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Dynamic responses of functionally graded and layered composite beams

O. Kırlangıç [1] and Ş.D. Akbaş*[2]

[1] *The General Directorate of Highways, Ankara, Turkey*
[2] *Department of Civil Engineering, Bursa Technical University, 16330, Bursa, Turkey*

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Abstract. This paper presents and compares the free and damped forced vibrations of layered and functionally graded composite beams. In the considered study, a cantilever beam subjected to a harmonic point load at the free end is investigated with layered and functionally graded materials. In the kinematics of the beam, the Timoshenko beam theory is used. The governing equations of problem are derived by using the Lagrange procedure. In the solution of the problem, the Ritz method is used. Algebraic polynomials are used with the trial functions for the Ritz method. In the obtaining of free vibration results, the eigenvalue procedure is implemented. In the solution of the damped forced vibration problem, the Newmark average acceleration method is used in the time history. In the damping effect, the Kelvin–Voigt viscoelastic model is used with the constitutive relations. In the numerical examples, the effects of material distribution parameter and dynamic parameters on the natural frequencies and forced vibration responses of functionally graded beams are obtained and compared with the results of the layered composite beam. Also, comparison studies are performed in order to validate the used formulations.

Keywords: functionally graded materials; layered materials; dynamic analysis; Timoshenko Beam Theory

1. Introduction

Fracture problem is one of the most important problems in layered composite structures because of the stress discontinuities at interfaces. In the layered structures, the cracks and delimitations frequently occur at the interfaces and it causes to serious failure results. In order to eliminate these problems in composite structures, functionally graded materials were used for get rid of discontinuities at interfaces surfaces and the minimize the stress concentrations. Functionally graded materials are special composites whose properties change gradually through one direction. In general, functionally graded materials consist of a mixture of ceramic and metal materials. In the last years, the functionally graded materials have been found in many engineering applications, such as aircrafts, space vehicles and biomedical sectors.

By increasing functionally graded structures, many researchers investigated the static, dynamic and stability analyses of functionally graded structures in last decades. In the literature, some investigations of dynamic analysis of functionally graded and layered structures are as follows; Palanivel (2006) performed the free vibration analysis of laminated composite beams by using two high-order shear deformation theory and finite elements method. Zenkour *et al*. (2010) investigated bending results of functionally graded viscoelastic sandwich beams embedded elastic foundation. Hein and Feklistova (2011) investigated vibration of axially functionally graded beams with different cross-sections and boundary conditions by using the Haar wavelet series. Eltaher *et al*. (2012) presented free vibration analysis of functionally graded nanobeams based on nonlocal elasticity theory by using finite element method. Babilio (2014) presented nonlinear dynamic responses of functionally graded beams under axial excitation based on Euler-Bernoulli beam theory. Tornabene *et al*. (2014) investigated static and vibration analysis of laminated doubly-curved shells and panels embedded in elastic foundation by using the generalized differential quadrature. Mohanty *et al*. (2015) investigated dynamic responses of functionally graded pre-twisted beams by using Timoshenko beam theory and finite element method. Akbaş (2014, 2015a, b, 2017a, 2018c, h, i, 2019a) presented dynamic analysis of functionally graded beams with different mechanical cases. Nguyen *et al*. (2016) analyzed static, buckling and vibration analysis of a laminated composite beams by using the high order beam theory. Hadji *et al*. (2017) analyzed wave propagation of functionally graded beams with higher order shear deformation theory. Mehar *et al*. (2018) investigated nonlinear dynamics of functionally graded carbon nanotube reinforced sandwich panels under temperature effects by using finite element method. Akbaş (2013, 2017b, 2018a, d, f, g, 2019b, f, g) investigated geometrically nonlinear analysis of composite beams such as functionally graded, laminated composites by using finite element method. Ghayesh (2018) analyzed forced nonlinear vibration of axially functionally graded micro beams by using coupled stress theory. Akbaş (2017c, 2018b, e, 2019c, 2019d, e) presented post-buckling, stability behavior of composite structures with functionally graded and laminated materials. Draiche *et al*. (2019) presented static analysis of laminated reinforced composite plates based on first-order shear

∗Corresponding author, Ph.D.,
E-mail: serefda@yahoo.com

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deformation theory by using the Navier method.

