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A novel model of fractional thermal and plasma transfer within a non-metallic plate

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Abstract. While in several publications the thermo-viscoelastic properties of solids have been documented, no attempt has been made to examine the action of coupled thermal and plasma wave in viscoelastic materials. In this paper, a new mathematical model for thermal and plasma transfer in an organic semiconductor was constructed with a time-fractional derivative of order $\alpha(0 < \alpha \leq 1)$ and a time-fractional integral of order $\beta(0 < \beta \leq 2)$, respectively. A two-dimensional problem is viewed for a half-space of viscoelastic thin-walled semiconductor whose surface is traction free and subjected to a heat flux with an exponentially decaying pulse. Laplace and Fourier's integral transforms are utilized. The carrier density, temperature, thermal stress, and viscoelastic displacement distributions have been obtained through the use of the theoretical model together with plasma and thermo-viscoelastic effects. The inversion technique for Fourier and Laplace transforms is carried out using a numerical technique based on Fourier series expansions. Comparisons are made with the results anticipated thru the coupled idea and generalized theory. The influence of the fractional-order parameter on all the regarded fields is examined.

Keywords: organic semiconductor; thermo-viscoelasticity; photothermal theory; fractional calculus; fourier transforms; laplace transforms

1. Introduction

The property of materials that possess both viscous and elastic properties when deformed is viscoelastic. Linear viscoelastic materials are rheological materials that show load time-temperature dependence. If their response is not only a function of the present input but also of the current and past input history, the characterization of the viscoelastic response can be expressed using integral (hereditary) convolution (Gross 1953, Meyers and Chawla 1999). The works of Gurtin and Sternberg (1962), Sternberg (1963) and Ilioushin (1968) are of special importance to these books. A model theory of thermal viscoelasticity has been proposed in the book of Ilioushin and Pobedria (1970).

In the analysis of solids, heat transfer is very important. About any impact on a solid would be followed by a heat and mass transportation transition and vice versa. The difference in the warmth conduction condition from diffusive to a wave type may moreover be influenced both through a minute thought of the warmness transport marvel or in a phenomenological route by methods for altering the traditional Fourier law of heat conduction proposed by Biot (1956). The first is expected to Cattaneo (1958), who determined a wave-type warmth condition through proposing another law of heat conduction to change the traditional Fourier law. Lord and Shulman (1967) were the first to construct a model of thermoelasticity ensuring that the velocity of waves is not infinite. For a summary, a presentation of generalized theories, one can refer to Ignaczak (1989) and Chandrasekharaiah (1998). Ezzat (2006) introduced a new model of generalized thermo-viscoelasticity equations for an electrically conductive isotropic medium permeated by a primary uniform magnetic field, taking into account the volume's rheological properties. As such, a handful of examinations: Sherief and Ezzat (1994), Ezzat (2001), Abbas (2006, 2014), Mukhopadhyay and Kumar (2009), Yu *et al*. (2013, 2014), Sharma and Sharma (2014), Sharma *et al*. (2013), Kumar and Abbas (2013), Abbas and Abo-Dahab (2014), Alzahrani and Abbas (2016), Lata *et al*. (2016), Lata (2018), Kumar *et al*. (2016a, b, 2018), Abbas and Kumar (2016), Itu *et al*. (2019), Sherief and Abd El-Latief (2016), Kumar and Devi (2017), Abbas and Marin (2017) and Othman *et al*. (2020), subject to these generalized hypotheses were inquired about.

The differential conditions including Riemann – Liouville differential administrators of partial request $0 < \alpha < 1$ appear to be critical in demonstrating numerous physical wonders (Kiriyakova 1994, Bagley and Torvik 1983, Adolfsson *et al*. 2005) and in this manner appear to justify a free investigation of their hypothesis parallel to the outstanding hypothesis of customary differential conditions. Caputo (1974) agreed with experimental outcomes when the usage of fractional derivatives to describe viscoelastic materials and mounted the link between fractional derivatives and linear viscoelasticity theory. For a survey of

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fractional calculus applications, one can refer to Podlubny (1999).

The study of anomalous diffusion, which is characterized by Kimmich's time-fractional diffusion-wave equation established by Kimmich (2002), has been spent a considerable amount of work recently

$$\rho C = \lambda I^{\beta} \nabla^2 C, \qquad 0 < \beta \leq 2 \tag{1}$$

where C is the concentration and the concept I^{β} is the Riemann–Liouville fractional integral, added as a herbal generalization of the popular n-fold repetitive integral I^{n} written in a convolution-type structure (Mainardi and Gorenflo 2000)

$$\left.\begin{aligned} I^{\beta} f(y,t) &= \frac{1}{\Gamma(\beta)} \int_0^t (t-\xi)^{\beta-1} f(\xi)\ d\xi \\ I^0 f(t) &= f(t) \end{aligned}\right\},$$
$$0 < \beta \leq 2.$$

In Kimmich's view, Eq. (1) defines various diffusion cases where $0 < \beta < 1$ corresponds to weak diffusion (subdiffusion), $\beta = 1$ corresponds to regular diffusion, $0 < \beta < 2$ corresponds to high diffusion (superdiffusion), and $\beta = 2$ corresponds to ballistic diffusion. It should be noted that the term diffusion is often used in a wider sense, including different transport phenomena. Condition (1) is the scientific model of a wide scope of basic physical marvels; for instance, subdiffusive vehicle happens in a wide scope of sorts of frameworks going from dielectrics and semiconductors through polymers to fractals, plates, permeable and arbitrary materials. Superdiffusion is similarly unprecedented and has been found in permeable glasses, polymer chains, natural frameworks, and transport of natural particles and nuclear bunches on the surface. One would perhaps depend on the abnormal warmth conduction in media where the bizarre dispersion is watched.

Fujita (1999) considered the heat flux equation in the form

$$q_i = -\ k\ I^{\beta-1} \nabla T, \qquad 1 < \beta \leq 2. \tag{2}$$

The Caputo heat wave formula was used by Povstenko (2005) to describe the fractional

$$q_i = -k\ I^{\beta-1} \nabla T, \qquad 0 < \beta \leq 2. \tag{3}$$

Sherief *et al*. (2010) presented a heat conduction equation in the form

$$q_i + \ \tau_o \frac{\partial^{\alpha} q_i}{\partial t^{\alpha}} = -k \nabla T, \qquad 0 < \alpha \leq 1 \tag{4}$$

Youssef (2010) developed a further heat conduction equation and took into account (2)-(4)

$$q_i + \ \tau_o \frac{\partial^{\alpha} q_i}{\partial t^{\alpha}} = -k I^{\beta-1} \nabla T, \qquad 0 < \alpha, \beta \leq 2. \tag{5}$$

Ezzat and El-Bary (2016) introduced a heat conduction equation in the form

$$q_i + \ \frac{\tau_o^{\alpha}}{\alpha!} \frac{\partial^{\alpha} q_i}{\partial t^{\alpha}} = -k \nabla T, \qquad 0 < \alpha \leq 1, \tag{6}$$

and established a unified mathematical model of the equations of generalized magneto-thermoelasticity based on fractional derivative heat transfer for isotropic perfect conducting media using the Taylor-Riemann series.

Ezzat and El-bary (2012) combined the results in Eqs. (5) and (6) and supposed that the heat conduction law takes the form

$$\begin{aligned} &q_i + \ \frac{\tau_o^{\alpha}}{\alpha!} \frac{\partial^{\alpha} q_i}{\partial t^{\alpha}} = -k I^{\beta-1} \nabla T, \\ &0 < \alpha \leq 1, \qquad 0 < \beta \leq 2, \end{aligned} \tag{7}$$

Gordon *et al*. (1965) discovered the first photothermal method when they found a laser-based intracavity sample that gave rise to photothermal bloom, the photothermal lens. Sometime later Kreuzer (1971) demonstrated that when laser light sources were used, photoacoustic spectroscopy could be used for the sensitive study. Photothermal methods were then used to measure temperature, thermal diffusivity, sound frequency, the velocity of bulk flow, surface thickness, and direct heat by Tam (1983, 1989). Todorovic (2003) studied the effect of plasma fields on micro thermoelastic structures. Their study was both theoretical and experimental. Opsal and Rosencwaig (1985) studied heat and plasma waves in Silicon. Their study focused on finding the depth that these waves will penetrate. Song *et al*. (2010) investigated the thermoelastic vibrations of the micro cantilever that has been excited by photons. Song *et al*. (2012) studied the plane waves reflection in a semiconductor medium in the generalized theory of thermoelasticity. Among the contributions to the generalized theory are Alzahrani and Abbas (2018), Lotfy *et al*. (2018), and Hobiny and Abbas (2018).

Organic semiconductors are solids with pi-bonded molecules or polymers made up of atoms of carbon and hydrogen and sometimes heteroatoms such as nitrogen, sulfur, and oxygen. Those exist in the form of amorphous thin films or molecular crystals. These are usually electrical insulators, but they become semiconducting when charges are either applied from suitable electrodes, after doping, or through photo excitation. Some organic semiconductors are what are called viscoelastic materials. For micromechanical structures, thermo-viscoelastic deformation and electronic deformation are prominent organic semiconductor deformations and the main guided mechanisms.

