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Influence of porosity on thermal buckling behavior of functionally graded beams

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Abstract. The interest of this work is the analysis of the effect of porosity on the nonlinear thermal stability response of power law functionally graded beam with various boundary conditions. The modelling was done according to the Euler-Bernoulli beam model where the distribution of material properties is imitated polynomial function. The thermal loads are assumed to be not only uniform but linear as well non-linear and the temperature rises through the thickness direction. The effects of the porosity parameter, slenderness ratio and power law index on the thermal buckling of P-FG beam are discussed.

Keywords: functionally graded material; thermal buckling; Euler beam theory; porosity parameter

1. Introduction

Functionally Graded Materials (FGMs) have been developed with a considerable interest in the durability for extremely high temperatures. These materials are fabricated by changing the percentage (volume fraction) content of two or more materials so that the new materials have the desired property. However, the FGMs are employed in aerospace vehicles, civil, mechanical, nuclear, optical, electronic, chemical, shipbuilding, and biomechanical industries (Abrate 2008, Akgöz and Civalek 2013, Eltaher *et al*. 2014, Arefi 2015a, b, Kar and Panda 2015, 2016, Akbaş 2015, Celebi *et al*. 2016, Akavci 2016, Ebrahimi and Shafiei 2016, Darabi and Vosoughi 2016, Turan *et al*. 2016, Karami *et al*. 2018, 2019, Madenci 2019, Safa *et al*. 2019, Nebab *et al*. 2019, Selmi 2020, Chami *et al*. 2020, Merzoug *et al*. 2020, Hadji 2020a, Chikh 2019, 2020, Ton-That 2020, Rachedi *et al*. 2020). Thus, several scientific's works have been published on the stability analysis of FGM structures subjected to thermal loads. Using classical plate model Javaheri and Eslami (2002) obtained the approximate solution under a variety of thermal loads. It presented the comparison of critical buckling temperature of functionally graded plates (FGPs) and conventional composite plates. This model (CPT) has been implemented for mechanical buckling analysis of FG-plates under uniaxial loading by Feldman and Aboudi (1997), Najafizadeh and Eslami (2002) examined the thermal stability of FG-circular plates under three different types of thermal loads. The linear and nonlinear stability equations were determined employing the variation formulations. Najafizadeh and Heydari (2004) analyzed the thermal buckling of functionally graded (FG) circular plates. Zhao *et al*. (2009) examined thermal and mechanical stability analysis of ceramic–metal FG-thin plate using FSDT formulations. Based on Euler-Bernoulli, Timoshenko and the parabolic shear deformation beam theory, Şimşek (2010) has studied the dynamic deflections and the stresses of simply-supported FG-beam subjected to a moving mass. On one hand, Ma and Lee (2011, 2012) analyzed both the linear and nonlinear free vibration of simply supported FG-beam under in plane thermal load based on the Euler-Bernoulli and Timoshenko beam theories Levyakov (2013, 2015) investigated analytically the nonlinear thermo-mechanical response of FG-beam with simply supported boundary conditions by considering the exact formulation of curvature based on Euler-Bernoulli and Timoshenko models. Kiani and Eslami (2010) studied the linear thermal buckling of functionally graded Euler-Bernoulli beam under different thermal loading for clamped-pinned, pinned-pinned, and pinned-roller boundary conditions. The thermo-mechanical buckling and nonlinear

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free vibrational behavior of FG Euler-Bernoulli beams embedded in nonlinear elastic medium with pinned-pinned and clamped-pinned boundary conditions is analyzed by Fallah and Aghdam (2012). Ebrahimi *et al*. (2015) studied the linear size dependent thermal buckling and free vibration of simply supported Timoshenko FG nanobeams subjected to in-plane thermal loading. Ebrahimi and Barati (2016) suggested analytical examination on linear thermal stability of FG nanobeams based on nonlocal TSDT. On the other hand, porous structures with functional properties have some similarities with the functionally graded materials. The porosity can cause a smooth or rough change in mechanical properties depends on some parameters such as distributions and volume fraction of the porosity. Therefore, it is very important to take into account the presence of micro-void in the form of porosity in the analysis of the FG structures behaviours under various loading types. The problem of buckling of the porous structures has been discussed by many authors. The buckling analysis of thin rectangular functionally graded (FG) plates under various loads was discussed by Mohammadi *et al*. (2010) based on the classical or first order shear deformation theory (FSDT). Jabbari *et al*. (2013, 2014) studied the porosity distribution effects on buckling analysis of thin circular FG-plate. Buckling analysis of metal foam porous beams was investigated by Chen *et al*. (2015) using a shear deformation beam model. Ebrahimi *et al*. (2016) considered the thermal effects on linear free vibrational behavior of pinned-pinned and clamped pinned FG Euler-Bernoulli porous beams. In the recent two years, the studies of the porous FG structures have increased considerably as the scientific research work of (Avcar 2019, Hadji *et al*. 2019, Ramteke *et al*. 2019, Abdulrazzaq *et al*. 2020a, Fenjan *et al*. 2020, Gafour *et al*. 2020, Hadji 2020b, Vinyas 2020, Thanh *et al*. 2020, Rahmani *et al*. 2020, She *et al*. 2020, Hadji and Avcar 2021).