In the literature, a comparison study between functionally graded and layered composite beams has not been investigated for dynamic analysis so far. In this study, dynamic responses which include damped forced vibration and free vibration, are obtained and compared in both functionally graded and layered cantilever beams. Timoshenko beam theory is used in the kinematics of the beams. The governing equations of problem is obtained by using the Lagrange procedure. In the solution of problem, the Ritz method is used.

In the damping effect, the Kelvin–Voigt viscoelastic model is used. In the free vibration analysis, the eigenvalue procedure is implemented. In the solution of the damped forced vibration problem, the Newmark average acceleration method is used in the time history. In the numerical results, the effects of material distribution parameter and dynamic parameters on the natural frequencies and forced vibration response of functionally graded beams are obtained and compared with the results of the layered composite beam.

2. Problem formulation

Consider a cantilever composite beam made of a mixture of two different materials as shown in layered and functionally graded distributions in Fig. 1. The composite beam subjected to a dynamic point load $Q(t)$ at free end. The geometry of the beams is indicated as the length L, the height h and width b. The dynamic point load $Q(t)$ is assumed to be sinusoidal harmonic in time domain as following

$$Q(t) = Q_0 \sin(\varpi t), \qquad 0 \le t \tag{1}$$

where, Q_0 is the amplitude of the dynamic load and ϖ is the frequency of the dynamic load. In the functionally graded beam, the material properties change along the height axis as shown Fig. 1(a). In layered beam, two layers are used as shown Fig. 1(b). The geometry properties of layers are equal to each other. In layered beam, the bottom and top layers of the beam is considered as Zirconia and Alumina, respectively, as shown Fig. 1.

The material properties (P) of the beam in case of functionally graded material change through height axis based on following power-law function distribution

$$P(Y) = (P_B - P_T)\left(\frac{Y}{h} + \frac{1}{2}\right)^n + P_T \tag{2}$$

where P_B and P_T are material properties of bottom and top surfaces, n is the power-law coefficient (material distribution parameter). According to the Eq. (2), when Y = -h/2, $P = P_T$, and when Y = h/2, $P = P_B$ when n = 0 material of beam gets homogenous full bottom material, and when $n = \infty$ material of beam gets homogenous full top material. In the layered beam with two layers, the material properties (P) of the beam is defined as follows

$$P(Y) = \begin{cases} P_B, & 0 \le Y \le 0.5h \\ P_T, & -0.5h \le Y \le 0 \end{cases} \tag{3}$$

The axial strain (ε_z) and shear strain (γ_{zy}) are given according to the Timoshenko beam theory as follows

$$\varepsilon_z = \frac{\partial u_0}{\partial z} - Y\frac{\partial \emptyset}{\partial z} \tag{4a}$$

$$\gamma_{zy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial z} \tag{4b}$$

where, u_0, v_0 and $\emptyset$ are axial displacement, vertical displacement and rotation, respectively. The constitute relation according to Kelvin-Voight viscoelastic model is given as follows

$$\sigma_z = E(Y)\left(\left[\frac{\partial u_0}{\partial z} - Y\frac{\partial \emptyset}{\partial z}\right] + \eta_1(Y)\frac{\partial}{\partial t}\left[\frac{\partial u_0}{\partial z} - Y\frac{\partial \emptyset}{\partial z}\right]\right) \tag{5a}$$

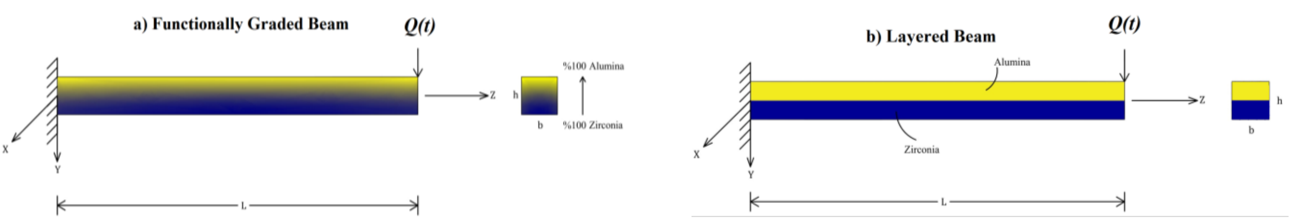
$$\sigma_{zy} = G(Y)K_s\left(\left[\frac{\partial v_0}{\partial z} - \emptyset\right] + \eta_2(Y)\frac{\partial}{\partial t}\left[\frac{\partial v_0}{\partial z} - \emptyset\right]\right) \tag{5b}$$