In the present work, we employ the state space approach to fractional two-dimensional problems of thermo-viscoelasticity. The resulting formulation together with the Laplace transform techniques is applied to a specific problem of a half-space subjected to a thermal shock loading which produces plane strain deformation inside the medium. The inversion of the Laplace transform is obtained numerically by using a method based on the Fourier expansion technique proposed by Honig and Hirdes (1984). The solutions are represented graphically for different values of fractional order.

2. Basic equations

The governing coupled equations of the linear photo-thermo-viscoelastic semiconductor with fractional order of heat transfer consist of the following:

(1) The equation of motion

$$\sigma_{ji,j} = \rho u_{i,tt}. \tag{8}$$

(2) The constitutive equation (Ezzat *et al*. 2001)

$$S_{ij}(x,t) = \int_0^t R(t-\xi)\frac{\partial e_{ij}(x,\xi)}{\partial \xi}d\xi = \hat{R}(e_{ij}), \tag{9}$$

where

$$\begin{aligned} &S_{ij} = \sigma_{ij} - \frac{\sigma_{kk}}{3}\delta_{ij}, \qquad \varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \\ &e_{ij} = \varepsilon_{ij} - \frac{e}{3}\delta_{ij}, \qquad e = \varepsilon_{kk}, \\ &x = (x_1, x_2, x_3), \end{aligned} \tag{10}$$

and $\hat{R}(t)$ is the relaxation modulus function such that $R(\infty) > 0$ given by

$$\hat{R}(t) = 2\mu\left(1 - A\int_0^t f(t)\ dt\right). \tag{11}$$

The function $f(t)$ is taken in the form

$$f(t) = e^{-\beta^* t}\, t^{a^*-1}, \tag{12}$$

where

$$0 < a^* < 1, \quad \beta^* > 0, \quad 0 \le A < \Gamma(a^*).$$

(3) The stress-strain relations are expressed as

$$\sigma = K[e\ - 3\alpha_N N - 3\alpha_T \vartheta]. \tag{13}$$

Substituting from Eq. (10) into Eq. (9) and using Eq. (13), we obtain (Hobiny and Abbas 2018)

$$\sigma_{ij} = \hat{R}(\varepsilon_{ij} - \frac{e}{3}\delta_{ij}) + [Ke - \gamma_N N - \gamma_T \vartheta]\delta_{ij}. \tag{14}$$

Eq. (8) together with Eq. (14) reduces to

$$\rho\ddot{u}_i = \hat{R}\ \left(\frac{1}{2}\nabla^2 u_i\ + \frac{1}{6}e_{,i}\right) + Ke_{,i} - \gamma_N N_{,i} - \gamma_T \vartheta_{,i}. \tag{15}$$

(4) The fractional heat equation (Ezzat *et al*. 2014)

$$\boldsymbol{k\vartheta_{,ii}} = \boldsymbol{\frac{\partial}{\partial t}\left(1 + \frac{\tau^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)(\rho C_E \vartheta + \gamma_T T_o e) - \frac{E_g}{v}N}, \tag{16}$$
$$\boldsymbol{0 < \alpha \le 1}.$$

(5) The integral fractional plasma equation (Abbas *et al*. 2017)

$$D_e I^{\beta-1} N_{,jj} = \dot{N}\ + \frac{N}{v}\ - \zeta\frac{\vartheta}{v}, \qquad 0 < \beta \le 2. \tag{17}$$

In the above equations, a dot denotes differentiation with respect to time, while a comma denotes material derivatives. The summation notation is used.

Limiting cases

(1) Coupled thermoelasticity theory (DCT)

The model of Eqs. (14)-(17) in the limiting case, $\tau = 0,\ \alpha = 0,\ \beta = 1,\ A = 0$, transforms to the work of Biot (1956) (CTE).

(2) Generalized thermoelasticity theory with one relaxation time (LS)

The model of Eqs. (14)-(17) in the limiting case,$\tau > 0,\ \alpha = 1,\ \beta = 1,\ A = 0$, transforms to the work of Lord-Shulman (1967) and Hobiny and Abbas (2018).

3. Formulation of the problem

We consider the rectangular Cartesian coordinate system (x, y, z) having origin on the surface y = 0, with y - axis pointing vertically into the medium and a homogenous, isotropic, viscoelastic semiconductor occupying the semi-space region. According to the assumption of two-dimensional motion, the field variables components are

$$\begin{aligned} &u = (u, w, 0), \qquad u = u(x, y, 0), \\ &w = w(x, y, 0), \quad \vartheta = \vartheta(x, y, 0), \quad N = N(x, y, 0). \end{aligned}$$

Let us introduce the following non-dimensional variables

$$\begin{aligned} &(x^*, y^*, u^*, w^*) = c_o\eta_o(x, y, u, w), \\ &\left(t*, v^*, t_p^*\right) = c_o^2\eta_o\left(t, v, t_p\right), \\ &c_o^2 = \frac{K}{\rho}, \qquad \tau^* = c_o^{2\alpha}\eta^\alpha\tau, \qquad \vartheta* = \frac{\gamma_T}{\rho c_o^2}\vartheta, \\ &N* = \frac{\gamma_N}{\rho c_o^2}N, \qquad \sigma_{ij}^* = \frac{\sigma_{ij}}{K}, \qquad \hat{R}* = \frac{2}{3K}\hat{R}. \end{aligned}$$

Using the above non-dimensional quantities, (dropping the asterisk for convenience), the governing Eqs. (11) and (14)-(17) can be written as

$$\frac{\partial^2 u}{\partial t^2} = \hat{R}\left(\frac{\partial^2 u}{\partial x^2} + \frac{3}{4}\frac{\partial^2 u}{\partial y^2} + \frac{1}{4}\frac{\partial^2 w}{\partial x \partial y}\right) + \frac{\partial e}{\partial x} - \frac{\partial N}{\partial x} - \frac{\partial \vartheta}{\partial x} \tag{18}$$

$$\frac{\partial^2 w}{\partial t^2} = \hat{R}\left(\frac{\partial^2 w}{\partial y^2} + \frac{3}{4}\frac{\partial^2 w}{\partial x^2} + \frac{1}{4}\frac{\partial^2 u}{\partial x \partial y}\right) + \frac{\partial e}{\partial y} - \frac{\partial N}{\partial y} - \frac{\partial \vartheta}{\partial y}, \tag{19}$$

$$\frac{\partial^2 e}{\partial t^2} = \left(1 + \hat{R}\right)\nabla^2 e - \nabla^2 N - \nabla^2 \vartheta, \tag{20}$$

$$\begin{aligned} &\nabla^2 \vartheta = \frac{\partial}{\partial t}\left(1 + \frac{\tau^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)(\vartheta + \varepsilon e) - \varepsilon_1 N, \\ &0 < \alpha \le 1, \end{aligned} \tag{21}$$

$$\begin{aligned} &I^{\beta-1}\nabla^2 N = \xi\dot{N} + \xi\frac{N}{v} - \varepsilon_2\frac{\vartheta}{v}, \\ &0 < \beta \le 2 \end{aligned} \tag{22}$$

$$\sigma_{xx} = \hat{R}\ \left(\frac{\partial u}{\partial x} - \frac{1}{2}\frac{\partial w}{\partial y}\right) + e - N - \vartheta, \tag{23}$$

$$\sigma_{yy} = \hat{R}\left(\frac{\partial w}{\partial y} - \frac{1}{2}\frac{\partial u}{\partial x}\right) + e - N - \vartheta, \tag{24}$$

$$\sigma_{zz} = -\frac{1}{2}\hat{R}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial y}\right) + e - N - \vartheta, \tag{25}$$

$$\sigma_{xy} = \frac{3}{4}\hat{R}\left(\frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\right), \tag{26}$$

$$\hat{R}(t) = \frac{4\mu}{3K}\left(1 - A\int_0^t e^{-\beta t}\, t^{a*-1} dt\right), \tag{27}$$

where

$\varepsilon_1 = \gamma_T E_g/(K\upsilon\gamma_N\eta_o C_E)$, $\xi = 1/D_e\eta_o$ and $\varepsilon_2 = \xi\zeta\alpha_N/\alpha_T$.