To conclude, we have noticed through our reading in the literature that the studies of porosity effect on the FG-beam are very interesting. For this, the aim of this investigation is to extend the Euler–Bernoulli beam theory proposed by Kiani and Eslami (2010) to the porous functionally graded (FG) beams under uniform, linear and nonlinear thermal loadings. The effective's properties of the porous FG-beam change according to a simple power law functions. Finally, several numerical results are presented and discussed to show the effect of the porosity parameter on the thermal stability of the FG-beam with various boundary conditions.

2. Theoretical formulations

2.1 Kinematics

Based on the Euler-Bernoulli assumption, the following displacement field can be obtained (Kiani and Eslami 2010)

$$\begin{aligned} u(x,z) &= u_0(x) - z\frac{\partial w_0}{\partial x} \\ v(x,z) &= 0 \\ w(x,z) &= w_0(x) \end{aligned} \tag{1}$$

Where $u_0(x,y)$, $w_0(x,y)$ are the two unknown displacement functions of middle surface of the beam in the *x* and *z* directions. The Von-Karman-type of geometric non-linearity is taken into consideration in the strain–displacement relations which are as follows

$$\varepsilon_x = \varepsilon_x^0 + z\, k_x \tag{2}$$

Where ε_x^0 and k_x are, respectively, the nonlinear longitudinal strain and curvature defined as

$$\varepsilon_x^0 = \frac{du_0}{dx} + \frac{1}{2}\left(\frac{d\,w_0}{dx}\right)^2 \quad \text{and} \quad k_x = -\frac{d^2 w_0}{dx^2} \tag{3}$$

2.2 Constitutive relations

Consider a FG rectangular beam with thickness h, length a and width b The Cartesian coordinate system is established so that $0 \le x \le l$, and $-\frac{h}{2} \le z \le +\frac{h}{2}$.

Functionally graded materials (FGMs) are composed of two kinds of materials: one is a metal and the other is ceramic. Here, Young's modulus E (z) varies continuously through the beams thickness by a polynomial material law. We will consider a non-homogeneity material with a porosity volume function, ξ $(0 \le \xi \le 1)$. In such a way, the efficient material properties, as Young's modulus E, the coefficient of thermal expansion α and thermal conductivity Kat a point are usually assumed to be given by the rule of mixture (Wattanasakulpong and Ungbhakorn 2014)

$$\begin{aligned} E(z) &= E_m + (E_c - E_m)\left(\frac{h+2z}{2h}\right)^p - (E_c + E_m)\frac{\xi}{2} \\ \alpha(z) &= \alpha_m + (\alpha_c - \alpha_m)\left(\frac{h+2z}{2h}\right)^p - (\alpha_c + \alpha_m)\frac{\xi}{2} \end{aligned} \tag{4}$$

$$K(z) = K_m + (K_c - K_m)\left(\frac{h+2z}{2h}\right)^p - (K_c + K_m)\frac{\xi}{2} \tag{5}$$

Where p is the volume fraction exponent. The value of p equal to zero represents a fully ceramic beam, whereas infinite p indicates a fully metallic beam. The distribution of the combination of ceramic and metal is linear for $p = 1$.

The constitutive relation of a FG beam under thermal and mechanical conditions using thermo-elasticity can be expressed as

$$\sigma_x = E(\varepsilon_x - \alpha(T - T_r)) \tag{6}$$

Where σ_x, T and T_r are, respectively, the axial stress, the temperature distribution through the thickness and the reference temperature. The axial force N, the bending moment M caused by thermal stress, respectively are written as

$$(N, M) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x\,(1, z)dz \tag{7}$$

By substituting Eqs. (4) and (6) into Eq. (7) we obtain

$$N = \overline{A}\left(\frac{du_0}{dx} + \frac{1}{2}\left(\frac{d\,w_0}{dx}\right)^2\right) - \overline{B}\frac{d^2w_0}{dx^2} - N_T M = \overline{B}\left(\frac{du_0}{dx} + \frac{1}{2}\left(\frac{d\,w_0}{dx}\right)^2\right) - \overline{C}\frac{d^2w_0}{dx^2} - M_T \tag{8}$$

In which $\overline{A}$, $\overline{B}$ and $\overline{C}$ are constants, they are written as

$$(\overline{A}, \overline{B}, \overline{C}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)(1, z, z^2)dz \tag{9}$$