where, E is the Young's Modulus, G shear Modulus, σ_z is the normal stress, σ_{zy} is the shear stresses and K_s is the shear correction factor, η_1 and η_2 are the damping ratios in bending and shearing, respectively as follows

$$\eta_1(Y) = \frac{c(Y)}{E(Y)}, \qquad \eta_2(Y) = \frac{c(Y)}{G(Y)} \tag{6}$$

where, c is the damping coefficient. The strain energy (U_i), the kinetic energy (K), the dissipation function (R) and potential energy of the external loads (U_e) are presented as follows

$$U_i = \frac{1}{2}\int_0^L \left[A_0\left(\frac{\partial u_0}{\partial z}\right)^2 - 2A_1\frac{\partial u_0}{\partial z}\frac{\partial \emptyset}{\partial z} + A_2\left(\frac{\partial \emptyset}{\partial z}\right)^2\right]dZ + \frac{1}{2}\int_0^L K_s B_0\left[\left(\frac{\partial v_0}{\partial z}\right)^2 - 2\frac{\partial v_0}{\partial z}\emptyset + \emptyset^2\right]dZ \tag{7a}$$



(a) Functionally graded distribution (b) Layered distribution

Fig. 1 A cantilever composite beam under a dynamic point load at free end

$$K = \frac{1}{2}\int_0^L \left(I_0 \left(\frac{\partial u_0}{\partial t}\right)^2 - 2I_1 \left(\frac{\partial u_0}{\partial t}\right)\left(\frac{\partial \emptyset}{\partial t}\right) + I_2 \left(\frac{\partial \emptyset}{\partial t}\right)^2 + I_0 \left(\frac{\partial v_0}{\partial t}\right)^2 \right) dZ \tag{7b}$$

$$R = \frac{1}{2}\int_0^L \left(C_0 \left(\frac{\partial}{\partial Z}\left(\frac{\partial u_0}{\partial t}\right)\right)^2 - 2C_1 \left(\frac{\partial}{\partial Z}\left(\frac{\partial u_0}{\partial t}\right)\right)\left(\frac{\partial}{\partial Z}\left(\frac{\partial \emptyset}{\partial t}\right)\right) + C_2 \left(\frac{\partial}{\partial Z}\left(\frac{\partial u_0}{\partial t}\right)\right)^2 + C_3 \left(\frac{\partial}{\partial Z}\left(\frac{\partial v_0}{\partial t}\right) - \left(\frac{\partial \emptyset}{\partial t}\right)\right)^2 \right) dZ \tag{7c}$$

$$U_e = -Q(t)\, v(z_p, t) \tag{7d}$$

where

$$(A_0, A_1, A_2) = \int_A E(Y)(1, Y, Y^2) dA, \quad B_0 = \int_A G(Y) dA,$$
$$(I_0, I_1, I_2) = \int_A \rho(Y)(1, Y, Y^2) dA \tag{8}$$
$$(C_0, C_1, C_2) = \int_A E(Y)\eta_1(Y)(1, Y, Y^2) dA, \quad C_3 = \int_A G(Y)\eta_2(Y) dA$$

The Lagrangian functional of the problem is presented as follows

$$I = K - (U_i + U_e) \tag{9}$$

In the solution of the problem in Ritz method, approximate solution is given as a series of *i* terms of the following form

$$u_0(z,t) = \sum_{i=1}^{\infty} \mathrm{a}_i(t)\alpha_i(z) \tag{10a}$$

$$v_0(z,t) = \sum_{i=1}^{\infty} \mathrm{b}_i(t)\beta_i(z) \tag{10b}$$

$$\emptyset(z,t) = \sum_{i=1}^{\infty} \mathrm{c}_i(t)\gamma_i(z) \tag{10c}$$

where a_i, b_i and c_i are the unknown coefficients, $\alpha_i(z,t)$, $\beta_i(z,t)$, $\gamma_i(z,t)$ are the coordinate functions depend on the boundary conditions over the interval [*0*, *L*]. The coordinate functions for the cantilever beam are given as algebraic polynomials

$$\alpha_i(z) = z^i \tag{11a}$$

$$\beta_i(z) = z^{(i+1)} \tag{11b}$$

$$\gamma_i(z) = z^i \tag{11c}$$

where *i* indicates the number of polynomials involved in the admissible functions.