4. Laplace and Fourier transforms

Taking Laplace transform defined by the relation

$$\bar{f}(x,y,s) = L\{f(x,y,t)\} = \int_0^\infty e^{-st} f(x,y,t)dt, \tag{28}$$

and the Fourier transform

$$\bar{f}^*(q,y,s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-iqx}\bar{f}(x,y,s)dx, \tag{29}$$

of both sides of Eqs. (18)- (27), denoting $\bar{R} = L\{R(t)\}$, we obtain

$$s^2\bar{u}^* = s\bar{R}\left(-q^2\bar{u}^* + \frac{3}{4}D^2\bar{u}^* + \frac{iq}{4}D\bar{w}^*\right) + iq(\bar{e}^* - \bar{N}^* - \bar{\vartheta}^*), \tag{30}$$

$$s^2\bar{w}^* = s\bar{R}\left(D^2\bar{w}^* - \frac{3}{4}q^2\bar{w}^* + \frac{iq}{4}D\bar{u}^*\right) + D(\bar{e}^* - \bar{N}^* - \bar{\vartheta}^*), \tag{31}$$

$$[s\bar{R}(D^2 - q^2) - s^2]\bar{e}^* = -(D^2 - q^2)(\bar{e}^* - \bar{N}^* - \bar{\vartheta}^*), \tag{32}$$

$$(D^2 - q^2)\bar{\vartheta}^* = s\left(1 + \frac{\tau^\alpha}{\alpha!}s^\alpha\right)(\bar{\vartheta}^* + \varepsilon\bar{e}^*) - \varepsilon_1\bar{N}^*, \quad 0 < \alpha \le 1, \tag{33}$$

$$\left(D^2 - q^2 - s^{\beta-1}\xi\left(s + \frac{1}{\upsilon}\right)\right)\bar{N}^* = -\frac{\varepsilon_2 s^{\beta-1}}{\upsilon}\bar{\vartheta}^*, \quad 0 < \beta \le 2, \tag{34}$$

$$\bar{\sigma}^*{}_{xx} = s\bar{R}\left(iq\bar{u}^* - \frac{1}{2}D\bar{w}^*\right) + \bar{e}^* - \bar{N}^* - \bar{\vartheta}^*, \tag{35}$$

$$\bar{\sigma}^*{}_{yy} = s\bar{R}\left(D\bar{w}^* - \frac{iq}{2}\bar{u}^*\right) + \bar{e}^* - \bar{N}^* - \bar{\vartheta}^*, \tag{36}$$

$$\bar{\sigma}^*{}_{zz} = \left(1 - \frac{1}{2}s\bar{R}\right)\bar{e}^* - \bar{N}^* - \bar{\vartheta}^*, \tag{37}$$

$$\bar{\sigma}^*{}_{xy} = \frac{3}{4}s\bar{R}\,(D\bar{u}^* + iq\bar{w}^*), \tag{38}$$

where

$$\bar{e}^* = iq\bar{u}^* + D\bar{w}^*, \tag{39}$$

and

$$D = \frac{\partial}{\partial y}, \quad \bar{R}(s) = L\{\hat{R}(t)\} = \frac{4\mu}{3K}\left(\frac{1}{s} - \frac{A\Gamma(a^*)}{s(s+\beta)^{a^*}}\right), \quad s > 0$$

where all the initial state functions are equal to zero.

Eliminating $\bar{e}^*$ and $\bar{N}^*$ between Eqs. (32), (33) and (34), we obtain

$$(D^6 - a_1D^4 + a_2D^2 - a_3)\bar{\vartheta}^*(q,y,s) = 0, \tag{40}$$

where

$$a_1 = 3q^2 + b,$$
$$a_2 = 3q^4 + 2bq^2 - as^2(b + as^2) + c,$$
$$a_3 = q^6 + bq^4 + c(q^2 + as^2)$$
$$b = s\left(1 + \frac{\tau^\alpha}{\alpha!}s^\alpha\right) + s^{\beta-1}\xi\left(s + \frac{1}{\upsilon}\right) - as^2,$$
$$c = s^\beta\xi\left(s + \frac{1}{\upsilon}\right)\left(1 + \frac{\tau^\alpha}{\alpha!}s^\alpha\right),$$
$$a = \frac{1}{1 + s\bar{R}}.$$

In a similar manner, we can show that $\bar{e}^*$ and $\bar{N}^*$ satisfy the equations

$$(D^6 - a_1D^4 + a_2D^2 - a_3)\bar{e}^*(q,y,s) = 0, \tag{41}$$

$$(D^6 - a_1D^4 + a_2D^2 - a_3)\bar{N}^*(q,y,s) = 0, \tag{42}$$

Eq. (40) can be factorized as

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)\bar{\theta}^*(q,y,s) = 0, \tag{43}$$

where $k_i^2\ (i = 1,2,3)$ are the roots of the following characteristic equation

$$k^6 - a_1k^4 + a_2k^2 - a_3 = 0. \tag{44}$$

The solution of Eq. (43) bounded for $y \ge 0$ has the form

$$\bar{\theta}^*(q,y,s) = \sum_{i=1}^{3}\zeta_i(q,s)e^{-k_iy}, \tag{45}$$

where $\zeta_i = \zeta_i(q,s)$, $i = 1,2,3$ are parameters depending on s and q. Note that the roots with negative real parts are not included in Eq. (45) since they are unbounded as $y \to \infty$. It is worth mentioning here that the roots k_1, k_2 and k_3 are functions of q and s.

Similarly, the solution of Eqs. (41) and (42) can be written as

$$\bar{e}^*(q,y,s) = \sum_{i=1}^{3}\zeta_i'(q,s)e^{-k_iy} \tag{46}$$

$$\bar{N}^*(q,y,s) = \sum_{i=1}^{3} \zeta_i^{''}(q,s) e^{-k_i y} \tag{47}$$

where ζ_i, $\zeta_i^{'}$ and $\zeta_i^{''}$ are parameters depending on q and s.

By substituting from Eqs. (45)–(47) into Eqs. (32), (33) and (34), we get

$$\zeta_i^{'} = A_i\ \zeta_i, \tag{48}$$

$$\zeta_i^{''} = B_i\ \zeta_i, \tag{49}$$

where

$$A_i = \frac{\left[k_i^2 - q^2 - s\left(1 + \frac{\tau^\alpha}{\alpha^!} s^\alpha\right)\right]\left[k_i^2 - q^2 - s^{\beta-1}\xi\left(s+\frac{1}{v}\right)\right] - \frac{\varepsilon_1\varepsilon_2}{v} s^{\beta-1}}{\varepsilon s\left(1 + \frac{\tau^\alpha}{\alpha^!} s^\alpha\right)\left(k_i^2 - q^2 - s^{\beta-1}\xi\left(s+\frac{1}{v}\right)\right)},$$

and

$$B_i = -\frac{\varepsilon_1\varepsilon_2 s^{\beta-1}}{v\left(k_i^2 - q^2 - s^{\beta-1}\xi\left(s+\frac{1}{v}\right)\right)}.$$

Substituting from Eq. (48) into (46) and Eq. (49) into (47), we have

$$\bar{e}^*(q,y,s) = \sum_{i=1}^{3} A_i\ \zeta_i e^{-k_i y}, \tag{50}$$

$$\bar{N}^*(q,y,s) = \sum_{i=1}^{3} B_i\ \zeta_i e^{-k_i y}. \tag{51}$$

Substituting Eq. (39) into Eq. (30), we obtain

$$(D^2 - \varphi)\bar{u}^*(q,y,s) = iq(\alpha_o \bar{\vartheta}^* + \alpha_o \bar{N}^* - \alpha_1 \bar{e}^*), \tag{52}$$

where

$$\alpha_o = \frac{4}{3s\bar{R}}, \quad \varphi = q^2 + \alpha_o s^2 \quad \text{and} \quad \alpha_1 = \alpha_o + \frac{1}{3}.$$

Substituting Eqs. (45), (50) and (51) into the R.H.S of Eq. (52) and solving the resulting differential equation, we get

$$\bar{u}^*(q,y,s) = Ce^{-\sqrt{\varphi}y} - iq\sum_{i=1}^{3} \frac{\zeta_i}{k_i^2 - \varphi}[\alpha_1 A_i - \alpha_o(1 + B_i)]e^{-k_i y}. \tag{53}$$

Substituting Eqs. (50) and (53) into Eq. (39) and integrating the resulting equation, we get

$$\bar{w}^*(q,y,s) = \frac{iq}{\sqrt{\varphi}} Ce^{-\sqrt{\varphi}y} - \sum_{i=1}^{3} \frac{\zeta_i}{k_i(k_i^2 - \varphi)}\ [A_i(k_i^2 - \varphi - \alpha_1) + \alpha_o q^2(1 + B_i)]e^{-k_i y}. \tag{54}$$

The stress components can be obtained by substituting from the above equations into Eqs. (35)-(38). Then

$$\bar{\sigma}^*{}_{xx} = \frac{2q}{\alpha_o} Ce^{-\sqrt{\varphi}y} + \sum_{i=1}^{3} \zeta_i \left[\left(1 + \frac{s\bar{R}}{2}\left(\frac{3\alpha_1}{k_i^2 - \varphi} - 1\right)\right)A_i - \left(1 + \frac{q^2}{k_i^2 - \varphi}\right)(1 + B_i)\right]e^{-k_i y}, \tag{55}$$

$$\bar{\sigma}^*{}_{yy} = -\frac{3iq}{2} Ce^{-\sqrt{\varphi}y} + \sum_{i=1}^{3} \zeta_i \left[\left(1 + s\bar{R} + \frac{3q^2\alpha_1}{2(k_i^2 - \varphi)}\right)A_i - \left(1 + \frac{3q^2\alpha_o}{2(k_i^2 - \varphi)}\right)(1 + B_i)\right]e^{-k_i y}, \tag{56}$$

$$\bar{\sigma}_{zz}{}^*(q,y,s) = \sum_{i=1}^{3} \left[\left(1 - \frac{s\bar{R}}{2}\right)A_i - (1 + B_i)\right]\ \zeta_i e^{-k_i y}, \tag{57}$$

$$\bar{\sigma}^*{}_{xy}(q,y,s) = \left(\frac{q^2 - \varphi}{\sqrt{\varphi}}\right)Ce^{-\sqrt{\varphi}y} + iq\sum_{i=1}^{3} \frac{\zeta_i}{k_i(k_i^2 - \varphi)}\ [A_i((1 + \alpha_1)k_i^2 - \varphi - \alpha_1) - \alpha_o(k_i^2 - q^2)(1 + B_i)]e^{-k_i y}. \tag{58}$$

The above approach gives the solution of the problem in the transformed domain in terms of four constants ζ_i, $i = 1, 2, 3$ and C which can be obtained from the boundary conditions of the articulate problem under consideration.