Where

$$\overline{A} = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)dz = h\left[E_m + \frac{(E_c - E_m)}{(p+1)} - (E_c + E_m)\frac{\xi}{2}\right]$$
$$\overline{B} = \int_{-\frac{h}{2}}^{\frac{h}{2}} zE(z)dz = h^2(E_c - E_m)\left[\frac{1}{(p+2)} - \frac{1}{(2p+2)}\right]$$
$$\overline{C} = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2E(z)dz = h^3\left[\frac{1}{12}E_m + (E_c - E_m)\left(\frac{1}{(p+3)} - \frac{1}{(p+2)} + \frac{1}{(4p+4)}\right) - \frac{(E_c + E_m)}{12}\frac{\xi}{2}\right] \tag{10}$$
$$N_T = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)\alpha(z)(T - T_r)dz$$
$$M_T = \int_{-\frac{h}{2}}^{\frac{h}{2}} zE(z)\alpha(z)(T - T_r)dz$$

Total potential energy of the FGM beam may be expressed as follows

$$\delta\,U = \int_0^l \int_A \left(\sigma_x(\varepsilon_x - \alpha(T - T_r))\right)dAdx \tag{11}$$

Substituting Eqs. (3) and (6) into Eq. (11) and integrating with respect to *z* and *y*, the total potential energy of the beam is given by

$$\delta\,U = \frac{b}{2}\left[\begin{array}{l} \int_0^l \overline{A}\left(\frac{du_0}{dx} + \frac{1}{2}\left(\frac{d\,w_0}{dx}\right)^2\right)^2 - 2\overline{B}\frac{d^2w_0}{dx^2}\left(\frac{du_0}{dx} + \frac{1}{2}\left(\frac{d\,w_0}{dx}\right)^2\right) \\ +\overline{C}\left(\frac{d^2w_0}{dx^2}\right)^2 - 2N_T\left(\frac{du_0}{dx} + \frac{1}{2}\left(\frac{d\,w_0}{dx}\right)^2\right) + 2M_T\left(\frac{d^2w_0}{dx^2}\right)dx \\ + \int_0^l \int_{-\frac{h}{2}}^{\frac{h}{2}} [E(z)\alpha(z)(T - T_r)]dx \end{array}\right] \tag{12}$$

To derive the equilibrium equations, the variational approach can be used. Assume that the total functional of U is Ψ. In this case Euler's equations are expressed as

$$\begin{aligned} &\frac{\partial\Psi}{\partial u_0} - \frac{d}{dx}\left(\frac{\partial\Psi}{\partial\left(\frac{du_0}{dx}\right)}\right) = 0 \\ &\frac{\partial\Psi}{\partial w_0} - \frac{d}{dx}\left(\frac{\partial\Psi}{\partial\left(\frac{dw_0}{dx}\right)}\right) + \frac{d^2}{dx^2}\left(\frac{\partial\Psi}{\partial\left(\frac{d^2w_0}{dx^2}\right)}\right) = 0 \end{aligned} \tag{13}$$

Substituting Eq. (12) into Eq. (13), we obtain

$$\begin{aligned} &\overline{A}\left(\frac{d^2u_0}{dx^2} + \frac{d\,w_0}{dx}\frac{d^{\,2}w_0}{dx^2}\right) - \overline{B}\frac{d^{\,3}w_0}{dx^3} - \frac{d\,N_T}{dx} = 0 \\ &\overline{B}\left(\frac{d^3u_0}{dx^3} + \frac{d\,w_0}{dx}\frac{d^{\,3}w_0}{dx^3} + \left(\frac{d\,w_0}{dx}\right)^2\right) \\ &-\overline{C}\frac{d^4w_0}{dx^4} - \frac{d^2M_T}{dx^2} + \frac{d^2w_0}{dx^2}\left(\overline{A}\left(\frac{du_0}{dx} + \frac{1}{2}\left(\frac{d\,w_0}{dx}\right)^2\right)\right) \\ &-\overline{B}\frac{d^2w_0}{dx^2} - N_T = 0 \end{aligned} \tag{14}$$

The stability equations of the beam may be derived by the adjacent equilibrium criterion. Assume that the equilibrium state of the FGM beam under thermal loads is defined in terms of the displacement components $(u_0^0, u_0^1, w_0^0, w_0^1)$. The displacement components of a neigh boring stable state differ by (u_1, u_2, w_1, w_2) with respect to the equilibrium position. Thus, the total displacements of a neigh boring state are

$$\begin{aligned} u_0 &= u_0^0 + u_0^1 \\ w_0 &= w_0^0 + w_0^1 \end{aligned} \tag{15}$$