After substituting Eq. (10) into energy Eq. (7), and then using the Lagrange's equation gives the following equation

$$\frac{\partial I}{\partial q_i} - \frac{\partial}{\partial t}\frac{\partial I}{\partial \dot{q}_i} + Q_{Dk} = 0, \qquad Q_{Dk} = -\frac{\partial R}{\partial \dot{q}_i} \tag{12}$$

where q_i is the unknown coefficients which are a_i, b_i and c_i. Q_{Dk} indicates the generalized damping load. After implementing the Lagrange procedure, the motion equation of the problem is obtained as follows

$$[\mathrm{K}]\{\mathrm{q(t)}\} + [C]\{\dot{q}(t)\} + [\mathrm{M}]\{\ddot{\mathrm{q}}(\mathrm{t})\} = \{\mathrm{F(t)}\} \tag{13}$$

where $[K]$, $[C]$, $[M]$ and $\{\mathrm{F(t)}\}$ are the stiffness matrix, the damping matrix, mass matrix and load vector, respectively. The detail of these expressions are given as follows

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \tag{14}$$

where

$$K_{11} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L A_0 \frac{\partial \alpha_i}{\partial z}\frac{\partial \alpha_j}{\partial z} dz$$
$$K_{12} = 0,$$
$$K_{13} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L A_1 \frac{\partial \alpha_i}{\partial z}\frac{\partial \gamma_j}{\partial z} dz,$$
$$K_{21} = 0,$$
$$K_{22} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s B_0 \frac{\partial \beta_i}{\partial z}\frac{\partial \beta_j}{\partial z} dz,$$
$$K_{23} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s B_0 \frac{\partial \beta_i}{\partial z}\gamma_j\, dz, \tag{15}$$
$$K_{31} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L A_1 \frac{\partial \gamma_i}{\partial z}\frac{\partial \alpha_j}{\partial z} dz,$$
$$K_{32} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s B_0 \gamma_i \frac{\partial \beta_j}{\partial z} dz,$$
$$K_{33} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L A_2 \frac{\partial \gamma_i}{\partial z}\frac{\partial \gamma_j}{\partial z} + \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s B_0 \gamma_i \gamma_j\, dz,$$

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \tag{16}$$

where

$$C_{11} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L C_0 \frac{\partial \alpha_i}{\partial z}\frac{\partial \alpha_j}{\partial z} dz, \qquad C_{12} = 0, \tag{17}$$

$$C_{13} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L C_1 \frac{\partial\alpha_i}{\partial z}\frac{\partial\gamma_j}{\partial z}dz, \qquad C_{21} = 0,$$
$$C_{22} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s C_3 \frac{\partial\beta_i}{\partial z}\frac{\partial\beta_j}{\partial z}dz,$$
$$C_{23} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s C_3 \frac{\partial\beta_i}{\partial z}\gamma_j\, dz,$$
$$C_{31} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L C_1 \frac{\partial\gamma_i}{\partial z}\frac{\partial\alpha_j}{\partial z}dz, \tag{17}$$
$$C_{32} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s C_3 \gamma_i \frac{\partial\beta_j}{\partial z}dz,$$
$$C_{33} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L C_2 \frac{\partial\gamma_i}{\partial z}\frac{\partial\gamma_j}{\partial z} + \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L K_s C_3 \gamma_i\gamma_j\, dz,$$

$$[M] = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \tag{18}$$

where

$$M_{11} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L \mathrm{I}_0\, \alpha_i\alpha_j \mathrm{dz}, \qquad M_{12} = 0,$$
$$M_{13} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L \mathrm{I}_1\, \alpha_i\gamma_j \mathrm{dz}, \qquad M_{21} = 0,$$
$$M_{22} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L \mathrm{I}_0\, \beta_i\beta_j \mathrm{dz}, \qquad M_{23} = M_{32} = 0 \tag{19}$$
$$M_{31} = -\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L \mathrm{I}_1\, \gamma_i\alpha_j \mathrm{dz}$$
$$M_{33} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^L \mathrm{I}_2\, \gamma_i\gamma_j \mathrm{dz}$$