It is now possible to solve a broad class of two-dimensional problems of organic semiconductors with fractional order of heat and carrier density transfer.

5. Application

We consider a viscoelastic semiconductor occupying the semi-space region:

$$G = \{(x,y,z):\ y \geq 0,\ -\infty < x,\ z < \infty\}$$

and the other semi-space

$$G* = \{(x,y,z):\ y \leq 0,\ -\infty < x,\ z < \infty\}$$

is a vacuum. Let the surface of G is traction free and subjected to a heat flux with an exponentially decaying pulse.

The thermal and plasma boundary conditions at $y = 0$, are

$$-k\frac{\partial\vartheta(x,y,t)}{\partial y} = q_o \frac{t^2 e^{-\frac{t}{t_p}}}{16\ t_p^2}\ H(g - |y|) \quad \text{or} \quad \frac{\partial\bar{\vartheta}^*(q,y,s)}{\partial y} = -\frac{q_o}{8(st_p + 1)^3}\sqrt{\frac{2}{\pi}}\frac{\sin q\ g}{q}, \tag{59}$$

$$D_e \frac{\partial N(x,y,t)}{\partial y} - s_o N = 0$$
$$\text{or} \quad \left[\frac{\partial \bar{N}^*(q,y,s)}{\partial y} - \omega \bar{N}^*(q,y,s)\right]_{y=0} = 0. \tag{60}$$

The mechanical boundary conditions at $y = 0$, are

$$\sigma_{yy}(x,y,t) = \sigma_{xy}(x,y,t) = 0$$
$$\text{or} \quad \bar{\sigma}^*{}_{yy}(q,y,s) = \bar{\sigma}^*{}_{xy}(q,y,s), \tag{61}$$

where s_o is the speed of surface recombination, H is the step function of the Heaviside unit, q_o is a constant, t_p is a characteristic time of the pulsing heat flux and $\omega = s_o\xi/c_o$.

Using Eqs. (45), (51), (56) and (58) the conditions (59)-(61) reduce to

$$\sum_{i=1}^{3} k_i\zeta_i = \frac{q_o}{8(st_p+1)^3}\sqrt{\frac{2}{\pi}}\frac{sin\, q\, g}{q}, \tag{62}$$

$$\sum_{i=1}^{3} \zeta_i \left[\left(1 + s\bar{R} + \frac{3q^2\alpha_1}{2(k_i^2-\varphi)}\right)A_i - \left(1 + \frac{3q^2\alpha_o}{2(k_i^2-\varphi)}\right)(1+B_i)\right] = \frac{3iq}{2}C, \tag{63}$$

$$\sum_{i=1}^{3} \frac{\zeta_i}{k_i(k_i^2-\varphi)}\left[A_i((1+\alpha_1)k_i^2 - \varphi - \alpha_1) - \alpha_o(k_i^2 - q^2)(1+B_i)\right] = \left(\frac{q^2-\varphi}{iq\sqrt{\varphi}}\right)C, \tag{64}$$

$$\sum_{i=1}^{3} B_i\zeta_i\,(k_i - \omega) = 0. \tag{65}$$

By solving the four linear Eqs. (62)-(65), we can obtain the four constants $\zeta_i,\ i = 1,2,3$ and C.

This completes the solution of the problem in the Laplace –Fourier transformed domain.

We shall now outline the method used to invert the Laplace transforms in the above equations. Let $\bar{f}(x,y,s)$ be the Laplace transform of a function $f(x,y,t)$. The inversion formula for Laplace transforms can be written as Honig and Hirdes (1984)

$$f(x,y,t) = \frac{e^{ct}}{2\pi}\int_{-\infty}^{\infty} e^{ity}\,\bar{f}(x,y,c+iy)\,dy,$$

where c is an arbitrary real number greater than all the real parts of the singularities of $\bar{f}(x,y,s)$.

Expanding the function $h(x,y,t) = exp(-ct)f(x,y,t)$ in a Fourier series in the interval $[0, 2L]$, we obtain the approximate formula proposed by Honig and Hirdes (1984)

$$f(x,y,t) \approx f_N(x,y,t) = \frac{1}{2}c_0 + \sum_{k=1}^{N} c_k, \tag{66}$$
$$0 \le t \le 2L,$$

where

$$c_k(x,y,t) = \frac{e^{ct}}{L}Re\left[e^{\frac{ik\pi t}{L}}\,\bar{f}\left(x,y,c + \frac{ik\pi}{L}\right)\right], \tag{67}$$
$$k = 0,1,2,\ldots$$

Two methods are used to reduce the total error. First, the 'Korrektur' method is used to reduce the discretization error. Next, the ε-algorithm is used to reduce the truncation error and therefore to accelerate convergence.

The Korrektur-method uses the following formula to evaluate the function $f(x,y,t)$

$$f(x,y,t) = f_{NK}(x,y,t) = f_N(x,y,t) - e^{-2cL}f_{N'}(x,y,2L+t). \tag{68}$$

We shall now describe the ε-algorithm that is used to accelerate the convergence of the series in (66). Let N be an odd natural number and let $s_m = \sum_{k=1}^{m} c_k$, be the sequence of partial sums of (66). We define the ε-sequence by

$$\varepsilon_{0,m} = 0, \quad \varepsilon_{1,m} = s_m, \quad m = 1,2,3,\ldots$$

and

$$\varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + \frac{1}{(\varepsilon_{n,m+1} - \varepsilon_{n,m})},$$
$$n,m = 1,2,3,\ldots.$$

It can be shown from Ref. Honig and Hirdes (1984) that the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \ldots, \varepsilon_{N,1},\ldots$ converges to $f(x,y,t) - c_0/2$ faster than the sequence of partial sums.

6. Numerical results and discussions

We apply the above results to the Polymethyl Methacrylate (Plexiglas) material which has wide applications in industry and medicine. Following the values of physical constants are shown in Table 1.

Table 1: The physical constants

$\rho = 1.18 \times 10^3 g/cm^3$,	$C_E = 1.4x10^3\ J/kg.K$,
$\eta_0 = 3.36 \times 10^6 s/m^2$,	$T_o = 293\ K$,
$\alpha_T = 13 \times 10^{-5}\ K^{-1}$,	$\alpha_N = -9 \times 10^{-31}\ m^3$,
$c_o = 2200\ m/s$,	$\lambda = 453.7 \times 10^7 N/m^2$,
$\mu = 194 \times 10^7 N/m^2$,	$k = 0.55\ J/m.s.K$,
$s_o = 2ms^{-1}$,	$D_e = 2.5 \times 10^{-31}\ m^2s^{-1}$,
$n_o = 10^{20}m^{-3}$,	$A = 0.106$,
$\beta^* = 0.005$,	$\varepsilon = 0.12$,
$E_g = 1.12\ (eV)$,	$t_p = 1s$,
$\upsilon = 5 \times 10^{-5}s$,	$\tau = 2 \times 10^{-3}s$.

To compute the values of the functions, a numerical procedure was used to invert the double transforms in theabove expressions. First, a numerical method based on

Fourier expansion was used to invert the Laplace transforms proposed by Honig and Hirdes (1984). Next, the integrals appearing in the formula for the inverse of the Hankel transform were evaluated numerically by a subroutine "qint" from the book of Press *et al*. (1992). We use the Fortran 90 programming language on a personal computer with an I7 processor. Multiple parameters within the software specify the sum of the measurement (and therefore the run time). There is a 'nsig' parameter that is the number of essential digits that describe the relative error as $(10)^{-nsig}$, nsig = 5 is normal for us. Overall the program evaluates the temperature value in less than 2 min at 50 points (Sherief and Hussein 2018).