Substituting Eq. (15) into Eq. (14), we obtain

$$\overline{A}\left(\frac{d^2u_0^0}{dx^2} + \frac{d^2u_0^1}{dx^2} + \frac{dw_0^0}{dx}\frac{d^2w_0^0}{dx^2} + \frac{d\,w_0^0}{dx}\frac{d^2w_0^1}{dx^2} + \frac{dw_0^1}{dx}\frac{d^2w_0^0}{dx^2} + \frac{dw_0^1}{dx}\frac{d^2w_0^1}{dx^2}\right) \tag{16}$$

$$
\begin{aligned}
&-\overline{B}\left(\frac{d^3w_0^0}{dx^3}+\frac{d^3w_0^1}{dx^3}\right)-\frac{d\,N_T}{dx}=0\\
&\overline{B}\left(\frac{d^3u_0^0}{dx^3}+\frac{d^3u_0^1}{dx^3}+\frac{dw_0^0}{dx}\frac{d^3w_0^0}{dx^3}+\frac{dw_0^0}{dx}\frac{d^3w_0^1}{dx^3}+\frac{dw_0^1}{dx}\frac{d^3w_0^0}{dx^3}+\frac{dw_0^1}{dx}\frac{d^3w_0^1}{dx^3}\right)\\
&-\overline{C}\left(\frac{d^4w_0^0}{dx^4}+\frac{d^4w_0^1}{dx^4}\right)-\frac{d^2M_T}{dx^2}-N_T\left(\frac{d^2w_0^0}{dx^2}+\frac{d^2w_0^1}{dx^2}\right)\\
&+\overline{A}\left(\frac{d^2w_0^0}{dx^2}+\frac{d^2w_0^1}{dx^2}\right)\left(\frac{du_0^0}{dx}+\frac{du_0^1}{dx}+\frac{1}{2}\left(\frac{dw_0^0}{dx}\right)^2+\frac{1}{2}\left(\frac{dw_0^1}{dx}\right)^2+\left(\frac{dw_0^0}{dx}\right)\left(\frac{dw_0^1}{dx}\right)\right)=0
\end{aligned}
\tag{16}
$$

Where the superscript 1 refers to the state of stability and the superscript 0 refers to the state of equilibrium conditions and therefore drop out of the equations. The stability equations are obtained as

$$
\begin{aligned}
&\overline{A}\frac{d^2u_0^1}{dx^2}-\overline{B}\frac{d^3w_0^1}{dx^3}=0\\
&\overline{B}\frac{d^3u_0^1}{dx^3}-\overline{C}\frac{d^4w_0^1}{dx^4}-N_T\frac{d^2w_0^1}{dx^2}=0
\end{aligned}
\tag{17}
$$

Such as to displacements, the force resultants of a neighboring state may be related to the state of equilibrium. Using Eq. (8) into Eq. (15), we obtain

$$
\begin{aligned}
N_0^1&=\overline{A}\frac{du_0^1}{dx}-\overline{B}\frac{d^2w_0^1}{dx^2}\\
M_0^1&=\overline{B}\frac{du_0^1}{dx}-\overline{C}\frac{d^2w_0^1}{dx^2}
\end{aligned}
\tag{18}
$$

The stability equation of FG- beam under thermal loading is assumed to be given by eliminating (u_0^1) as

$$\frac{d^4w_0^1}{dx^4}+\lambda^2\frac{d^2w_0^1}{dx^2}=0 \tag{19}$$

Where

$$\lambda^2=\frac{\overline{A}N_T}{\overline{AC}-\overline{B}^2} \tag{20}$$

The parameter λ is a constant and the Eq. (19) is a linear homogeneous equation whose general solution is

$$w_0^1(x)=D_1\,sin(\lambda x)+D_2\,cos(\lambda x)+D_3x+D_4 \tag{21}$$

Where D_1, D_2, D_3 and D_4 are undetermined constants calculated via the boundary conditions.

$$
\begin{aligned}
&w_0^1(x)=D_1\,sin(\lambda x)+D_2\,cos(\lambda x)+D_3x+D_4\\
&u_0^1(x)=\frac{\overline{B}}{\overline{A}}\lambda D_1\,cos(\lambda x)-\frac{\overline{B}}{\overline{A}}\lambda D_2\,sin(\lambda x)+D_5x+D_6\\
&M_0^1(x)=\frac{\overline{AC}-\overline{B}^2}{\overline{A}}\lambda^2D_1\,sin(\lambda x)+\frac{\overline{AC}-\overline{B}^2}{\overline{A}}\lambda^2D_2\,cos(\lambda x)+\overline{B}D_5\\
&N_0^1(x)=\overline{A}D_5
\end{aligned}
\tag{22}
$$

The following boundary conditions are imposed at the edges for FGM beam

$$
\begin{aligned}
u_0^1(0)=w_0^1(0)&=\frac{dw_0^1}{dx}(0)\\
&=u_0^1(l)=w_0^1(l)=\frac{dw_0^1}{dx}(l)=0
\end{aligned}
\tag{23}
$$

$$
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 0\\
sin(\lambda l) & cos(\lambda l) & l & 1 & 0 & 0\\
\lambda & 0 & 1 & 0 & 0 & 0\\
\lambda\,cos(\lambda l) & -\lambda\,sin(\lambda l) & 1 & 0 & 0 & 0\\
\frac{\overline{B}}{\overline{A}}\lambda & 0 & 0 & 0 & 0 & 1\\
\frac{\overline{B}}{\overline{A}}\lambda\,cos(\lambda l) & -\frac{\overline{B}}{\overline{A}}\lambda\,sin(\lambda l) & 0 & 0 & l & 1
\end{bmatrix}
\begin{Bmatrix} D_1\\ D_2\\ D_3\\ D_4\\ D_5\\ D_6 \end{Bmatrix}=0
\tag{24}
$$