$$\{\mathrm{F(t)}\} = \sum_{j=1}^{n} Q\beta_j \tag{20}$$

The governing equation of motions Eq. (13) is solved numerically by using implicit Newmark average acceleration $(\alpha = 0.5\,, \beta = 0.25)$ method in the time domain. By this procedure, the dynamic problem is transferred to system of static problem in each step as following

$$[\bar{K}(t,X)]\{d_n\}_{j+1} = \{\bar{F}(t)\} \tag{21}$$

in which

$$[\bar{K}(t,X)] = [K] + \frac{[M]}{\beta\Delta t^2} + \frac{[C]\alpha}{\beta\Delta t} \tag{22a}$$

$$\{\bar{F}(t)\} = \{F(t)\}_{j+1} + [B_1]\{d_n\}_j + [B_2]\{\dot{d}_n\}_j + [B_3]\{\ddot{d}_n\}_j \tag{22b}$$

where

$$[B_1] = \frac{[M]}{\beta\Delta t^2} + \frac{[C]\alpha}{\beta\Delta t}\,,$$
$$[B_2] = \frac{[M]}{\beta\Delta t} + [C]\left(\frac{\alpha}{\beta} - 1\right), \tag{23}$$
$$[B_3] = [M]\left(\frac{1}{2\beta} - 1\right) + [C]\left(\frac{\alpha}{2\beta} - 1\right)$$

After evaluating $\{d_n\}_{j+1}$ at a time $t_{j+1} = t_j + \Delta t$, the acceleration and velocity vectors can be evaluated by

$$\{\ddot{d}_n\}_{j+1} = \frac{1}{\beta\Delta t^2}\left(\{d_n\}_{j+1} - \{d_n\}_j\right) - \frac{[M]}{\beta\Delta t}\{\dot{d}_n\}_j - \left(\frac{\alpha}{2\beta} - 1\right)\{\ddot{d}_n\}_j \tag{24a}$$

$$\{\dot{d}_n\}_{j+1} = \{\dot{d}_n\}_j + \Delta t\,(1-\alpha)\{\ddot{d}_n\}_j + \Delta t\,\alpha\,\{\ddot{d}_n\}_{j+1} \tag{24b}$$

3. Numerical results

In the numerical examples, vibration frequencies and dynamic responses are investigated for a cantilever beam for functionally graded and layered composite materials. In the numerical examples, the materials of the beams are selected as Zirconia and Alumina, and their distributions are shown in Fig. 1. The material parameters of these materials are given as follows. Zirconia: E = 151 GPa, ρ = 2702 kg/m^3, v = 0.2882, Alumina: E = 70 GPa, ρ = 3000 kg/m^3, v = 0.31. v indicates the Poisson's ratio. The geometry properties of the beam are selected as $b = 0.1$ m, h = 0.1 m and L = 1 m. In the numerical results, number of the series term is taken as 10. The amplitude of the dynamic load is selected as Q_0 = 30 kN. The damping ratio of Alumina is considered as $\eta_{Al} = 0.001$ and The damping ratio of Zirconia is selected as according to multiples of Aluminium value, such as $\eta_{zr}/\eta_{Al} = 2$. The damping ratio of each layer graded through height direction according to Eq. (2).

In order to validate the used formulations, the dimensionless fundamental frequency ($\omega_b = \omega/\sqrt{\mathrm{A}_0/\mathrm{I}_0}$) of a functionally graded cantilever beam are calculated for various E_B/E_T ratio for L/h = 20 compared with those of Yang and Chen (2008). It is seen from Table 1, the present results are in good agreement with that the results of Yang and Chen (2008).

In another comparison study, the maximum vertical displacements (at the free end) of the layered beam are obtained and compared with ANSYS Workbench 14 structural analysis program for L/h = 10, Q_0 = 30 kN,

Table 1 Dimensionless of the fundamental frequencies of a cantilever functionally graded beam for L/h = 20

E_B/E_T	Present	Yang and Chen (2008)
0.2	0.8291	0.83
1	0.8794	0.88
5	0.8289	0.83

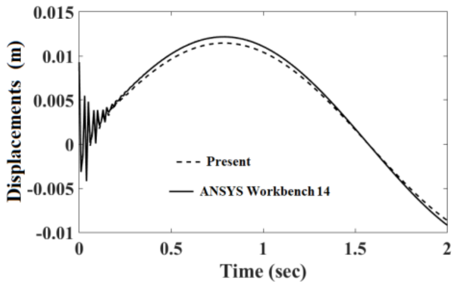


Fig. 2 Comparison study: Time responses of the layered beam for L/h = 10, Q_0 = 1000 kN, Ω = 2 rad/s