In the context of the photo-thermal theory under the fractional calculus, the analysis of the influence of fractional orders α, β on the behavior of an organic semiconductor plate was performed in the preceding pages. The computations were carried out for one value of time $t = 0.1$ and different values of fractional order parametersα with wide range $0 < \alpha \leq 1$ and β with wide range $0 < \beta \leq 2$, where $\alpha = 0$, $\beta = 1$ (corresponding to DCT-theory), $\alpha = 1.0$, $\beta = 1$ (corresponding to LS-theory) and $\alpha = 0.5$, $\beta = 0.5$ (corresponding to the new fractional model). For these results, we have two

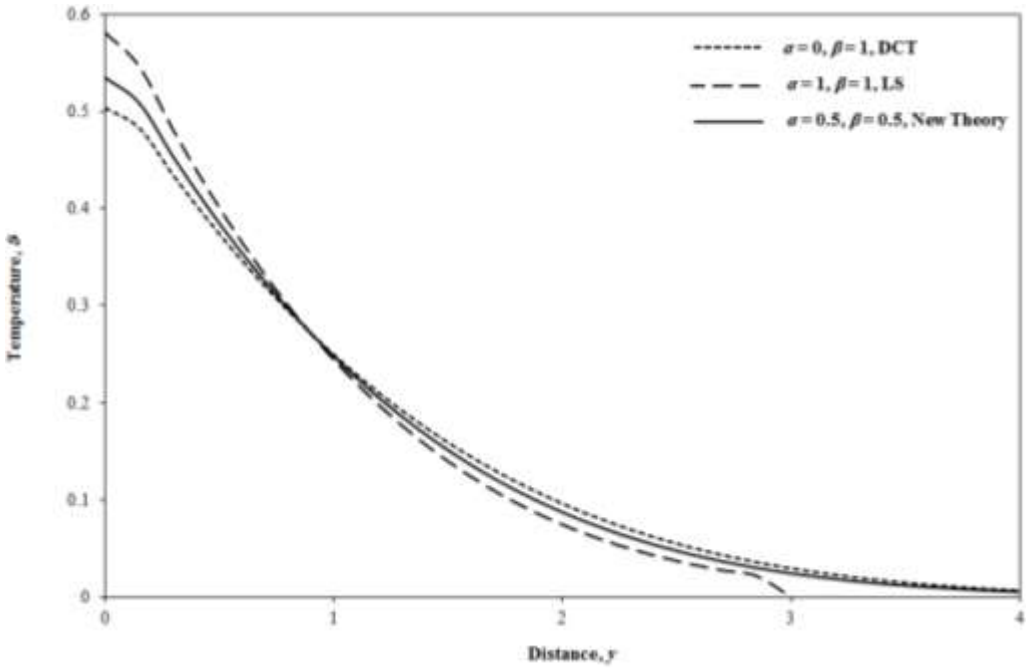

Fig. 1 The variation of temperature in an organic semiconductor plate for different theories at $x = 0.7$

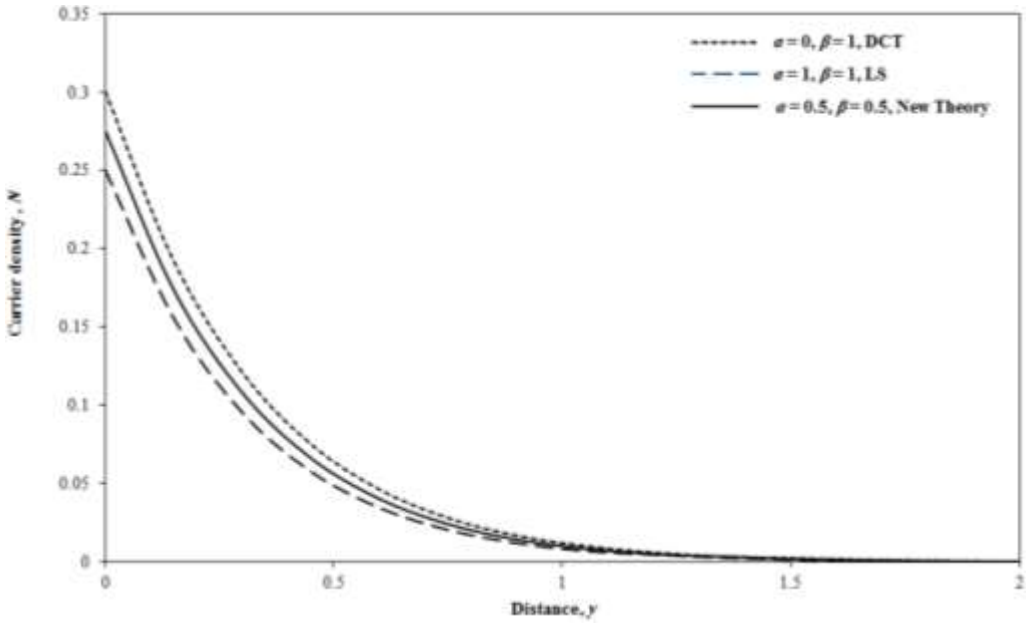

Fig. 2 The variation of carrier density in an organic semiconductor plate for different theories at $x = 0.7$

classes of graphs in relation to the distances x and y on all field quantities. The first group (Figs. 1-6) shows the effects of the fractional orders α, β on the temperature ϑ, the carrier density N, the horizontal displacement u and the vertical displacement component w, as well as the stress components σ_{yy}, σ_{xy} with respect to the distances y when $x = 0.7$, while the second group (Figs. 7-12) defines field quantity behavior with respect to distances x as $y = 0.2$. In Figs. 1-6, dotted lines represent the solutions for DCT-theory and dashed lines represent the solutions for LS–theory, while solid lines represent the solutions for the new fractional model. Comparisons are made with the findings that some previous theories have expected.

Fig. 1 shows the temperature distribution according to the new theory, which is compared with some previous theories. The temperature rises to the maximum value at the point $y = 0$, according to the values of α ,β and it gradually decreases with the increasing the distance $y(0 \leq y \leq 4)$. The important phenomenon in this figure showed that in a bounded region the temperature distribution in the fractional model is limited. The variations of this distribution do not occur beyond this area. This clearly shows the difference between the solution leading to the traditional use of the Fourier heat equation (Biot model, $\alpha = 1,\ \beta = 1$) and the use of generalized theories (LS-theory,$\alpha = 1,\ \beta = 1$) and the new fractional case ($0 < \alpha \leq$

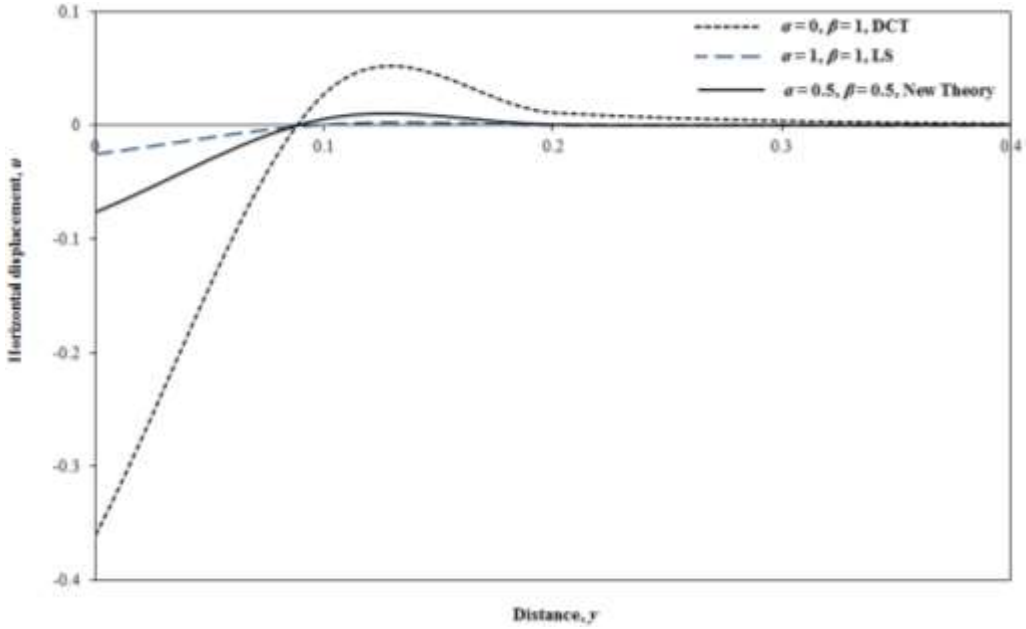

Fig. 3 The variation of horizontal displacement in an organic semiconductor plate for different theories at $x = 0.7$

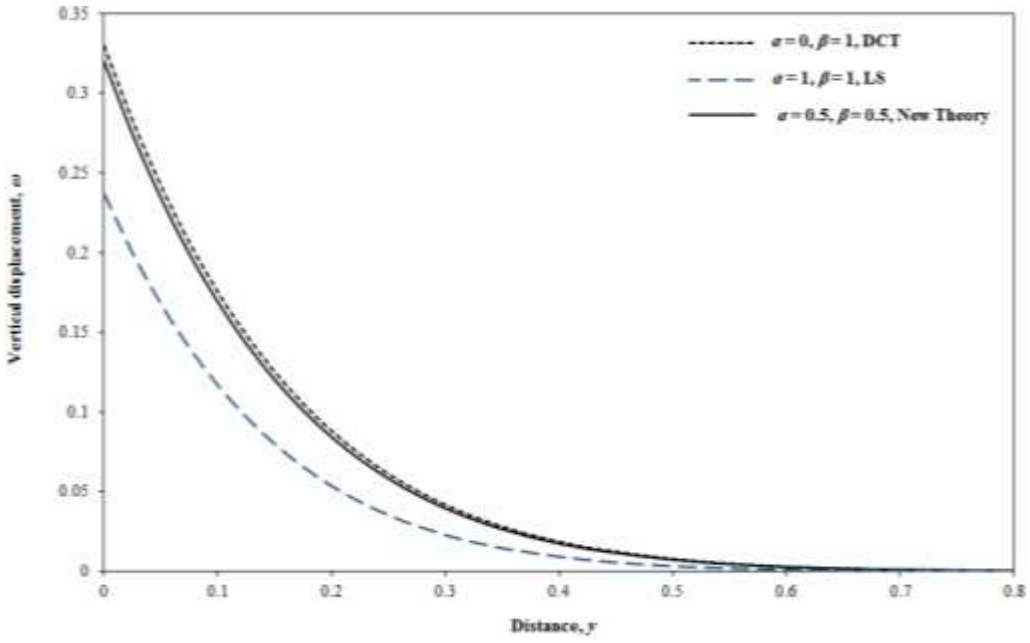

Fig. 4 The variation of vertical displacement in an organic semiconductor plate for different theories at $x = 0.7$

1, $0 < \beta \leq 2$).

