1 - E_2]\left[\frac{2z+h}{2h}\right]^q + E_2, \\ \nu_{FGM} &= [\nu_1 - \nu_2]\left[\frac{2z+h}{2h}\right]^q + \nu_2, \\ \rho_{FGM} &= [\rho_1 - \rho_2]\left[\frac{2z+h}{2h}\right]^q + \rho_2 \end{aligned} \tag{2}$$

The material parameters: E_1, E_2, ν_1, ν_2, and ρ_1, ρ_2 for constituents materials Stainless steel and Nickel at a temperature of 300 K. Toulokian (1967) stated the material properties C at high temperature environ, with temperature-dependents which is a function of temperature. In Eq. (3), the constants $(C_0, C_{-1}, C_1, C_2, C_3)$ are different for different material.

$$C = C_0(C_{-1}T^{-1} + C_1T + C_2T^2 + C_3T^3) \tag{3}$$

A FG-CS consisting of two constituent materials. In these Types, Nickel and Stainless steel are used as the interior surfaces and the exterior surface respectively, but their arrangement has profound influence on the formation of FG-CSs. The order of the FG constituent materials is reversed as Type-I and Type-II. At temperature 300K, for Stainless steel and Nickel, the material properties for FG-CS are: E, υ, ρ for Stainless steel are $2.07788 \times 10^{11}\,\frac{N}{m^2}$, 0.317756 and $8166\,\frac{Kg}{m^3}$and Nickel are $2.05098 \times 10^{11}\,\frac{N}{m^2}$, 0.3100, and 8900 $\frac{Kg}{m^3}$ (Loy *et al*. 1999).

The resulting fabric properties of a functionally graded material are expressed as

$$C = \sum_{i=1}^{\eta'} C_i V_{f_i} \tag{4}$$

Here C_i denotes the fabric property and V_{fi}, the volume fraction of the *i*th FG material in that order. $\eta'$is the number of functionally graded constituent fabric. When volume fractions of these ingredient materials are sum up, then their result is written as

$$\sum_{i=1}^{p} V_{f_i} = 1 \tag{5}$$

3. Theoretical formulation of shell problem

An orthogonal system $(x,\ \theta,\ t)$ is setup for the reference surface (middle surface) as shown in Fig. 1. The x, θ co-ordinate are assumed to be along longitudinal and circumferential direction, respectively and z - co-ordinates are taken in its radial directions. The space of the ring-stiffeners $\int$ on the shell may or may not have equal spaces according to the adjustment of the shell. From one end of the FG-CSs, the measurement of the k^{th} stiffener is positioned at $x = a_k L$ with width b_k and rectangular cross section depth d_k. The stiffeners may be assembled from diverse materials and also from the parent shell material. ν_k, G_k, E_k, ρ_k, stands for Poisson's ratio, shear modulus, Young's modulus and stiffener's mass density, respectively.

The strain and kinetic energies is given for vibrating cylindrical shell

$$S = \frac{R}{2}\int_0^L \int_0^{2\pi} \begin{pmatrix} A_{11}e_{11}^2 + 2e_1e_2A_{12} + 2e_1k_1B_{11} + 2e_1k_2B_{12} + e_2^2A_{22} \\ +2e_2k_2B_{12} + 2e_2k_2B_{22} + \gamma^2A_{66} + 4\tau\gamma B_{66} + k_1^2D_{22} \\ +2k_1k_2D_{12} + k_2^2D_{22} + 4\tau^2D_{66} \end{pmatrix} d\theta dx \tag{6}$$

The kinetic energy (K.E) of vibratingshell is articulated as

$$K = \frac{1}{2}\int_O^L \int_0^{2\pi} \rho_T \left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right] Rd\theta dx, \tag{7}$$

whereas $(\kappa_1,\ \kappa_2,\tau)$ and $(e_1,\ e_2,\ \gamma)$ are referenced as surface curvatures and surface strains.

A_{ij} (extensional stiffness), B_{ij} (coupling stiffness) and D_{ij} (bending stiffness) such that (i, j = 1~6) and these stiffness's A_{ij}, B_{ij}, D_{ij} are given as

$$\{A_{ij}, B_{ij}, D_{ij}\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij}\{1, z, z^2\} dz \tag{8}$$

$$[Q] = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{pmatrix} \tag{9}$$

Q_{ij} is reduced stiffness for isotropic materials with conjunction of E and v are written as

$$Q_{11} = \frac{E}{1 - v^2} = Q_{22}, \quad Q_{12} = \frac{vE}{1 - v^2}, \quad Q_{66} = \frac{E}{2(1 + v)} \tag{10}$$

The mass density relation ρ_T is expressed as

$$\rho_T = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho\, dz \tag{11}$$

The strain energy S in modified form is taken from Sander's theory and can be elaborated as

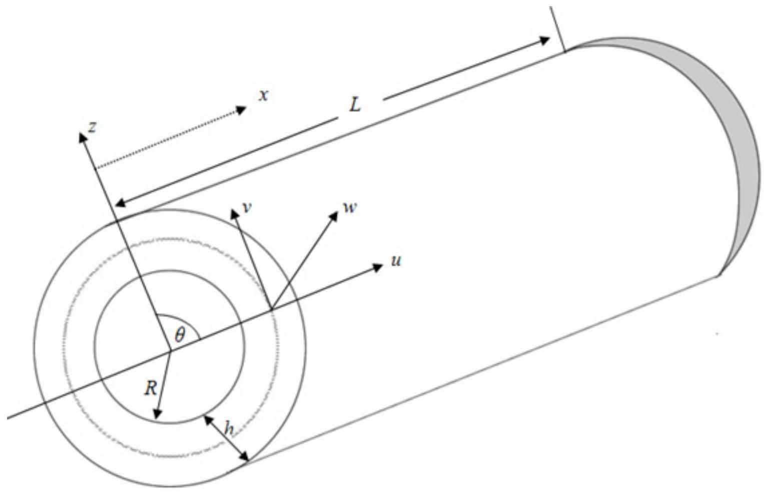


Fig. 1 Geometry of cylindrical shell

$$S=\frac{R}{2}\int_0^L\int_0^{2\pi}\left[A_{11}\left(\frac{\partial u}{\partial x}\right)^2+\frac{A_{22}}{R^2}\left(\frac{\partial v}{\partial\theta}-w\right)^2+\frac{2A_{12}}{R}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial\theta}-w\right)+A_{66}\left(\frac{\partial v}{\partial x}+\frac{1}{R}\frac{\partial u}{\partial\theta}\right)^2\right.$$
$$+2B_{11}\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2}+\frac{2B_{12}}{R^2}\left(\frac{\partial^2 w}{\partial\theta^2}+w\right)\left(\frac{\partial u}{\partial x}\right)+\frac{2B_{12}}{R}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial v}{\partial\theta}-w\right)+\frac{2B_{22}}{R^3}\left(\frac{\partial v}{\partial\theta}-w\right)\left(\frac{\partial^2 w}{\partial\theta^2}+w\right)$$
$$+B_{66}\left(\frac{\partial^2 w}{\partial x\partial\theta}+\frac{3}{4}\frac{\partial v}{\partial x}-\frac{1}{4R}\frac{\partial u}{\partial\theta}\right)\left(\frac{\partial v}{\partial x}+\frac{1}{R}\frac{\partial u}{\partial\theta}\right)+D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\frac{D_{22}}{R^4}\left(\frac{\partial^2 w}{\partial\theta^2}+w\right)^2$$
$$\left.+\frac{2D_{12}}{R^2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial^2 w}{\partial\theta^2}+w\right)+4D_{66}\left(\frac{\partial^2 w}{\partial x\partial\theta}+\frac{3}{4}\frac{\partial v}{\partial x}-\frac{1}{4R}\frac{\partial u}{\partial\theta}\right)^2\right]d\theta dx \tag{12}$$

For the k^{th} ring-stiffener the strain energy S_k is given by Galletly (1955)

$$S_k=\int_0^{2\pi}\left\{\frac{E_kI_{zk}}{2(R+e_k)}\left(\frac{\partial w_k}{\partial x}+\frac{1}{R+e_k}\frac{\partial^2 u_k}{\partial\theta^2}\right)^2+\frac{E_kI_{xk}}{2(R+e_k)^3}\left(w_k+\frac{\partial^2 w_k}{\partial\theta^2}\right)^2+\frac{E_kA_k}{2(R+e_k)}\right.$$
$$\left.\left(\frac{\partial v_k}{\partial\theta}-w_k\right)^2+\frac{G_kJ_k}{2(R+e_k)}\left(-\frac{\partial^2 w_k}{\partial x\partial\theta}+\frac{1}{R+e_k}\frac{\partial u_k}{\partial\theta}\right)^2\right\}d\theta \tag{13}$$

where e_k denote the eccentricity of the ring stiffener and for externally eccentric stiffener it is expressed as

$$e_k=\frac{(h+d_k)}{2} \tag{14}$$

Whereas for concentric stiffener it is zero i.e.