The solutions of Eq. (24) should be non-zero while the thermal buckling of FG-beams occurs, that is, determinants of coefficient matrices equal to zero. Expanding the determinants and simplifying equations to obtain the bifurcation conditions of thermal buckling.

$$\lambda l(2-2\,cos(\lambda l)-\lambda l\,sin(\lambda l))=0 \tag{25}$$

Finally, the critical thermal force of the beam, N_{Tcr} for all cases of boundary conditions in light of Eq. (25), can be

Table 1 Values of D for different boundary conditions

Boundary conditions	CC	SS	CR	SR	CS
D	39.47842	9.86960	9.86960	2.46740	20.19077

expressed as follows

$$N_{Tcr} = D\frac{\overline{AC} - \overline{B}^2}{\overline{A}(l)^2} \tag{26}$$

Where D is a constant and depends on the type of boundary conditions (clamped-clamped, simply supported-simply supported, clamped-roller edges, simply supported-roller edges, clamped-simply supported). The values of the constant D for various boundary condisions is abstracted in the Table 1.

3. Thermal buckling solution

3.1 Buckling of FGM beams under uniform temperature rise

The beam reference temperature is assumed to be T_r. The temperature is uniformly raised to a final value T in which the beam buckles. By substituting $T = T_r + \Delta T$ into Eq. (10) the result is

$$N_T = \Delta T h \overline{\Pi} \tag{27}$$

$$\overline{\Pi} = \begin{bmatrix} E_m\alpha_m(1-\xi) + \dfrac{(E_c - E_m)\alpha_m + E_m(\alpha_c - \alpha_m)}{(p+1)} + \dfrac{(E_c - E_m)(\alpha_c - \alpha_m)}{(2p+1)} \\ -\dfrac{(E_c - E_m)(\alpha_c + \alpha_m)}{(p+1)}\left(\dfrac{\xi}{2}\right) - \dfrac{(E_c + E_m)(\alpha_c - \alpha_m)}{(p+1)}\left(\dfrac{\xi}{2}\right) \\ -(E_m\alpha_c + E_c\alpha_m)\left(\dfrac{\xi}{2}\right) + (E_c + E_m)(\alpha_c + \alpha_m)\left(\dfrac{\xi}{2}\right)^2 \end{bmatrix} \tag{28}$$

Using Eqs. (26) and (27) one can easily obtain the critical buckling temperature change, ΔT_{cr}, and is defined as follows

$$\Delta T_{cr} = D\frac{\overline{AC} - \overline{B}^2}{\overline{A\Pi}(l)^2 h} \tag{29}$$

3.2 Buckling of FGM beams under linear temperature across the thickness

We assume that the temperature of the top surface is T_t and the temperature varies from T_t, according to the power law variation through-the-thickness, to the bottom surface temperature T_b in which the beam buckles. Due to the increments of transversely temperature inside FGM beams are assumed to be the function of thickness coordinate z, the increments $T = T(z)$. In this case, the temperature through the- thickness is given by

$$T(z) = T_t + \Delta T\left(\frac{h + 2z}{2h}\right) \tag{30}$$

Where the buckling temperature difference

$$\Delta T = T_b - T_t \tag{31}$$

3.3 Buckling of FGM beams under non-linear temperature across the thickness

The FGM beams are subjected to transversely non-linear temperature rise, and the increments of temperature on top surface and bottom surface are T_t and T_b, respectively. Four sides of the beam are adiabatic with environment. Due to the increments of transversely temperature inside FGM beams are assumed to be the function of thickness coordinate z, the increments $T = T(z)$ satisfy the following one-dimensional thermal conduction equation

$$\frac{d}{dz}\left[\left(K(z)\frac{dT}{dz}\right)\right] = 0 \tag{32}$$

Where $K(z)$ is given by Eq. (4) This model ignores the time of heat conduction, and the change of temperature due to work produced by the deformations is also neglected.

However, the non-linear temperature fields can be obtained easily by using the boundary conditions as

$$T(z) = T_t + \left(\frac{\Delta T}{\Omega}\right)\left[\sum_{i=0}^{5}\frac{(-1)^i}{(ip+1)}\left(\frac{(K_c - K_m)}{K_m}\right)^i\left(\frac{h + 2z}{2h}\right)^{(ip+1)}\right] \tag{33}$$

With

$$\Omega = \sum_{i=0}^{5}\frac{(-1)^i}{(ip+1)}\left(\frac{(K_c - K_m)}{K_m}\right)^i \tag{34}$$

4. Numerical results and discussion

In this study, various numerical examples are presented and discussed for verifying the efficiency of the present theory in predicting buckling stability of FG beams with various boundary conditions under uniform, linear and nonlinear thermal loadings through the thickness. It is assumed that the functionally graded beam is made of a mixture of aluminum and alumina. The Young modulus and coefficient of thermal expansion for aluminum are $E_m =$

70 GPa, $\alpha_m = 23 \times 10^{-6}$/°C and $K_m = 204$ W/m°K and for alumina are $E_c = 380$ GPa, $\alpha_c = 7{,}4 \times 10^{-6}/°C$ and $K_c = 10{,}4$ W/m°K, respectively. It is assumed that the temperature difference between the metal-rich surface of the FGM and reference temperature is $T_m - T_r = 5$℃ (Kiani and Eslami 2010).