Fig. 2 represents the variation of carrier density along to the distance $y(0 \leq y \leq 1.8)$. The carrier density is observed to have maximum values on $y = 0$, according to the values of α, β and decreases as y increases to near zero on $y = 1.8$.

Fig. 3 displays the difference of the horizontal displacement u of the distance y. It is noticed that the horizontal displacement attains maximum negative values and gradually increases until it attains a peak value at a particular location in close proximity to y = 0 and then continuously decreases to zero. It is noted that the horizontal displacement u approaches maximum negative values and increases slowly until it reaches a peak value in close proximity to y = 0 at a particular location and then eventually decreases to zero. Fig. 4 exhibits the space variation of the vertical displacement ω distribution. It is found that the vertical displacement ω has the same behavior as carrier density field except the wide range of y ($0 \leq y \leq 0.8$).

Figs. 5 and 6 are plotted to show the variation of stress components σ_{yy}, σ_{xy} against y. It observed that that the magnitude of normal stress σ_{yy} and shear stress σ_{xy}` always started from zero, which fulfilled the boundary condition and then increased by increasing the distance y to y = 0.72 and y = 0.19, respectively. After this increase, the gap y to close to zero decreases.

Next, the functions were evaluated as functions of x on

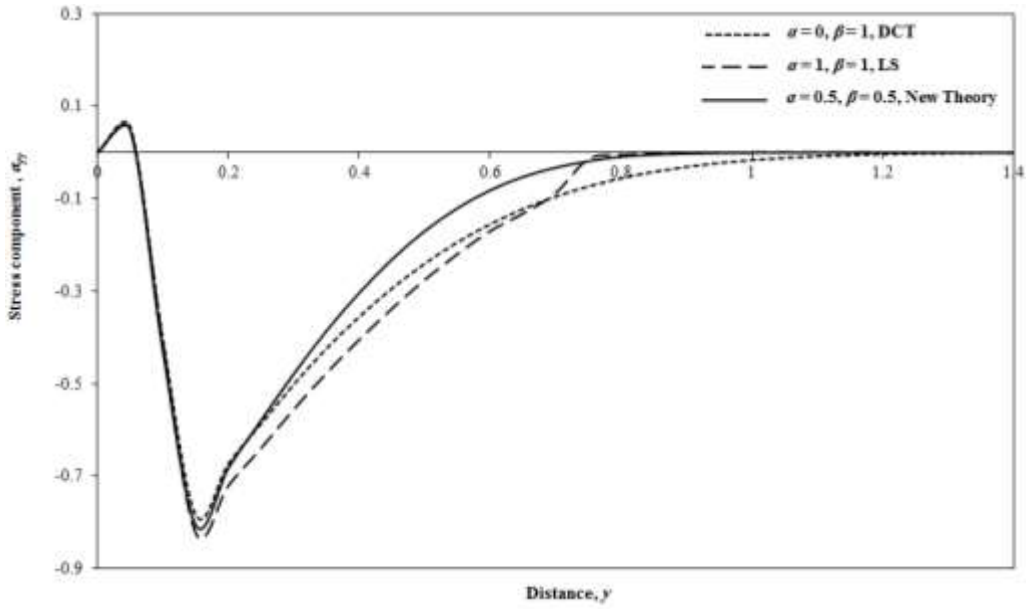

Fig. 5 The variation of stress component σ_{yy} in an organic semiconductor plate for different theories at x = 0.7

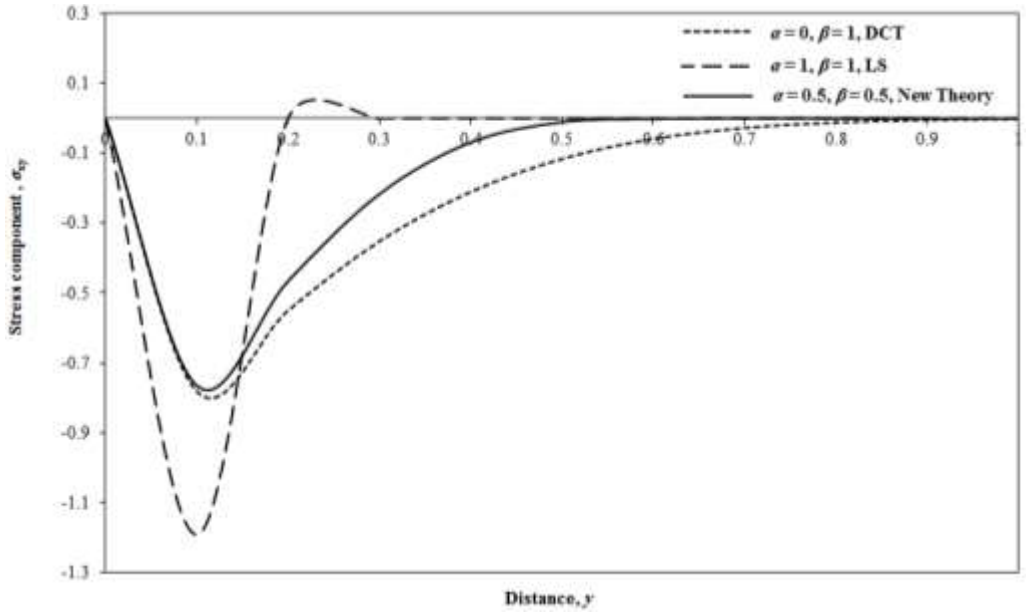

Fig. 6 The variation of stress component σ_{xy} in an organic semiconductor plate for different theories at x = 0.7

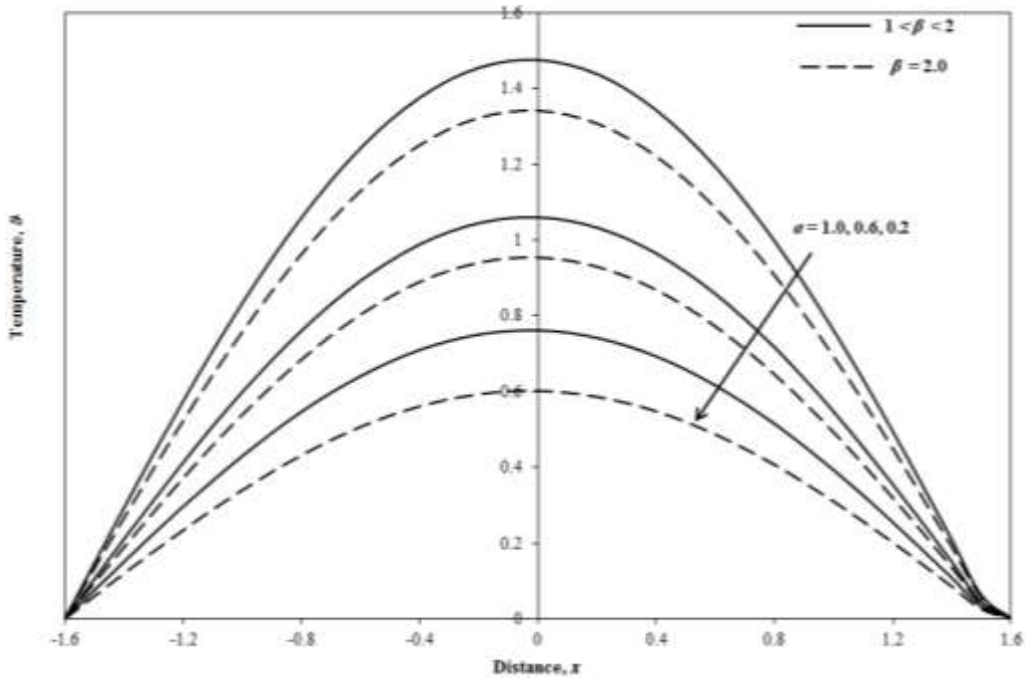

Fig. 7 The variation of temperature for different values of fractional order α and β at $y = 0.2$

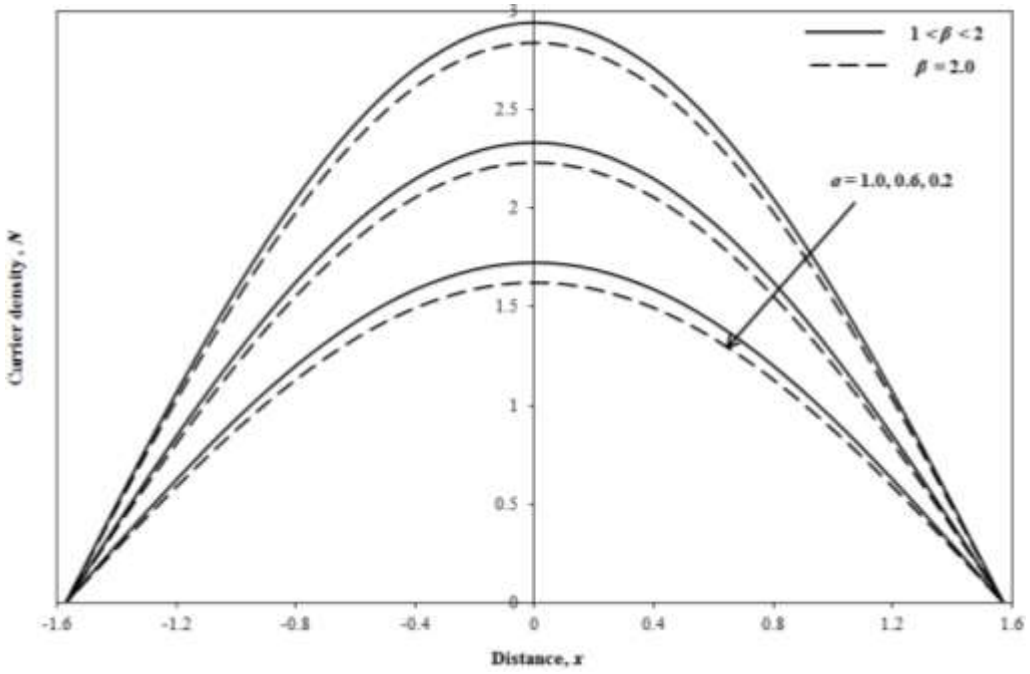

Fig. 8 The variation of carrier density for different values of fractional order α and β at $y = 0.2$

the plane surface $y = 0$.