$$e_k=0 \tag{15}$$

For internally eccentric stiffener it is given by

$$e_k=-\frac{(h+d_k)}{2} \tag{16}$$

I_{zk} , I_{xk} the second moment of areas and A_k , J_k signifies the cross-section areas and torsional constant given by the following equations

$$I_{zk}=\frac{b_k^3d_k}{12},\quad I_{zk}=\frac{b_kd_k^3}{12},\quad A_k=b_kd_k,$$
$$J_k=\frac{1}{3}\left[1-\frac{192b_k}{\pi^5d_k}\sum_{n=1,3,5}^{\infty}\frac{1}{n^5}tanh\frac{n\pi d_k}{2b_k}\right]b_k^3d_k \tag{17}$$

The kinetic energy of the k^{th} stiffener i.e., K_k is given by

$$K_k=\frac{1}{2}\rho_k\int_o^{2\pi}\left\{A_k\left[\left(\frac{\partial u_k}{\partial t}\right)^2+\left(\frac{\partial v_k}{\partial t}\right)^2+\left(\frac{\partial w_k}{\partial t}\right)^2\right]\right.$$
$$\left.+(I_{xk}+I_{zk})\left(\frac{\partial^2 w_k}{\partial t\partial x}\right)^2\right\}(R+e_k)d\theta \tag{18}$$

At the position of the stiffener the associations between the displacements (u_k,v_k,w_k) of the k^{th} stiffener and the displacements (u,v,w) of the shell at $x=a_kL$, from geometrical considerations, are expressed as

$$u_k=u+e_k\frac{\partial w_k}{\partial x} \tag{19}$$

$$v_k=v\left(1+\frac{e_k}{R}\right)+\frac{e_k}{R}\frac{\partial w}{\partial\theta} \tag{20}$$

$$w_k=w \tag{21}$$

The Lagrangian functional can be obtained with the coalescence energies (strain and kinetic) with the combination of k^{th} stiffener in the following shape

$$\Pi=\left[K+\sum_{k=1}^{\int}K_k\right]-\left[S+\sum_{k=1}^{\int}S_k\right] \tag{22}$$

3.2 Modal displacement form

The unknown functions involving the shell dynamical equations are functions of shape linear variables. The independent variables are separated by employing prescribed method. They are supposed in the form of the product of separate functions of independent variables. The displacement components are represented in the form (Flügge 1962, Forsberg 1964, Warburton 1965)

$$\begin{aligned}u&=A_mU(x)\cos(n\theta)\cos(\omega t)\\ v&=B_mV(x)\sin(n\theta)\cos(\omega t)\\ w&=C_mW(x)\cos(n\theta)\cos(\omega t)\end{aligned} \tag{23}$$

Where A_m,B_m,C_m are taken as the displacement amplitudes in x,θ and z directions. The angular frequency and circumferential wave number are represented by ω and n respectively.

4. Methodology

The present analysis is carried out for simply supported, clamped-clamped and clamped-free BCs to examine the frequency spectrum of ring-stiffened FG-CSs. Over the past several years vibration of shell and plate structures of various configurations and boundary conditions have been extensively studied (Wuite and Adali 2005, Xuebin 2008, Lam and Loy 1998). Excellent reviews of such studies are available in Leissa (1973). The present technique is very powerful for the prediction of vibration of shells with ring-stiffened. The Lagrangian energy functional Π is framed by adding energy expressions for the FG-CSs and ring stiffeners with regard to shell frequency by RRM. The shell frequency equation is solved to extract the shell frequency and mode shapes.

$$\Pi = \left[K + \sum_{k=1}^{\int} K_k\right] - \left[S + \sum_{k=1}^{\int} S_k\right] \tag{24}$$

The assumed forms of u, v, w in the Eq. (23) and the values of their partial derivatives are substituted in the Eqs. (7), (12), (13) and (18) for energy expressions S, K, S_k and K_k respectively and after their substitution we get the following expression for energy functional Π as

$$\begin{aligned}
\prod = \frac{R\pi}{2}\int_0^L \Bigg[& A_{11}A_m^2\left(\frac{dU}{dx}\right)^2 + \frac{2}{R}A_{12}(A_mB_mn + A_mC_m)V\left(\frac{dU}{dx}\right) - 2A_mC_mB_{11}\left(\frac{\partial U}{\partial x}\right)^2 \\
& + \frac{2}{R^2}B_{12}(n^2A_mC_m + nB_mA_m)V\left(\frac{dU}{dx}\right) + A_{22}\frac{(Bn+C)^2}{R^2}V^2 - B_{12}\frac{2(BCn+C^2)}{R}V\left(\frac{dU}{dx}\right) \\
& + \frac{2B_{22}}{R^3}(B_mC_mn^3 + C_m^2n^2 + B_mC_mn + B_m^2n^2)V^2 + A_{66}\left(B_m^2 + \frac{n^2A_m^2}{R^2} - \frac{2B_mA_mn}{R}\right)U^2 \\
& + 4B_{66}\left(\frac{nB_mC_m}{R} - \frac{n^2A_mC_m}{R^2} + \frac{B_m^2}{R} - \frac{nB_mA_m}{R^2}\right)U^2 + D_{11}C_m^2\left(\frac{dU}{dx}\right)^2 - \frac{2D_{12}}{R^2}(n^2C_m^2 + nB_mC_m)V\left(\frac{dU}{dx}\right) \\
& + \frac{D_{22}}{R^4}(n^2C_m + nB_m)^2V^2 + \frac{4D_{66}}{R^2}(nC_m + B_m)^2U^2 - \left\{\frac{R\omega^2\rho_T\pi}{2}(A_m^2U^2 + B_m^2V^2 + C_m^2W^2)\right\}\Bigg] dX \\
& + \pi\Bigg[\frac{E_kI_{zk}}{2(R+e_k)^3}U^2(C_m^2(R+e_k)^2 + n^4(A_m^2 + C_m^2e_k^2 + 2A_mC_me_k) - 2n^2(A_mC_m + C_m^2e_k)(R+e_k)) \\
& + \frac{E_kI_{Xk}}{2(R+e_k)^3}V^2C_m^2(1-n^2)^2 + \frac{E_kA_k}{2(R+e_k)}V^2((1+\frac{e_k}{R})^2n^2B_m^2 + (1+\frac{n^2e_k}{R})^2C_m^2 \\
& - 2nB_mC_m\left(1+\frac{e_k}{R}\right)\left(1+\frac{n^2e_k}{R}\right))\Bigg] - \frac{\omega^2\rho_k\pi}{2}(R+e_k)\Bigg[A_kU^2(A_m + C_me_k)^2 \\
& + A_kV^2\left\{(C_m^2n^2 + B_m^2 - 2B_mC_mn)\frac{e_k^2}{R^2} + B_m^2 + 2B_m\frac{e_k}{R}(B_m - C_mn)\right\} + C_m^2V^2 + (I_{Xk} + I_{zk})C_m^2U^2\Bigg]
\end{aligned} \tag{25}$$

4.1 Derivation of generalized eigenvalue problem

The RRM is applied to formulate the generalized eigenvalue problem. This is a direct approach gives rapid results. Necessary conditions for extrema of the functional $\Pi_{\max}$ give in the following Eq. (25).

$$\frac{\partial \Pi_{\max}}{\partial A_m} = \frac{\partial \Pi_{\max}}{\partial B_m} = \frac{\partial \Pi_{\max}}{\partial C_m} = 0 \tag{26}$$

After solving the Eq. (26) with its full form, the following eigenvalue is

$$\Big[[[S] + [S_k]] - \omega^2[[K] + [K_k]]\Big]X = [0] \tag{27}$$

or

$$\big[[C] - \omega^2[D]\big]X = [0] \tag{28}$$

where

$$[C] = \big[[S] + [S_k]\big],$$
$$[D] = \big[[K] + [K_k]\big],$$

This is an eigenvalue problem and is resolved by the help of the MATLAB software. MATLAB software is very helpful and capable tool that gives a quick solution of the problem. Both eigenvalues and eigenvectors are obtained by a simple command. For extracting the frequencies and mode shapes of the shell, the eigenvalues and eigenvector are formed. Where $[S]$, $[K]$ are designated by stiffness and the mass matrices of the FG-CSs respectively and $[S_k]$, $[K_k]$ are k^{th} ring stiffener matrices with $X = [A_m, B_m, C_m]^T$. The expression for matric entries of $[C]$ and [D] are given in Appendix-I.