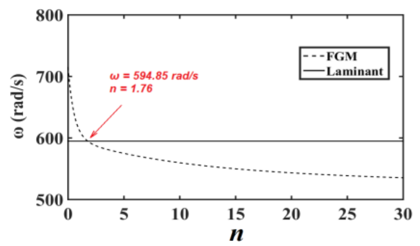
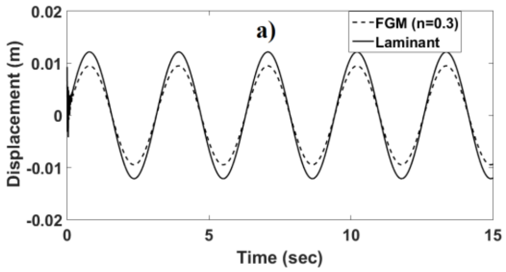
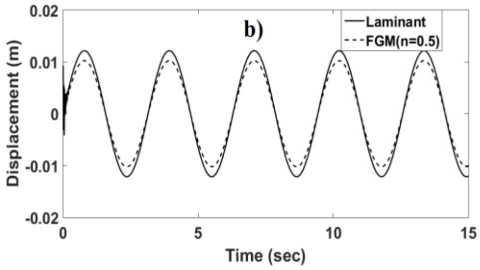


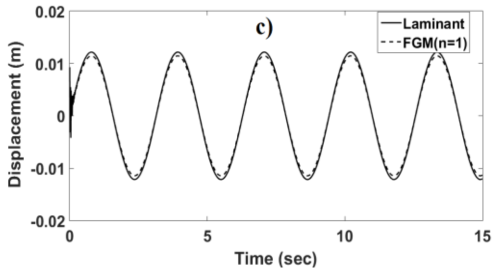
Fig. 3 The natural frequencies of the functionally graded and layered composite cantilever beams for different material distribution parameters



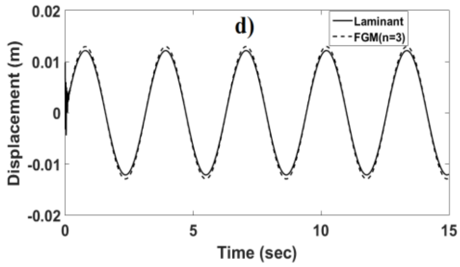
(a) For n = 0.3



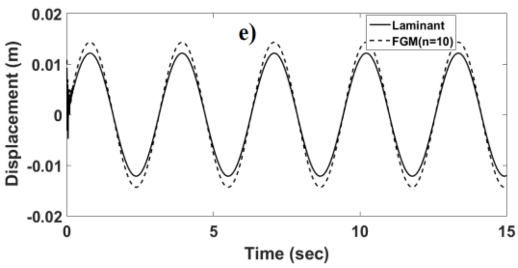
(b) For n = 0.5



(c) For n = 0.1



(d) For n = 3



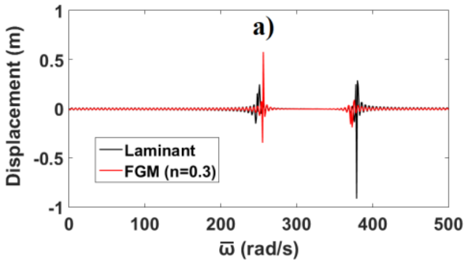
(e) For n =10

Fig. 4 Time responses of the functionally graded and layered composite cantilever beams for different material distribution parameters

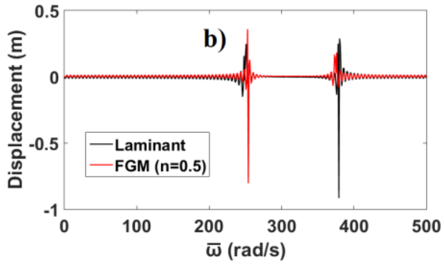
ϖ = 2 rad/s in Fig. 2. It is seen from Fig. 2, that results of this study agree well with results of ANSYS Workbench14.