Figs. 7 and 8 display the temperature ϑ and carrier density N distributions for wide range of $x(-1.6 \leq x \leq 1.6)$ at different values of the differential fractional order $0 < \alpha \leq 1$ in the limiting value of the integral fractional order $1 < \beta < 2$ (superconductivity) and $\beta = 2$ (ballistic conductivity)as in Ref. Kimmich (2002) . We have noticed that the increasing of the value of the parameter α causes decreasing in the temperature and carrier density. The temperature increment and carrier density increment are continuous function, which means that the particles transport the heat and plasma to the other particles easily and this makes the decreasing rate of the temperature greater than the other ones.

Fig. 9 exhibits the space variation of the horizontal displacement u distribution against x at different values of the differential fractional order α ($\alpha = 1.0,0.7,0.3$) and β ($1 < \beta < 2$). It is observed that the horizontal displacement u has the same behavior as temperature field except the wide range of $x(-4 \leq x \leq 4)$.

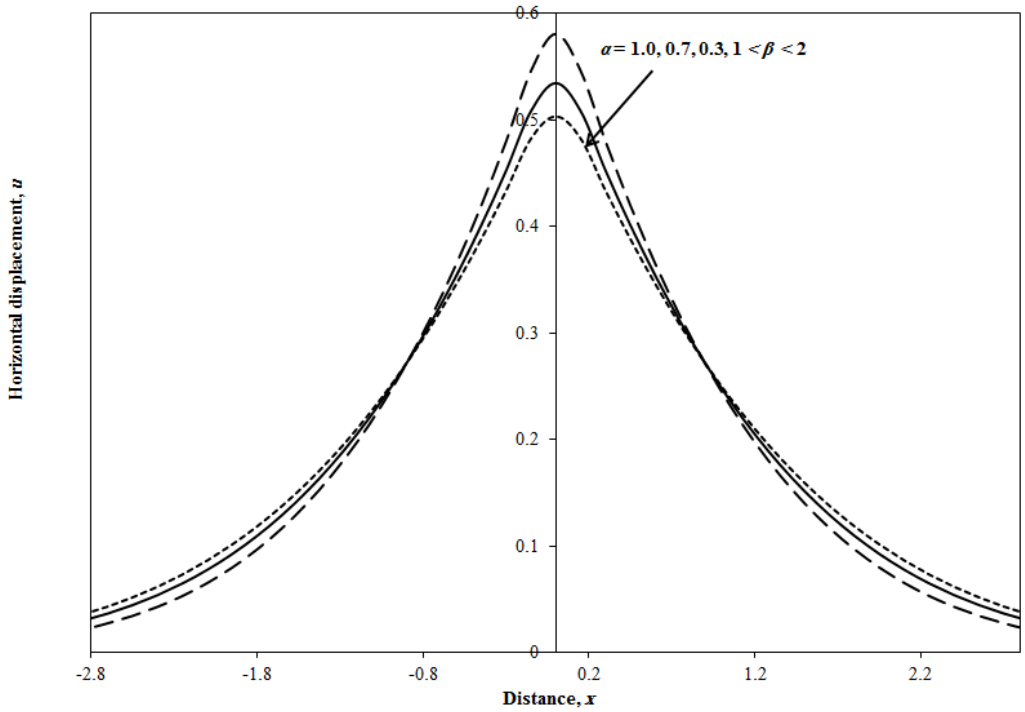

Fig. 9 The variation of horizontal displacement for different values of fractional order α at $y = 0.2$

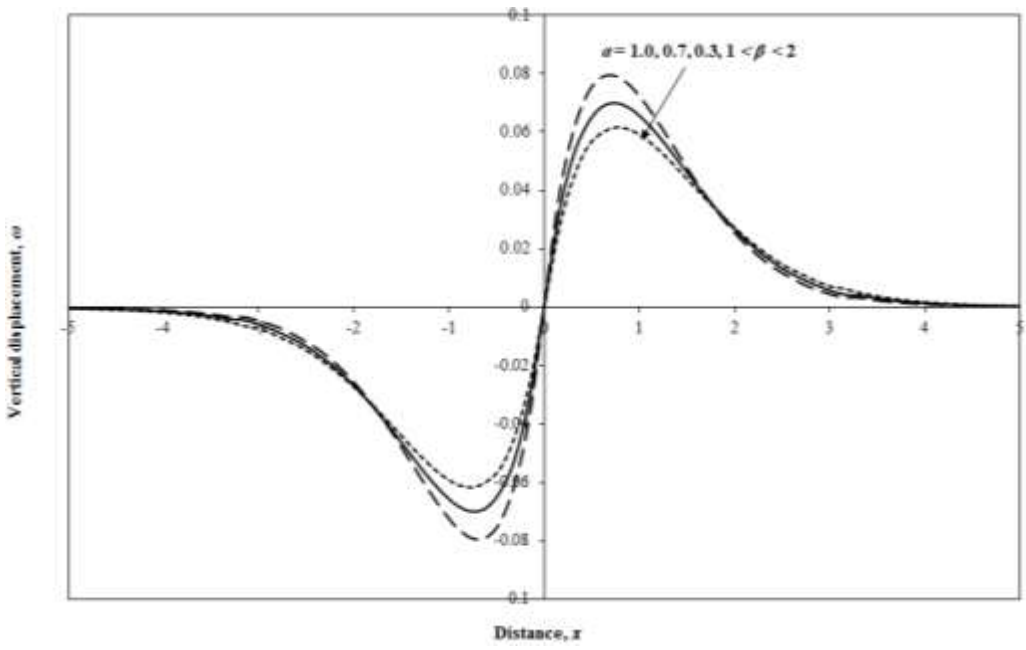

Fig. 10 The variation of vertical displacement for different values of fractional order α at $y = 0.2$

Fig. 10 predicts the variation of vertical displacement ω against x. It found that the vertical component of the displacement starts to decrease at the beginning and end of the heated surface $(-5 \leq x \leq 5)$ and has a minimum value in the center of the heated surface, after which it starts to increase and reaches a limit close to the edges $x = \pm 5$ and then decreases to zero. We noticed that vertical displacement ω field has been affected by the time-fractional parameter α; where the increasing of the value of the parameter α causes decreasing in the field.

Figs. 11 and 12 are displayed to show the variation of stress components σ_{yy}, σ_{xy} with respect to the distance x. From these figures we learned that thermal and mechanical waves are consistent capacities, smooth and enter unchanged state based on time-fractional parameters estimation.

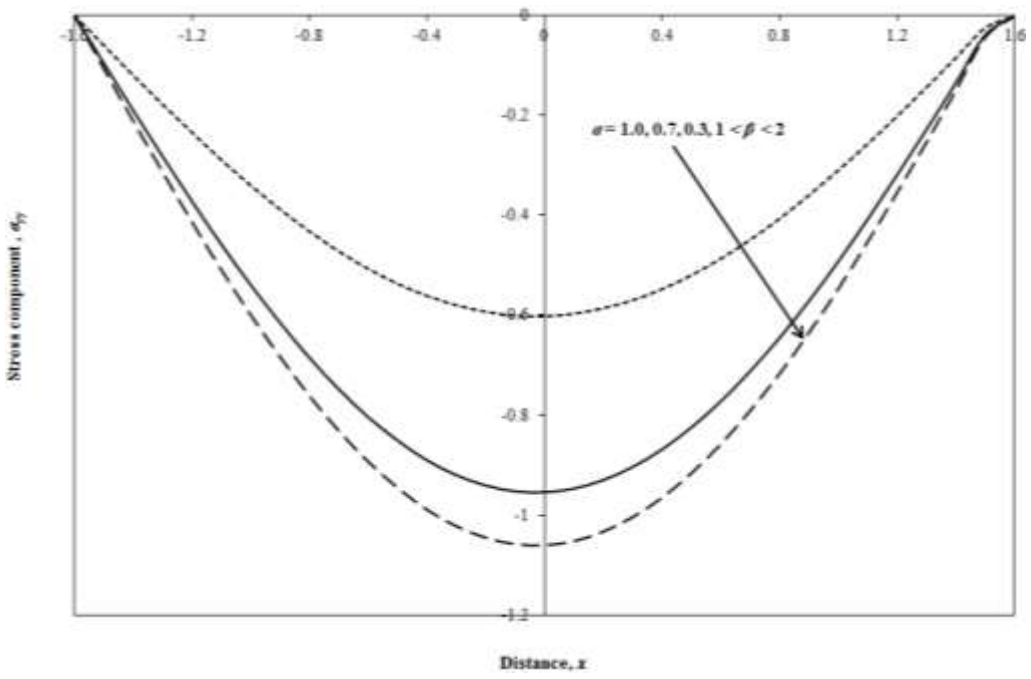

Fig. 11 The variation of stress component σ_{yy} for different values of fractional order α at $y = 0.2$