5. Numerical results and discussions

For the convergence rate of FG-CSs, the non-dimensional frequency parameters enumerated in the current work, i.e., using RRM, are happened to be in a good consistency along with the so-called exact results furnished by Loy *et al.* (1999), those were established by working out with the deformation theory provided in Table 1. The proposed model based on RRM can incorporate in order to accurately predict the acquired results of material data point. The frequencies are described for non-dimensional frequency parameters as: $\xi = \omega R\sqrt{(1-\nu^2)\rho/E}$ as shown in Table 1 and positive coherence is achieved. The percentage difference is negligible as n = 1, 3, 4 are 0.006%, 0.01%, 0.002% and at n = 2 by 0.0061% and present RRM result are lower than equivalent results executed by Loy *et al.* (1999). The frequency parameters for circumferential wave numbers n = 5, 6 are same with

Table 1 Convergence of RRM frequencies Loy *et al*. (1999)

L/R	h/R	Method	N					
			1	2	3	4	5	6
20	0.01	Loy *et al*. (1999)	0.016102	0.009382	0.022105	0.042095	0.06801	0.09973
		RRM	0.016101	0.009378	0.022103	0.042094	0.04209	0.09973

Table 2 Convergence of RRM frequencies Warburton (1965)

n	Method	m					
		1	2	3	4	5	6
2	Warburton (1965)	2040.8	5637.6	8935.3	11405	13245	14775
	RRM	2043.7	5631.9	8926.4	11399.4	13243.7	14779.9
3	Warburton (1965)	2199.3	4041.9	6620	9124	11357	13384
	RRM	2194.4	4031.2	6605.9	9108.4	11343.4	13374.9

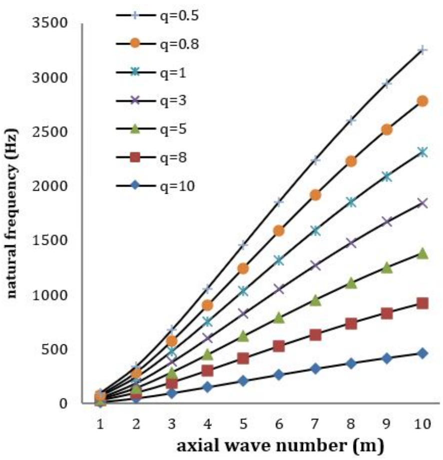
Table 3 Convergence of frequencies of SS-SS FG-CSs with 14 evenly spaced ring stiffeners versus d/b (stiffener's depth-to-width ratio) Swaddiwudhipong *et al*. (1995)

d/b	1.33414			2.6628			3.9942		
N	2	3	4	2	3	4	2	3	4
Swaddiwudhipong *et al*. (1995)	0.084	0.067	0.107	0.083	0.109	0.203	0.087	0.159	0.304
RRM	0.082	0.064	0.106	0.082	0.107	0.200	0.085	0.153	0.301

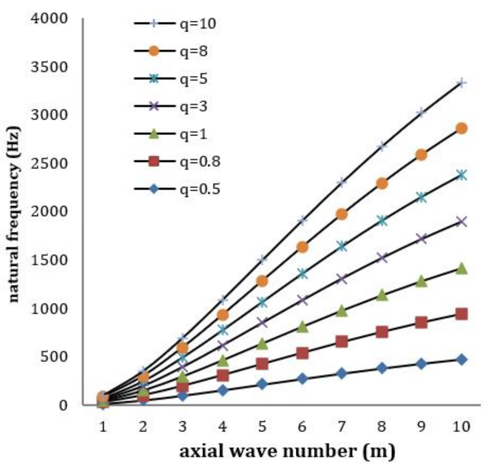
the outcomes of Loy *et al*. (1999). In Table 2, the frequencies (Hz) for FG-CSs are weighed against those calculated for a SS-SS end conditions by Warburton (1965). The frequencies are taken for circumferential modes n = 2, 3 and m = 1 ~ 6. Table 3 shows the natural frequency results versus depth to width ratios d/b are well matched those evaluated by Swaddiwudhipong *et al*. (1995) with 14 evenly spaced ring stiffeners Galletly (1955) SS-SS shells. It is observed that present results have smallest frequency values as compared with Swaddiwudhipong *et al*. (1995). It is observed frequency increases on increasing the ratio (d/b). The proposed model based on RRM can incorporate in order to accurately predict the acquired results of material data point.

5.1 Frequency analysis without ring supports

In Fig. 2, frequencies of vibrating FG-CSs are plotted versus the axial wave number, m. These frequencies have been examined for the wave number, n = 1. Like others cases the frequency is decreased for Type-I and it is increased for the Type-II when the value of 'q' is increased and decreased respectively. Fig. 3 portrays the variations of frequencies with circumferential wave number, n for Type-I and-II cylindrical shells. These variations of the natural frequency (Hz) have been calculated for the following

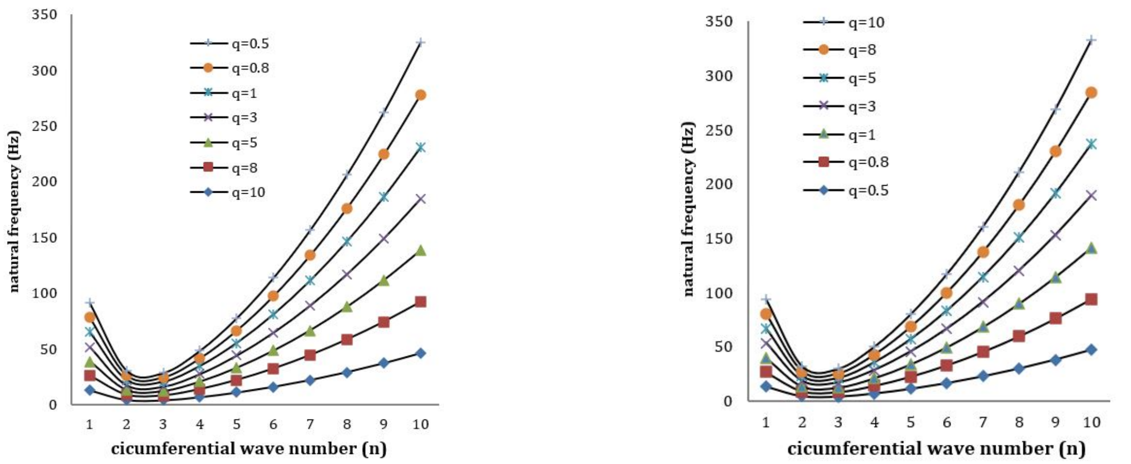


Type-I



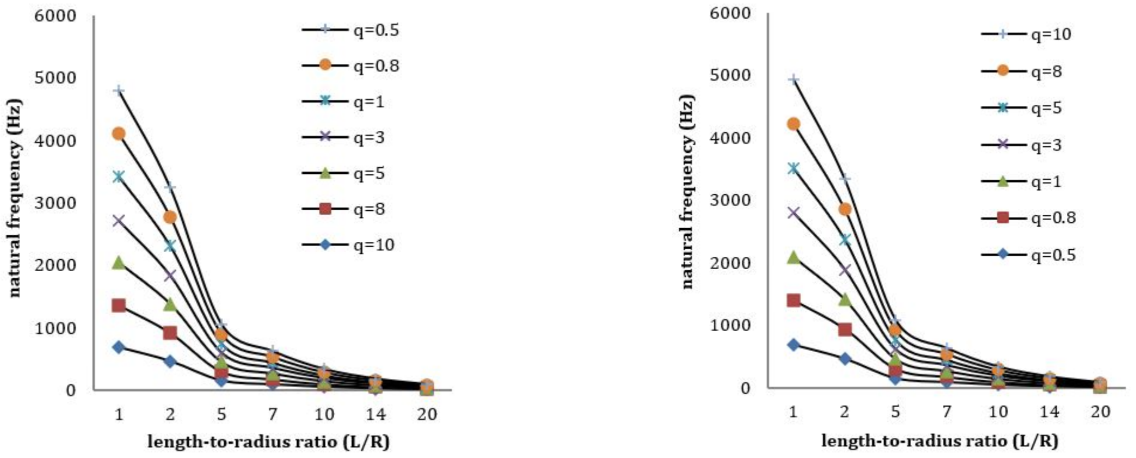
Type-II

Fig. 2 Frequency variation versus m for SS-SS condition (n = 1, h/R = 0.002 m, L/R = 20 m)