Table 2 present the comparisons of the critical buckling temperature for a perfect CR FG beam under uniform, linear and non-linear temperature rise for different values power law index p. It can be seen from the results that the critical buckling temperature is in inverse relation with material index p and this is confirmed for FG beams under uniform, linear and non-linear temperature rise. It can be also observed that the biggest values of the critical buckling temperature are obtained for nonlinear thermal loading.

Table 3 shows a comparison of the critical buckling temperature of perfect and porous FG beams under uniform temperature with different boundary conditions and index values (p). As can be seen, the critical load of the buckling decreases with the increase of the power law index. It is

Table 2 Critical buckling temperature for a CR FG beam under uniform, linear and non-linear temperature rise for different values of power law index p with porosity coefficient $\xi = 0$, $(l/h = 10)$

Temperature load	p						
	p = 0.2	p = 1	p = 2	p = 4	p = 5	p = 6	p = 10
Uniform	803.9224	516.3450	457.7688	463.8468	472.2712	478.5915	485.5025
Linear	1659.4296	959.0102	797.0252	788.0296	804.3103	786.3787	851.4813
Non linear	1961.4450	1331.19	1093.1184	795.7086	1005.90	961.185	984.1343

Table 3 Critical buckling temperature of FG beam under uniform temperature rise with various boundary conditions and different values of porosity coefficient ξ and power law index p $(l/h = 10)$

Boundary conditions	$\xi = 0$			$\xi = 0.1$			$\xi = 0.2$		
	p = 0.2	p = 2	p = 8	p = 0.2	p = 2	p = 8	p = 0.2	p = 2	p = 8
CC	3215.6913	2065.3809	1831.0763	3802.6672	2264.6707	1922.9919	4660.2554	2496.4028	1945.5546
CS	1644.6272	1056.3146	936.4823	1944.8291	1158.2390	983.4912	2383.4323	1276.7556	995.0308
SS	803.9224	516.3450	457.7688	950.6663	566.1674	480.7476	1165.0633	624.1004	486.3884
CR	803.9224	516.3450	457.7688	950.6663	566.1674	480.7476	1165.0633	624.1004	486.3884
SR	200.9806	129.0862	129.0862	237.6666	141.5418	120.1869	291.2658	156.0251	121.5971

Table 4 Critical buckling temperature of FG beam under uniform temperature rise with various boundary conditions and different values of porosity coefficient ξ and thickness ratio l/h $(p = 20)$

Boundary conditions	$\xi = 0$			$\xi = 0.1$			$\xi = 0.2$		
	l/h = 5	l/h = 10	l/h = 20	l/h = 5	l/h = 10	l/h = 20	l/h = 5	l/h = 10	l/h = 20
CC	7436.1353	1859.0338	464.7584	8418.8224	2104.7056	526.1764	9681.6879	2420.4220	605.1055
CS	3803.1233	950.7808	237.6952	4305.7069	1076.4267	269.1067	4951.5845	1237.8961	309.4740
SS	1859.0329	464.7582	116.1895	2104.7045	526.1761	131.5440	2420.4207	605.1052	151.2763
CR	1859.0329	464.7582	116.1895	2104.7045	526.1761	131.5440	2420.4207	605.1052	151.2763
SR	464.7582	116.1895	29.0474	526.1761	131.54403	32.8860	605.1052	151.2763	37.8191

Table 5 Critical buckling temperature for a SS FG beam under linear temperature rise for different values of power law index p, porosity coefficient ξ and thickness ratio l/h

l/h	$\xi = 0$				$\xi = 0.1$				$\xi = 0.2$			
	p = 0.2	p = 1	p = 3	p = 8	p = 0.2	p = 1	p = 3	p = 8	p = 0.2	p = 1	p = 3	p = 8
5	6668.8746	3403.8395	3134.3316	3387.6414	8003.5875	4199.4816	3136.1300	3514.0690	10041.1018	4572.4486	2863.4218	3290.1637
15	731.7546	370.1961	340.5934	368.6222	879.9208	458.3493	341.0778	383.0202	1106.0916	499.8930	311.2109	358.7658
25	256.7850	127.5046	117.0944	127.1007	310.0275	159.0587	117.4736	132.5363	391.2907	174.0885	107.0340	124.2540
35	125.9260	60.6407	55.5181	60.5590	153.016	76.6011	55.8684	63.5254	194.3558	84.3261	50.7812	59.6436
45	72.0746	33.1246	30.1781	33.1756	88.4023	42.6679	30.5165	35.1259	113.3126	47.3868	27.6319	33.0549