In Fig. 3, the natural frequencies of functionally graded and layered beams are obtained and compared. In the natural frequencies of the functionally graded beam, different values of the power-law coefficient (material distribution parameter) n are used in order to compare with the layered beam.

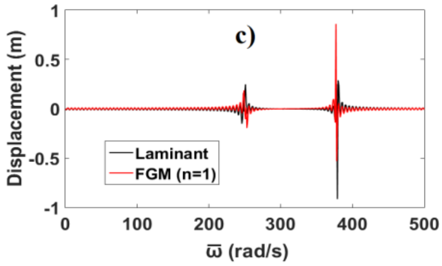
As seen from Fig. 3 that, the natural frequencies of functionally graded beam decrease by increasing the material graduation parameter (n). Because Young's Modulus of the beams decrease according to Eq. (2) and position of the materials, the stiffness and the natural frequencies of the beam decreases. The natural frequencies of the layered beams are bigger than the results of functionally graded parameter for $n > 1.76$ for this example. If the value of functionally graded graduation parameter selects less than n = 1.76 for this example, the natural frequencies of functionally graded material is bigger than the layered beam's. So, using the functionally graded material has a superiority over layered beams for $n < 1.76$ for this example. It is noted that this value is obtained for presented example. It depends on material and geometry parameters. So, it varies according to different examples.



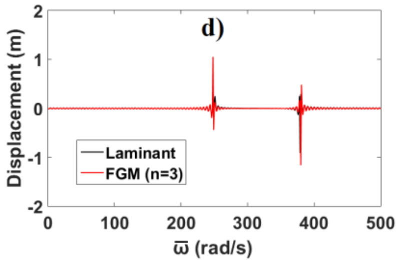
(a) For n = 0.3



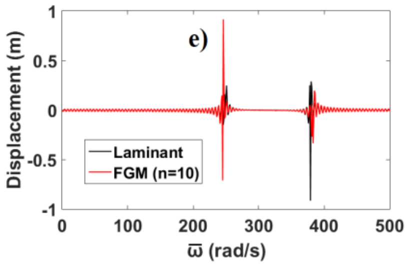
(b) For n = 0.5



(c) For n = 0.1



(d) For n = 3



(e) For n =10

Fig. 5 The relationship between of the displacements and the frequency of the dynamic load ($\overline{w}$) of the functionally graded and layered composite cantilever beams for different material distribution parameters

The presented results show that the material graduation parameter plays important role on the vibration characters of the functionally graded materials.

Fig. 4 illustrates the time responses of the vertical maximum displacements (at the free-end of the cantilever beam) in functionally graded and layered beams for L/h = 10 and $\overline{\varpi}$ = 2 rad/s.

As shown, by increasing the material graduation parameter (n), the amplitude of displacements increases. It is stated before that the stiffness of the functionally graded beam decreases with increasing in the n parameter. As the results of the free vibration results, the dynamic displacements of the layered beams are bigger than the results of functionally graded parameter in small values of n. With increasing in the n parameter, the dynamic displacements of functionally graded material is bigger than the values of the layered beam. It is concluded from these results, with suitable choosing of n parameter, functionally graded materials have more advantages than layered materials in regards to mechanical responses.

Fig. 5 presents the frequency of the dynamic load ($\overline{w}$)-maximum vertical displacements relation for different values of functionally graded material distribution parameters for L/h = 10 and t = 0.2 s.

It is seen from Fig. 5, the resonance frequencies can be observed in the vertical asymptote. With increasing in the n parameter, the resonance frequencies of the functionally graded beams decrease. In smaller values of n parameters, the resonance frequencies of layered beams smaller than those of the functionally graded beams. In higher values of n parameters, this situation changed entirely.

4. Conclusions

The free vibration and damped forced vibration analysis of the functionally graded and layered beams are investigated. The Timoshenko beam theory and the Kelvin–Voigt viscoelastic model is used. Equations are solved by applying the Ritz method. In the dynamic solution, the Newmark average acceleration method is used in the time history. In the numerical examples, the difference between functionally graded and layered beams is investigated on the free and forced vibration responses. The natural frequencies, dynamic displacements and resonance responses are obtained and compared. Some comparison studies are presented.

Numerical results show that the dynamic responses of the functionally graded and layered beams are very different each other. With different functionally graded distribution, the difference between dynamic responses of functionally graded and layered beams differ significantly.

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