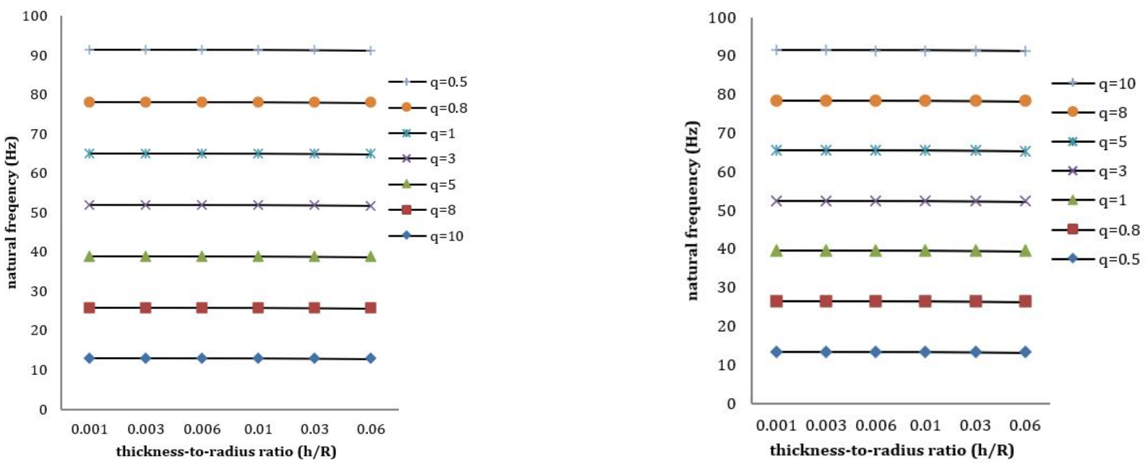
Type-I Type-II

Fig. 3 Frequency variation versus n for SS-SS condition (m = 1, h/R = 0.002 m, L/R = 20 m)



Type-I Type-II

Fig. 4 Frequency variation versus L/R for SS-SS condition (m = 1, n = 1, h/R = 0.002)



Type-I Type-II

Fig. 5 Frequency variation versus h/R for SS-SS condition (m = 1, n = 1, L/R = 20)

volume fraction exponents: q = 0.5, 0.8, 1, 3, 5, 8, 10 for the two Types of FGM cylindrical shells. The influence of q is opposite on frequencies of two shells. For Type-I and- II cylindrical shells, frequencies decrease and increase with q, due to interchange of order of FG constituents material. In Type-I, the natural frequency decreases when q is increased

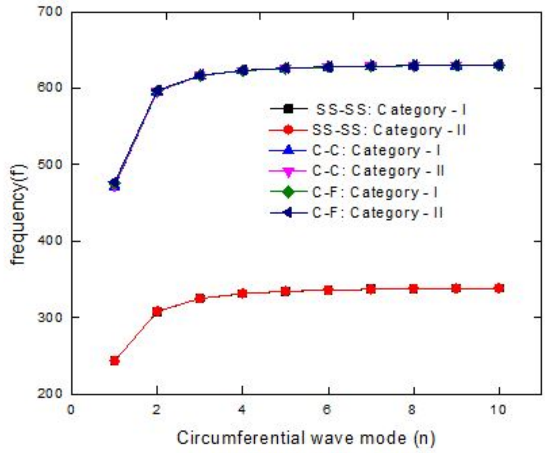


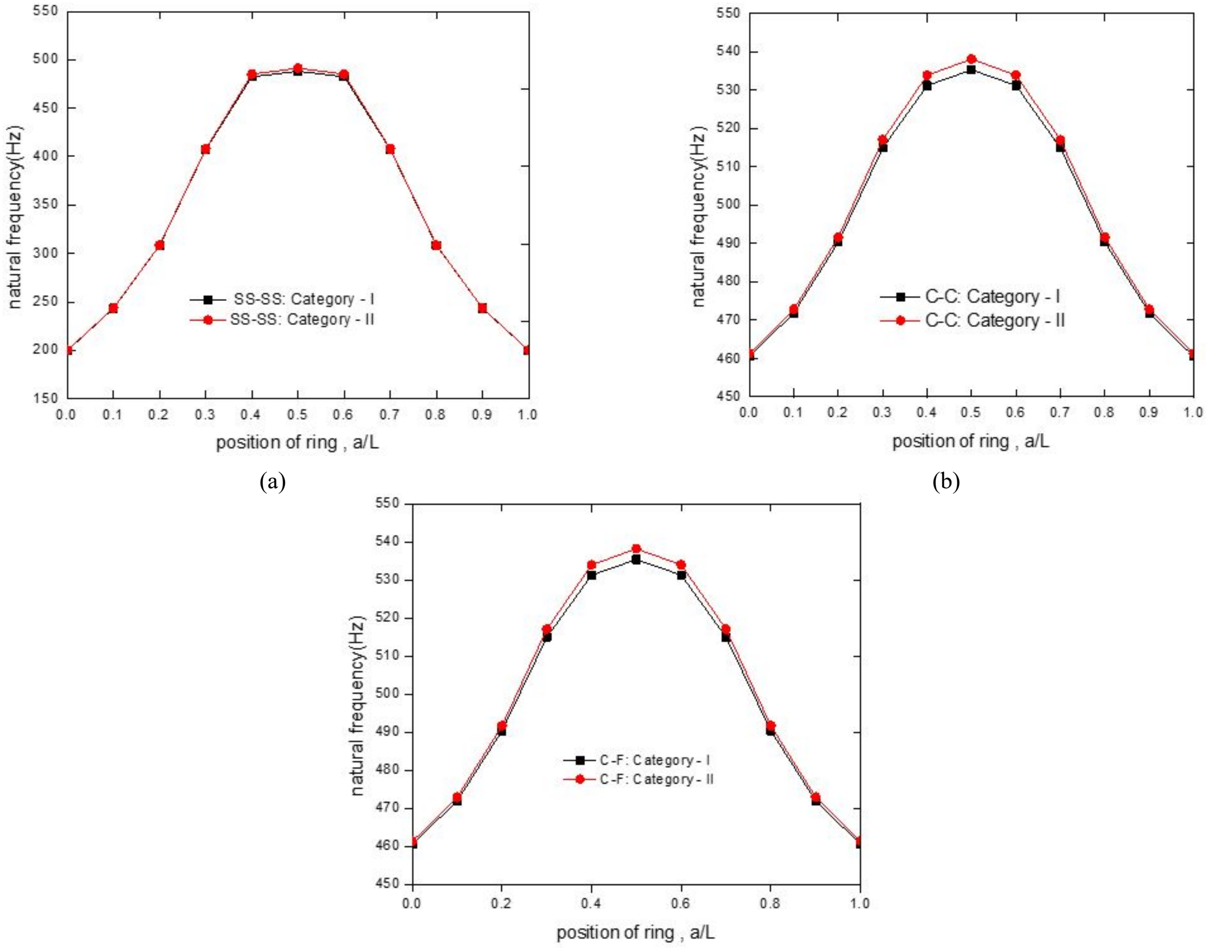
Fig. 6 Frequency pattern versus n for three boundary conditions for FG-CS with ring support (m = 1, L= 20m, h = 0.002m, R = 1m, q = 0.5, a = 0.3)

while on the other hand, this behavior is reversed for Type-II cylindrical shells. Fig. 4, depict the variations of frequencies for Type-I and- II FG-CS against L/R *for* both types. For Type-I, the frequency decreases by increasing the value of q but for Type- II, this change is reversed i.e., it increases with increase in values of q. But frequencies decrease as L/R is increased for both types cylindrical shells. For large L/R, the frequencies curves merges with each other. Fig. 5 displays a graphical view of frequencies for both Type- I and- II against the value of height-to-radius ratio h/R. For both types of cylindrical shell, the frequencies increase linearly as h/R is increased but for various values of 'q' the frequency diminishes for Type- I and increase for Type- II.

3.2 Frequency analysis with ring supports

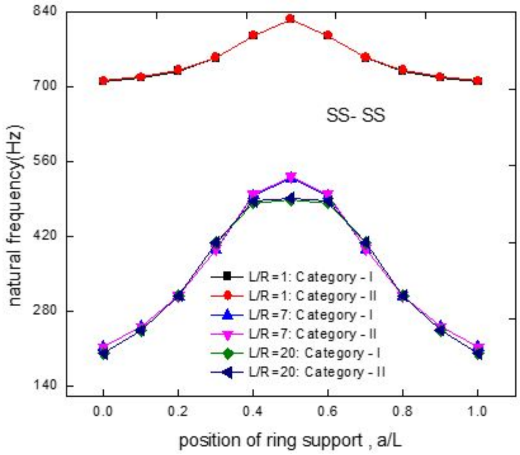
Here frequencies for both types of FG-CSs with ring supports are presented in following figures. The frequency variation with the position of the ring support at a = 0.3L for the edge conditions: SS- SS, C-C and C-F for both FG-CS as shown in Fig. 6. The frequencies increase significantly from n = 1~10 and for other wave number, the frequencies increase linearly.