Table 6 Critical buckling temperature for a SS FG beam under non-linear temperature rise for different values of power law index p, porosity coefficient ξ and thickness ratio l/h

l/h	$\xi=0$				$\xi=0.1$				$\xi=0.2$			
	$p=0.2$	$p=1$	$p=3$	$p=8$	$p=0.2$	$p=1$	$p=3$	$p=8$	$p=0.2$	$p=1$	$p=3$	$p=8$
5	7882.6064	5363.8274	4153.6144	4010.3236	9488.4622	5843.2816	4142.5153	4146.0908	11961.7043	6386.9652	3764.3410	3859.2009
15	864.9335	584.4105	451.3542	436.3786	1043.1691	637.7606	450.5298	451.9082	1317.6582	698.2690	409.1272	420.8147
25	303.5197	202.0572	155.17341	150.4630	367.5457	221.3189	155.1710	156.3736	466.1345	243.1733	140.7101	145.7438
35	148.8445	96.7149	73.5726	71.6903	181.4045	106.5850	73.7966	74.9508	231.5311	117.7898	66.7585	69.9590
45	85.1921	53.3642	39.9920	39.2736	104.8032	59.3694	40.3092	41.4436	134.9864	66.1916	36.3257	38.7718

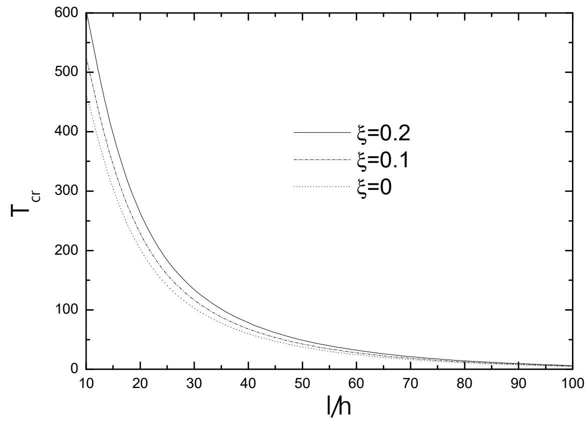

Fig. 1 Variation of critical buckling temperature under a uniform temperature load with thickness ratio and porosity coefficient ξ for a CR FG beam $(p=20)$

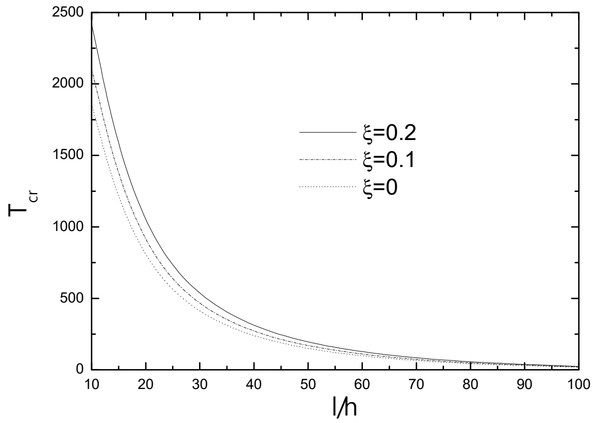

Fig. 2 Variation of critical buckling temperature under a uniform temperature load with thickness ratio and porosity coefficient ξ for a CC FG beam $(p=20)$

apparent that the critical bucking temperatures for the SS boundary conditions are in excellent agreement with CR boundary conditions for any value of the thermal load.

Table 4 illustrates a comparison of the critical buckling temperature of perfect and porous FG beams under uniform temperature with various boundary conditions and l/h ratio. It can be seen that the critical buckling thermal load decreases with the increase of l/h ratio, the increasing of the porosity parameter lead to increase the critical buckling temperature. The lower values of Critical buckling temperature are obtained for SR FG-beam.

Critical buckling temperature of FG beam under linear and non-linear temperature rise for different values of power law index p, porosity coefficient ξ and slenderness ratio l/h is illustrated in Tables 5 and 6, respectively.

In these cases, boundary conditions are assumed to be simply supported. In these tables, the critical buckling temperature decreases for $p \prec 3$ and increases for $p \succ 3$. It is apparent that in both cases of thermal loading, the bucking temperature of the FG beam decreases with the increase of l/h ratio. For nonlinear distribution of the temperature across the thickness, the critical buckling temperature decreases with the increase of the power law index p. It can be conclude that the maximum critical buckling temperature is obtained for porous FG-beam with $\xi=0.2$ for $p \leq 1$.