Fig. 7 depicts the frequency variations versus ring support for Types-I and-II cylindrical shells. These variations of frequencies are drawn with three types of end conditions. As a/L is enhanced for these boundary

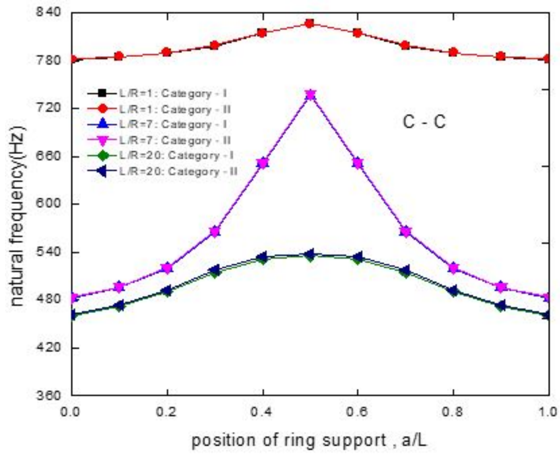


(a) (b) (c)

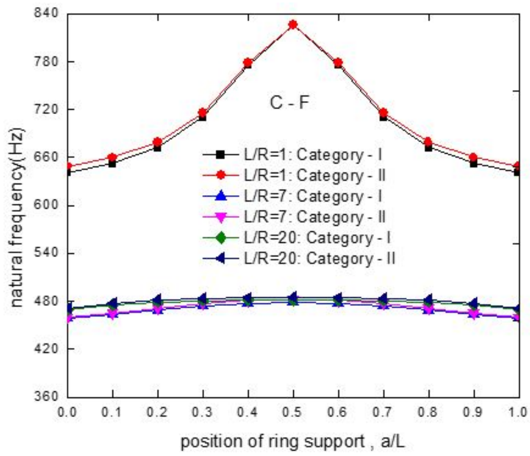
Fig. 7 Variations of frequencies versus the ring support a/L under various boundary conditions for FGM cylindrical shells (m = 1, n = 1, L = 20 m, h = 0.002 m, R = 1 m, q = 0.5) conditions, the frequencies go up. At a/L (= 0.5) all the frequencies are higher and at a/L (= 0.6~0.9), the frequencies decreases. The frequencies are same at a/L = 0, 1 and rust itself a bell shape. In these figures, the C-F frequencies are lower than that of C-C and SS-SS. These frequencies have a great impact on the vibration of FG-CSs. It is inferred this frequency behavior with position of the ring supports has paramount influence on the vibrations of FG-CSs.



(a)

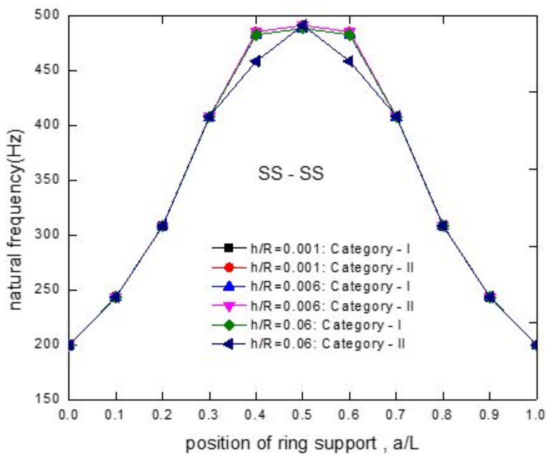


(b)

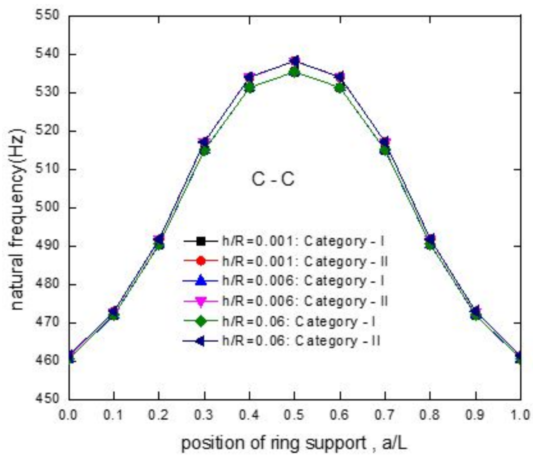


(c)

Fig. 8 Frequency variations at different ratios of L/R of FG-CSs with the position of ring support (n = 1, m = 1, h = 0.002 m, R = 1 m, q = 0.5)

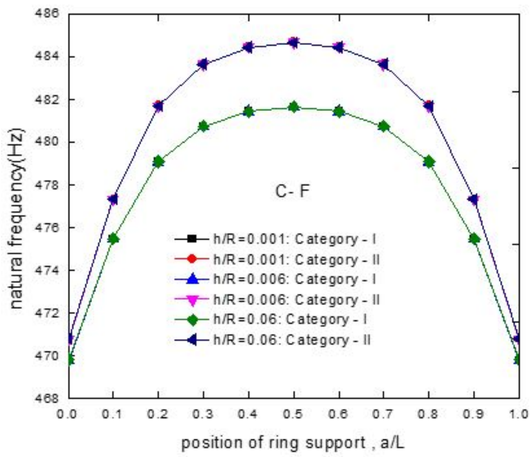


(a)



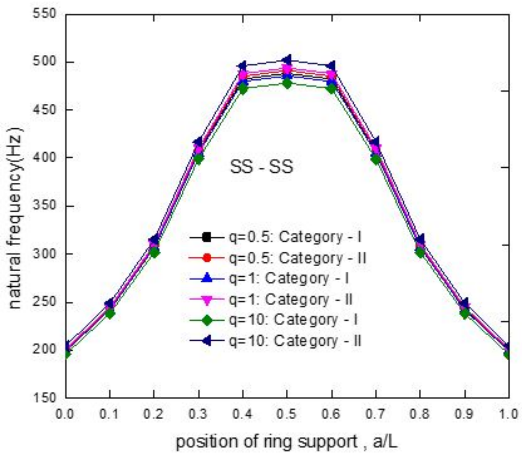
(b)

Fig. 9 Variations of frequencies at different ratios of h/R of FGM cylindrical shells with the position of ring supports

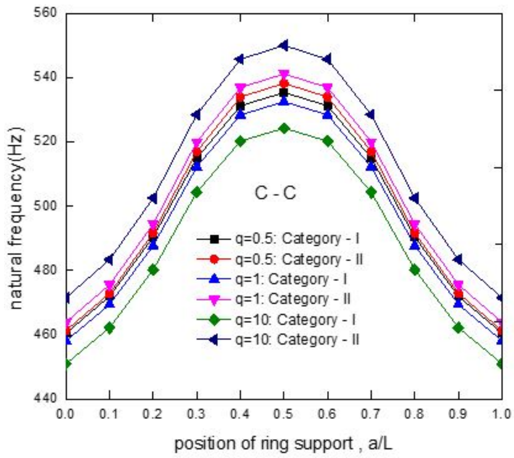


(c)

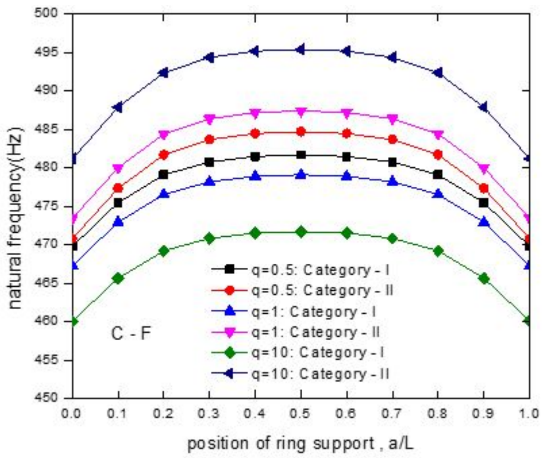
Fig. 9 Continued



(a)



(b)



(c)

Fig. 10 Variations of frequencies at different ratios of q of FGM cylindrical shells with the position of ring supports (n = 1, m = 1, L = 20 m, h = 0.002 m, R = 1 m)

In Figs. 8(a), (b), and (c), frequencies for Type-I and-II FG-CSs are sketched versus with position a/L of ring supports for three values of L/R = 1, 7, 20. These results have been evaluated for three boundary conditions, respectively. Their graphical view presents that affect ring support positions on vibration frequencies is pronounced at large values of L/R ratios where at L/R = 1, their influence has reduced.

Figs. 9(a), (b), and (c) demonstrate the frequencies of FG-CSs with three different boundary conditions against positions of ring supports. Frequency variations for these boundary conditions have observed slightly different with parameter (L = 20, R = 1 m, q = 0.5). The SS-SS frequencies increase and decrease symmetrically around the mid location of ring supports. For rest of two end conditions, the variations of frequencies are observed to in linear style. Figs. 10(a), (b), and (c) depict the frequencies of FG-CS for Type-I and-II cylindrical shells with specified boundary conditions. These variations have been plotted against the locations of ring supports for three.

For three conditions, frequency variations show different behavior with these values of this law. The frequencies are visible for simply supported condition in the case of ring supports. The frequency first increases and gain maximum value in the midway of the shell length and then lowers down. For clamped-clamped conditions, variations of frequencies are also like that for simply supported end conditions. For clamped-free conditions, frequency variations appear to be seen linear manner. For simply supported-simply supported conditions, a symmetrical behavior for natural frequencies is seen with the mid location of ring supports. For clamped-free conditions, a linear behavior of frequency variations is observed around the mid location of the ring supports.

6. Conclusions

In this analytical study, vibrations of functionally grade cylindrical shells have been investigated for the distribution of material composition of material with two different categories. Theoretical study gives a prediction to estimate experimental frequencies and avoids any future risk to a physical system. Here the Rayleigh-Ritz technique has been applied to derive the shell frequency equation. Terms of ring supports have been introduced by a polynomial function that bears the degree equal to the number of ring supports. These results have been obtained for circumferential wave mode, ratios of length- and- thickness-to-radius. Variations of frequencies with the locations of ring supports have been analyzed placed round the circumferential direction. The position of a ring support has been taken along the shell length. It is seen that frequencies increases on inducting of ring-stiffeners. As the position of a ring is changed from one end of the shell to other one, the frequency first increases and obtains its maximum value at the shell mid length position and then decreases. Its values at both ends are similar. This procedure can be applied to vibration characteristics of FG-shell using various volume fraction laws with ring supports.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

References

Alazzawy, W.I. (2008), "Static and Dynamic Analysis of Stiffened Plate Used in Machine Tool Column", *J. Eng.*, **14**(4), 3099-3111.

Alazzawy, W.I. and Jweeg, M.J. (2010), "A study of free vibration and fatigue for cross-ply closed cylindrical shells using General Third shell Theory (GTT)", *J. Eng.*, **16**(2), 5170-5184.

AlSaleh, R.J. and Fuggini, C. (2020), "Combining GPS and accelerometers' records to capture torsional response of cylindrical tower", *Smart Struct. Syst., Int. J.*, **25**(1), 111-122. https://doi.org/10.12989/sss.2020.25.1.111

Amabili, M. (1999), "Vibration of circular tubes and shells filled and partially immersed in dense fluids", *J. Sound Vib.*, **221**(4), 567-585. https://doi.org/10.1006/jsvi.1998.2050

Amabili, M., Pellicano, F. and Paidoussis, M.P. (1998), "Nonlinear vibrations of simply supported, circular cylindrical shells, coupled to quiescent fluid", **12**(7), 883-918. https://doi.org/10.1006/jfls.1998.0173

Ansari, R. and Rouhi, H. (2015), "Nonlocal Flügge shell model for the axial buckling of single-walled Carbon nanotubes: An analytical approach", *Int. J. Nano Dimens.*, **6**(5), 453-462. https://doi.org/10.7508/IJND.2015.05.002

Arani, A.G., Kolahchi, R. and Esmailpour, M. (2016), "Nonlinear vibration analysis of piezoelectric plates reinforced with carbon nanotubes using DQM", *Smart Struct. Syst., Int. J.*, **18**(4), 787-800. https://doi.org/10.12989/sss.2016.18.4.787

Arefi, M. and Zenkour, A.M. (2017), "Nonlinear and linear thermo-elastic analyses of a functionally graded spherical shell using the Lagrange strain tensor", *Smart Struct. Syst., Int. J.*, **19**(1), 33-38. https://doi.org/10.12989/sss.2017.19.1.033

Arnold, R.N. and Warburton, G.B. (1949), "Flexural vibrations of the walls of thin cylindrical shells having freely supported ends", *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, **197**(1049), 238-256. http://dx.doi.org/10.1098/rspa.1949.0061

Asghar, S., Hussain, M. and Naeem, M. (2019), "Non-local effect on the vibration analysis of double walled carbon nanotubes based on Donnell shell theory", *Physica E: Low-dimens. Syst. Nanostruct.*, **116**, 113726. https://doi.org/10.1016/j.physe.2019.113726

Bisen, H.B., Hirwani, C.K., Satankar, R.K., Panda, S.K., Mehar, K. and Patel, B. (2018), "Numerical study of frequency and deflection responses of natural fiber (Luffa) reinforced polymer composite and experimental validation", *J. Natural Fibers*, 1-15. https://doi.org/10.1080/15440478.2018.1503129

Boussoula, A., Boucham, B., Bourada, M., Bourada, F., Tounsi, A., Bousahla, A.A. and Tounsi, A. (2019), "A simple nth-order shear deformation theory for thermomechanical bending analysis of different configurations of FG sandwich plates", *Smart Struct. Syst., Int. J.*, **25**(2), 197-218. https://doi.org/10.12989/sss.2020.25.2.197

Boussoula, A., Boucham, B., Bourada, M., Bourada, F., Tounsi, A., Bousahla, A.A. and Tounsi, A. (2020), "A simple nth-order shear deformation theory for thermomechanical bending analysis of different configurations of FG sandwich plates", *Smart Struct. Syst., Int. J.*, **25**(2), 197-218. https://doi.org/10.12989/sss.2020.25.2.197

Chi, S.H. and Chung, Y.L. (2006a), "Mechanical behavior of functionally graded material plates under transverse load—Part I: Analysis", *Int. J. Solids Struct.*, **43**(13), 3657-3674. https://doi.org/10.1016/j.ijsolstr.2005.04.010

Chi, S.H. and Chung, Y.L. (2006b), "Mechanical behavior of functionally graded material plates under transverse load—part II: numerical results", *Int. J. Solids Struct.*, **43**, 3657-3691. https://doi.org/10.1016/j.ijsolstr.2005.04.010

Chung, H., Turula, P., Mulcahy, T.M. and Jendrzejczyk, J.A.

(1981), "Analysis of cylindrical shell vibrating in a cylindrical fluid region", *Nuclear Eng*. *Des*., **63**(1), 109-1012. https://doi.org/10.1016/0029-5493(81)90020-0

Dewangan, H.C., Panda, S.K. and Sharma, N. (2020a), "Experimental Validation of Role of Cut-Out Parameters on Modal Responses of Laminated Composite-A Coupled Fe Approach", *Int*. *J*. *Appl*. *Mech*., **12**(6), 2050068. https://doi.org/10.1142/S1758825120500684

Dewangan, H.C., Sharma, N., Hirwani, C.K. and Panda, S.K. (2020b), "Numerical eigenfrequency and experimental verification of variable cutout (square/rectangular) borne layered glass/epoxy flat/curved panel structure", *Mech*. *Based Des*. *Struct*. *Mach*., 1-18. https://doi.org/10.1080/15397734.2020.1759432

Dong, S.B. (1977), "A block-stodola eigen solution technique for large algebraic systems with non-symmetrical matrices", *Int*. *J*. *Numer*. *Methods Eng*., **11**, 247. https://doi.org/10.1002/nme.1620110204

Ergin, A. and Temarel, P. (2002), "Free vibration of a partially liquid-filled and submerged, horizontal cylindrical shell", *J*. *Sound Vib*., **254**(5), 951-965. https://doi.org/10.1006/jsvi.2001.4139

Flügge, W. (1962), *Stresses in Shells*, (2nd Edition), Springer-Verlag, Berlin, Germany.