Figs. 1 and 2 show the effects of the ratio slenderness (l/h) on the critical buckling temperature of a porous and non-porous FG beam under a uniform thermal load. It is observed that with the increase of the l / h ratio, the critical buckling temperature decreases regardless of the porosity coefficient. From these figures, it can be seen that the maximum critical buckling temperature is obtained with a porosity coefficient equal to $\xi=0.2$.

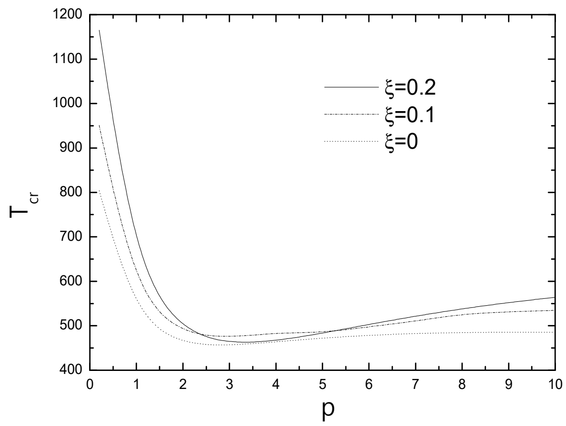

Fig. 3 Variation of critical buckling temperature with gradient index and porosity coefficient ξ under a uniform temperature load for a CR FG beam $(l/h=10)$

Figs. 3, 4 and 5 illustrate the effect of the porosity coefficient on the critical buckling temperature as a function of the power law index under a uniform, linear and non-linear temperature load for the case of a porous FG beam.

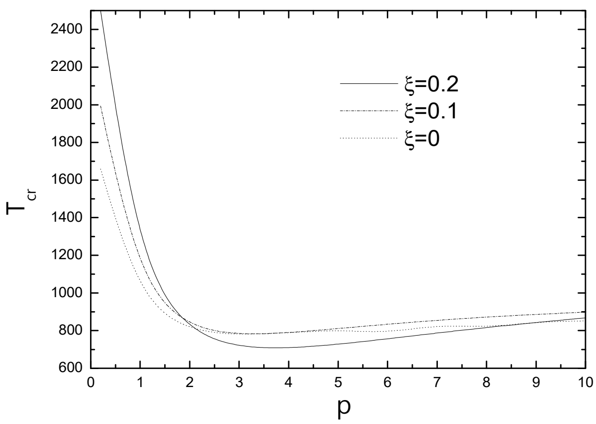

Fig. 4 Variation of critical buckling temperature with gradient index and porosity coefficient ξ under a linear temperature load for a CR FG beam $(l/h = 10)$

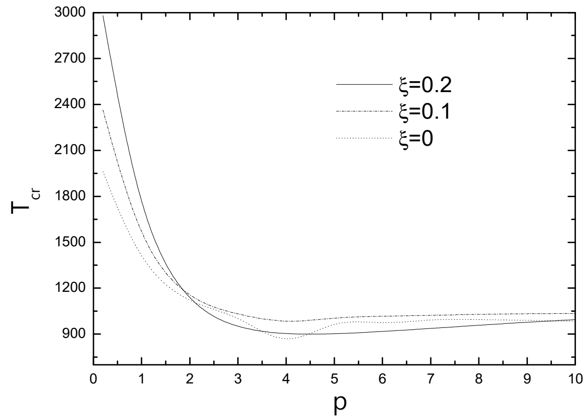

Fig. 5 Variation of critical buckling temperature with gradient index and porosity coefficient ξ under a non-linear temperature load for a CR FG beam $(l/h = 10)$

We observe that the critical buckling temperature decreases with the material index for until $p \prec 3$ and increases for $p \succ 3$. The biggest values of the critical buckling temperature are obtained for porous FG beam with $\xi = 0.2$ and the variation between the three distributions increases for the case of small values of the power index p. It is clear that the critical buckling temperature for the beam under a non-linear temperature change is higher than that for the beam under uniform temperature change. While the critical buckling temperature for the beam under linear temperature change is intermediate to the two previous thermal loading cases.

5. Conclusions

This article deals nonlinear thermal buckling analysis of porous FG beams with various combinations of boundary conditions under uniform, linear and non-linear thermal loadings distribution through the thickness. the plate is modelled based on the Euler–Bernoulli beam theory. The presence of the porosity in the materials is considered in the in the calculations. From the results it appears that the phenomenon of porosity in the materials is a significant parameter on thermal stability of FG structures. Finally, this new result can be used for comparison with other beam models developed in the future. An improvement of the present model will be considered in the future work to consider other type of materials and structures (Attia 2017, Abdulrazzaq *et al*. 2020b, Timesli 2020, Cuong-Le *et al*. 2020, Tayeb *et al*. 2020, Boulal *et al*. 2020, Madenci and Özütok 2020, Nebab *et al*. 2020, Daraei *et al*. 2020, Cao *et al*. 2020, Farokhian and Salmani-Tehrani 2020, Khadimallah and Hussain 2020, Karami *et al*. 2020, Tabasi *et al*. 2020).

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