Flügge, W. (1967), *Stresses in Shells*, (2nd Edition), Springer-Verlag, Berlin, Germany. https://www.springer.com/gp/book/9783662010280

Forsberg, K. (1964), "Influence of boundary conditions on modal characteristics of cylindrical shells", *J*. *Am*. *Inst*. *Aeronaut*. *Astronaut*., **2**, 182-189. https://arc.aiaa.org/doi/abs/10.2514/6.1964-77

Galletly, G.D. (1955), "On the in-vacuo vibrations of simply supported, ring-stiffened cylindrical shells", US National Congress of Applied Mechanics. http//www.vacuo-vibrations-supported-ring-stiffened-cylindrical/dp/B0007FTWBQ

Gasser, L.F.F. (1987), "Free vibrations on thin cylindrical shells containing liquid", M.S. Thesis, Federal University of Rio de Janerio, peccoppe-ufrj, Rio de Janerio, Portugal. [In Portuguese]

Ghobaei-Arani, M., Jabbehdari, S. and Pourmina, M.A. (2018), "An autonomic resource provisioning approach for service-based cloud applications: A hybrid approach", *Future Gener. Comput. Syst.*, **78**, 191-210. https://doi.org/10.1016/j.future.2017.02.022

Goncalves, P.B. and Batista, R.C. (1987), "Frequency response of cylindrical shells partially submerged or filled with liquid", *J. Sound Vib.*, **113**(1), 59-70. https://doi.org/10.1016/S0022-460X(87)81340-8

Goncalves, P.B. and Batista, R.C. (1988), "Non-linear vibration analysis of fluid-filled cylindrical shells", *J. Sound Vib*., **127**(1), 133-143. https://doi.org/10.1006/jsvi.2001.4139

Jiang, J. and Olson, M.D. (1994), "Vibrational analysis of orthogonally stiffened cylindrical shells using super elements", *J. Sound Vib*., **173**, 73-83. https://doi.org/10.1006/jsvi.1994.1218

Jweeg, M.J. and Alazzawy, W.I. (2007), "A suggested analytical solution for laminated closed cylindrical shells using General Third Shell Theory (GTT)", *Al-Nahrain J. Eng. Sci*., **10**(1), 11-26.

Jweeg, M.J. and Majeed, W.I. (2020), "Free vibration Analysis solution for laminated truncated conical shells using high orde theory", *Proceedings of the 6th Sc Conference of the College of Engineering*, *University of Baghdad*, Volume 3, pp. 208-225.

Kareem, M.G. and Majeed, W.I. (2019), "Transient dynamic analysis of laminated shallow spherical shell under low-velocity impact", *J. Mater*. *Res*. *Technol*., **8**(6), 5283-5300. https://doi.org/10.1016/j.jmrt.2019.08.050

Koizumi, M. (1997), "FGM Activities in Japan", *Composites*. https://doi.org/10.1016/S1359-8368(96)00016-9

Krommer, M., Vetyukova, Y. and Staudigl, E. (2016), "Nonlinear modelling and analysis of thin piezoelectric plates: buckling and post-buckling behavior", *Smart Struct*. *Syst*., *Int*. *J*., **18**(1), 155-181. https://doi.org/10.12989/sss.2016.18.1.155

Lam, K.Y. and Loy, C.T. (1998), "Influence of boundary conditions for a thin laminated rotating cylindrical shell", *Compos*. *Struct*., **41**(3-4), 215-228. https://doi.org/10.1016/S0263-8223(98)00012-9

Lee, S.Y., Huynh, T.C., Dang, N.L. and Kim, J.T. (2019), "Vibration characteristics of caisson breakwater for various waves, sea levels, and foundations", *Smart Struct*. *Syst*., *Int*. *J*., **24**(4), 525-539. https://doi.org/10.12989/sss.2019.24.4.525

Leissa, A.W. (1973), "Vibration of shells". https://ntrs.nasa.gov/search.jsp?R=19730018197

Love, A.E.H. (1888), "XVI. The small small free vibrations and deformation of thin elastic shell", *Phil*. *Trans*. *R*. *Soc*. *London*, **A179**, 491-549. https://doi.org/10.1098/rsta.1888.0016

Loy, C.T. and Lam, K.Y. (1997), "Vibration of cylindrical shells with ring supports", *J*. *Mech*. *Eng*., **39**, 455-471. https://doi.org/10.1016/S0020-7403(96)00035-5

Loy, C.T., Lam, K.Y. and Reddy, J.N. (1999), "Vibration of functionally graded cylindrical shells", *Int*. *J*. *Mech*. *Sci*., **41**(3), 309-324. https://doi.org/10.1016/S0020-7403(98)00054-X

Marcel Dekker, Books: https://www.abebooks.com/book-search/title/vibrations-shells-plates/author/soedel-werner/

Mehar, K. and Panda, S.K. (2016a), "Geometrical nonlinear free vibration analysis of FG-CNT reinforced composite flat panel under uniform thermal field", *Compos*. *Struct*., **143**, 336-346. https://doi.org/10.1016/j.compstruct.2016.02.038

Mehar, K. and Panda, S.K. (2016b), "Free vibration and bending behaviour of CNT reinforced composite plate using different shear deformation theory", *Proceedings of IOP Conference Series*: *Materials Science and Engineering*, **115**(1), 012014.

Mehar, K. and Panda, S.K. (2018a), "Dynamic response of functionally graded carbon nanotube reinforced sandwich plate", *Proceedings of IOP Conference Series*: *Materials Science and Engineering*, Vol. 338, No. 1, p. 012017.

Mehar, K. and Panda, S.K. (2018b), "Thermal free vibration behavior of FG-CNT reinforced sandwich curved panel using finite element method", *Polym*. *Compos*., **39**(8), 2751-2764. https://doi.org/10.1002/pc.24266

Mehar, K. and Panda, S.K. (2018c), "Elastic bending and stress analysis of carbon nanotube-reinforced composite plate: Experimental, numerical, and simulation", *Adv*. *Polym*. *Technol*., **37**(6), 1643-1657. https://doi.org/10.1002/adv.21821

Mehar, K. and Panda, S.K. (2018d), "Thermoelastic flexural analysis of FG-CNT doubly curved shell panel", *Aircr*. *Eng*. *Aerosp*. *Technol*., **90**(1), 11-23. https://doi.org/10.1108/AEAT-11-2015-0237

Mehar, K. and Panda, S.K. (2018e), "Nonlinear finite element solutions of thermoelastic flexural strength and stress values of temperature dependent graded CNT-reinforced sandwich shallow shell structure", *Struct*. *Eng*. *Mech*., *Int*. *J*., **67**(6), 565-578. https://doi.org/10.12989/sem.2018.67.6.565

Mehar, K. and Panda, S.K. (2019), "Multiscale modeling approach for thermal buckling analysis of nanocomposite curved structure", *Adv*. *Nano Res*., *Int*. *J*., **7**(3), 181-190. https://doi.org/10.12989/anr.2019.7.3.181

Mehar, K., Panda, S.K., Dehengia, A. and Kar, V.R. (2016), "Vibration analysis of functionally graded carbon nanotube reinforced composite plate in thermal environment", *J*. *Sandw*. *Struct*. *Mater*., **18**(2), 151-173. https://doi.org/10.1177/1099636215613324

Mehar, K., Panda, S.K. and Mahapatra, T.R. (2017a), "Thermoelastic nonlinear frequency analysis of CNT reinforced functionally graded sandwich structure", *Eur. J. Mech.-A/Solids*, **65**, 384-396. https://doi.org/10.1016/j.euromechsol.2017.05.005

Mehar, K., Panda, S.K., Bui, T.Q. and Mahapatra, T.R. (2017b), "Nonlinear thermoelastic frequency analysis of functionally graded CNT-reinforced single/doubly curved shallow shell panels by FEM", *J. Thermal Stress.*, **40**(7), 899-916. https://doi.org/10.1080/01495739.2017.1318689

Mehar, K., Panda, S.K. and Mahapatra, T.R. (2017c), "Theoretical and experimental investigation of vibration characteristic of carbon nanotube reinforced polymer composite structure", *Int. J. Mech. Sci.*, **133**, 319-329. https://doi.org/10.1016/j.ijmecsci.2017.08.057

Mehar, K., Panda, S.K. and Patle, B.K. (2017d), "Thermoelastic vibration and flexural behavior of FG-CNT reinforced composite curved panel", *Int. J. Appl. Mech.*, **9**(4), 1750046. https://doi.org/10.1142/S1758825117500466

Mehar, K., Mahapatra, T.R., Panda, S.K., Katariya, P.V. and Tompe, U.K. (2018a), "Finite-element solution to nonlocal elasticity and scale effect on frequency behavior of shear deformable nanoplate structure", *J. Eng. Mech.*, **144**(9), 04018094. https://doi.org/10.1061/(ASCE)EM.1943-7889.0001519

Mehar, K., Panda, S.K. and Mahapatra, T.R. (2018b), "Thermoelastic deflection responses of CNT reinforced sandwich shell structure using finite element method", *Scientia Iranica*, **25**(5), 2722-2737. https://doi.org/10.24200/SCI.2017.4525

Mehar, K., Panda, S.K. and Patle, B.K. (2018c), "Stress, deflection, and frequency analysis of CNT reinforced graded sandwich plate under uniform and linear thermal environment: A finite element approach", *Polym. Compos.*, **39**(10), 3792-3809. https://doi.org/10.1002/pc.24409

Mehar, K., Panda, S.K. and Mahapatra, T.R. (2018d), "Nonlinear frequency responses of functionally graded carbon nanotube-reinforced sandwich curved panel under uniform temperature field", *Int. J. Appl. Mech.*, **10**(3), 1850028. https://doi.org/10.1142/S175882511850028X

Mehar, K., Panda, S.K., Devarajan, Y. and Choubey, G. (2019), "Numerical buckling analysis of graded CNT-reinforced composite sandwich shell structure under thermal loading", *Compos. Struct.*, **216**, 406-414. https://doi.org/10.1016/j.compstruct.2019.03.002

Moazzez, K., Googarchin, H.S. and Sharifi, S.M.H. (2018), "Natural frequency analysis of a cylindrical shell containing a variably oriented surface crack utilizing line-spring model", *Thin-Wall. Struct.*, **125**, 63-75. https://doi.org/10.1016/j.tws.2018.01.009

Naeem, M.N., Ghamkhar, M., Arshad, S.H. and Shah, A.G. (2013), "Vibration analysis of submerged thin FGM cylindrical shells", *J. Mech. Sci. Technol.*, **27**(3), 649-656. https://doi.org/10.1007/s12206-013-0119-6

Najafizadeh, M.M. and Isvandzibaei, M.R. (2007), "Vibration of (FGM) cylindrical shells based on higher order shear deformation plate theory with ring support", *Acta Mechanica*, **191**, 75-91. https://doi.org/10.1007/s00707-006-0438-0

Poplawski, B., Mikulowski, G., Pisarski, D., Wiszowaty, R. and Jankowski, L. (2019), "Optimum actuator placement for damping of vibrations using the Prestress-Accumulation Release control approach", *Smart Struct. Syst.*, *Int. J.*, **24**(1), 27-35. https://doi.org/10.12989/sss.2019.24.1.027

Ramteke, P., Mehar, K., Sharma, N. and Panda, S. (2020a), "Numerical Prediction of Deflection and Stress Responses of Functionally Graded Structure for Grading Patterns (Power-Law, Sigmoid and Exponential) and Variable Porosity (Even/Uneven)", Scientia Iranica.

Ramteke, P.M., Mahapatra, B.P., Panda, S.K. and Sharma, N. (2020b), "Static deflection simulation study of 2D Functionally graded porous structure", *Materials Today: Proceedings*, **33**, 5544-5547. https://doi.org/10.1016/j.matpr.2020.03.537

Sadoughifar, A., Farhatnia, F., Izadinia, M. and Talaeetaba, S.B. (2020), "Size-dependent buckling behaviour of FG annular/circular thick nanoplates with porosities resting on Kerr foundation based on new hyperbolic shear deformation theory", *Struct. Eng. Mech.*, *Int. J.*, **73**(3), 225-238. https://doi.org/10.12989/sem.2020.73.3.225

Sewall, J.L. and Naumann, E.C. (1968), "An experimental and analytical vibration study of thin cylindrical shells with and without longitudinal stiffeners", *National Aeronautic and Space Administration*; *for sale by the Clearinghouse for Federal Scientific and Technical Information*, Springfield, Va. https://ntrs.nasa.gov/search.jsp?R=19680024266%202020-06-07T18:48:40+00:00Z

Shah, A.G., Mahmood, T. and Naeem, M.N. (2009), "Vibrations of FGM thin cylindrical shells with exponential volume fraction law", *Appl. Mathe. Mech.*, **30**(5), 607-615. https://doi.org/10.1007/s10483-009-0507-x

Sharma, C.B. (1974), "Calculation of natural frequencies of fixed-free circular cylindrical shells", *J. Sound Vib.*, **35**(1), 55-76. https://doi.org/10.1016/0022-460X(74)90038-8

Sharma, C.B. and Johns, D.J. (1971), "Vibration characteristics of a clamped-free and clamped-ring-stiffened circular cylindrical shell", *J. Sound Vib.*, **14**(4), 459-474. https://doi.org/10.1016/0022-460X(71)90575-X

Sharma, C.B., Darvizeh, M. and Darvizeh, A. (1998), "Natural frequency response of vertical cantilever composite shells containing fluid", *Eng. Struct.*, **20**(8), 732-737. https://doi.org/10.1016/S0141-0296(97)00102-8

Sharma, P., Singh, R. and Hussain, M. (2019), "On modal analysis of axially functionally graded material beam under hygrothermal effect", *Proceedings of the Institution of Mechanical Engineers*, *Part C*: *J. Mech. Eng. Sci.*, **234**(5), 1085-1101. https://doi.org/10.1177/0954406219888234.

Sodel, W. (1981), "Vibration of shell and plates", In: *Mechanical Engineering Series*, New York, USA.

Sofiyev, A.H. and Avcar, M. (2010), "The stability of cylindrical shells containing an FGM layer subjected to axial load on the Pasternak foundation", *Eng.*, **2**, 228-236. https://doi.org/10.4236/eng.2010.24033

Suresh, S. and Mortensen, A. (1997), "Functionally graded metals and metal-ceramic composites: Part 2 Thermomechanical behaviour", *Int. Mater. Rev.*, **42**, 85-116. https://doi.org/10.1179/imr.1995.40.6.239

Swaddiwudhipong, S., Tian, J. and Wang, C.M. (1995), "Vibration of cylindrical shells with ring supports", *J. Sound Vib.*, **187**(1), 69-93. https://doi.org/10.1006/jsvi.1995.0503

Tohidi, H., Hosseini-Hashemi, S.H. and Maghsoudpour, A. (2018), "Size-dependent forced vibration response of embedded micro cylindrical shells reinforced with agglomerated CNTs using strain gradient theory", *Smart Struct. Syst.*, *Int. J.*, **22**(5), 527-546. https://doi.org/10.12989/sss.2018.22.5.527

Toulokian, Y.S. (1967), *Thermo Physical Properties of High Temperature Solid Materials*, New York: Macmillan. https://apps.dtic.mil/dtic/tr/fulltext/u2/649947.pdf

Wang, C. and Lai, J.C.S. (2000), "Prediction of natural frequencies of finite length circular cylindrical shells", *Appl. Acoust.*, **59**(4), 385-400. https://doi.org/10.1016/S0003-682X(99)00039-0

Wang, C.M., Swaddiwudhipong, S. and Tian, J. (1997), "Ritz method for vibration analysis of cylindrical shells with ring-stiffeners", *J. Eng. Mech.*, **123**, 134-143. https://doi.org/10.1061/(ASCE)0733-9399(1997)123:2(134)

Warburton, G.B. (1965), "Vibration of thin cylindrical shells", *J. Mech. Eng. Sci.*, **7**, 399-407.

https://doi.org/10.1243/JMES_JOUR-1965-007-062-02

Wuite, J. and Adali, S. (2005), "Deflection and stress behavior of nanocomposite reinforced beams using a multiscale analysis", *Compos*. *Struct*., **71**(3-4), 388-396. https://doi.org/10.1016/j.compstruct.2005.09.011

Xiang, Y., Ma, Y.F., Kitipornchai, S. and Lau, C.W.H. (2002), "Exact solutions for vibration of cylindrical shells with intermediate ring supports", *Int*. *J*. *Mech*. *Sci*., **44**(9), 1907-1924. https://doi.org/10.1016/S0020-7403(02)00071-1

Xuebin, L. (2008), "Study on free vibration analysis of circular cylindrical shells using wave propagation", *J*. *Sound Vib*., **311**, 667-682. https://doi.org/10.1016/j.jsv.2007.09.023

Yeh, J.Y. (2016), "Vibration characteristic analysis of sandwich cylindrical shells with MR elastomer", *Smart Struct*. *Syst*., *Int*. *J*., **18**(2), 233-247. https://doi.org/10.12989/sss.2016.18.2.233

Zahrai, S.M. and Kakouei, S. (2019), "Shaking table tests on a SDOF structure with cylindrical and rectangular TLDs having rotatable baffles", *Smart Struct*. *Syst*., *Int*. *J*., **24**(3), 391-401. https://doi.org/10.12989/sss.2019.24.3.391

Zhang, X.M. (2002), "Frequency analysis of submerged cylindrical shells with the wave propagation approach", *J. Mech. Sci.*, **44**, 1259-1273. https://doi.org/10.1016/S0020-7403(02)00059-0

Zhang, X.M., Liu, G.R. and Lam, K.Y. (2001), "Coupled vibration of fluid-filled cylindrical shells using the wave propagation approach", *Appl*. *Acoust*., **62**, 229-243. https://doi.org/10.1016/S0003-682X(00)00045-1

CC