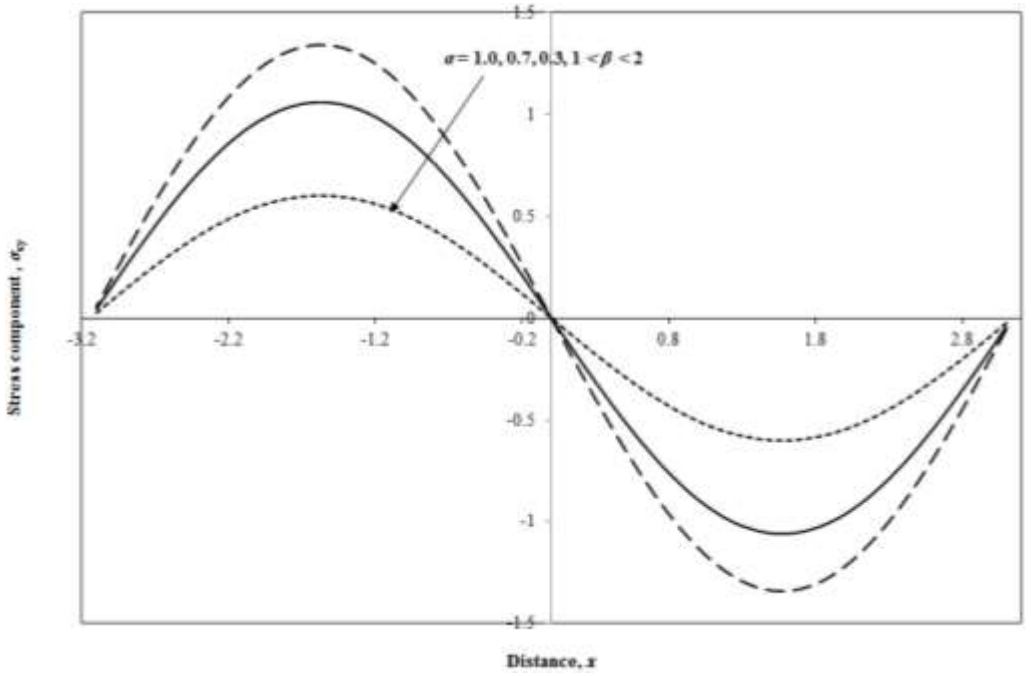

Fig. 12 The variation of stress component σ_{xy} for different values of fractional order α at $y = 0.2$

7. Conclusions

(1) A field of materials science related to the design, synthesis, characterization, and application of small organic molecules or polymers that exhibit desirable electronic properties such as electrical conductivity. In contrast to conventional inorganic and semiconductor conductors, organic electronic materials are made from small (carbon-based) organic molecules or polymers using synthetic strategies developed in the context of organic chemistry and polymer chemistry. One of the promised benefits of organic electronics is its potentially lower cost compared to traditional inorganic electronics.

(2) The attractive properties of organic semiconductors include the electrical conductivity that can vary with the concentrations of impurities. As for metals, they are characterized by mechanical flexibility. Some of them have high thermal stability.

(3) Any organic semiconductors are called viscoelastic

compounds. The influential organic semi-conductive deformations and the key driven mechanisms are the thermos-viscoelastic deformation and electronic deformation for micromechanical structures.

(4) All previous studies of semiconductors in the coupled and generalized theories were based on the consideration of the elastic medium and did not deal with the viscous elastic medium. In this paper, we dealt with the study of fractional heat and plasma propagation in viscoelastic semiconductors that have not been shown in literature before.

(5) Fractional calculus has been widely used to alter many existing models of physical systems. The use of fractional derivatives for viscoelastic materials and established the connections between fractional derivatives and the theory of linear viscoelasticity was good agreement with experimental findings from Caputo (1974) and Caputo and Mainardi (1971). The fractional model is used for a two-dimensional organic problem half-space semi-conductor.

(6) The results of all the functions for the new fractional model are distinctly different from those obtained in coupled (Biot 1956) and generalized theory (Lord and Shulman 1967).

(7) The fractional orders α and β have a significant effect on all fields in all figures.

(8) The waves for all distributions are observed to enter a stable state depending on the value of fractional orders α and β.

(9) The speed of the wave propagation in fractional thermo-viscoelastic variable fields according to the new model is finite like in the generalized theories and coincides with the physical behaviors of viscoelastic materials.

(10) The values of the distributions of all the physical quantities reach a steady state with an increase of the distance y and satisfy the boundary conditions.

(11) The waves quickly cut the y-axis when $\alpha = \beta = 1$ than when $0 < \alpha, \beta < 1$.

(12) The particles transport the heat and plasma to the other particles easily and this makes the decreasing rate of the temperature and carrier density greater than the other ones.

References

Abbas, I.A. (2006), "Natural frequencies of a poroelastic hollow cylinder", *Acta Mecc*., **186**(1-4), 229-237. https://doi.org/10.1007/s00707-006-0314-y

Abbas, I.A. (2014), "Nonlinear transient thermal stress analysis of thick-walled FGM cylinder with temperature-dependent material properties", *Mecc*., **49**(7), 1697-1708. https://doi.org/10.1007/s11012-014-9948-3

Abbas, I.A. and Abo-Dahab, S.M. (2014), "On the numerical solution of thermal shock problem for generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity", *J. Comput. Theor. Nanosci*., **11**(3), 607-618. https://doi.org/10.1166/jctn.2014.3402

Abbas, I.A. and Kumar, R. (2016), "2D deformation in initially stressed thermoelastic half-space with voids", *Steel Compos. Struct*., *Int. J*., **20**(5), 1103-1117. https://doi.org/10.12989/scs.2016.20.5.1103

Abbas, I.A. and Marin, M. (2017), "Analytical solution of thermoelastic interaction in a half-space by pulsed laser heating", *Phys. E*., **87**(3), 254-260. https://doi.org/10.1016/j.physe.2016.10.048

Adolfsson, K., Enelund, M. and Olsson, P. (2005), "On the fractional order model of visco-elasticity", *Mech. Time-Depend. Mat*., **9**(1), 15-34. https://doi.org/10.1007/s11043-005-3442-1

Alzahrani, F.S. and Abbas, I.A. (2016), "The effect of magnetic field on a thermoelastic fiber-reinforced material under GN-III theory", *Steel Compos. Struct*., *Int. J*., **22**(2), 369-386. https://doi.org/10.12989/scs.2016.21.4.791

Alzahrani, F.S. and Abbas, I.A. (2018), "Generalized photo-thermo-elastic interaction in a semiconductor plate with tworelaxation times", *Thin-Wall. Struct*., **129**, 342-348. https://doi.org/10.1016/j.tws.2018.04.011

Bagley, R.L. and Torvik, P.J. (1983), "A theoretical basis for the application of fractional calculus to viscoelasticity", *J. Rheol*., **27**(3), 201-210. https://doi.org/10.1122/1.549724

Biot, M.A. (1956), "Thermoelasticity and irreversible thermo-dynamics", *J. Appl. Mech. Tech. Phys*., **27**(3), 240-253. https://doi.org/10.1063/1.1722351

Caputo, M. (1974), "Vibrations on an infinite viscoelastic layer with a dissipative memory", *J. Acous. Soc. Am*., **56**(3), 897-904. https://doi.org/10.1121/1.1903344

Caputo, M. and Mainardi, F. (1971), "A new dissipation model based on memory mechanism", *Pure Appli. Geophys*., **91**, 134-147. https://doi.org/10.1007/BF00879562

Cattaneo, C. (1958), "Sur une forme de l'équation de la Chaleur éliminant le paradoxe d'une propagation instantaneée", *C.R. Acad. Sci. Paris*, **247**(3), 431-433.

Chandrasekharaiah, D.S. (1998), "Hyperbolic thermoelasticity: a review of recent literature", *Appl. Mech. Rev*., **51**(12), 705-729. https://doi.org/10.1115/1.3098984

Ezzat, M.A. (2001), "Free convection effects on perfectly conducting fluid", *Int. J. Eng. Sci*., **39**(7), 799-819. https://doi.org/10.1016/S0020-7225(00)00059-8

Ezzat, M.A. (2006), "The relaxation effects of the volume properties of electrically conducting viscoelastic material", *J. Mater. Sci. Eng. B*., **130**(1-3), 11-23. https://doi.org/10.1016/j.mseb.2006.01.020

Ezzat, M.A. and El-Bary, A.A. (2012), "MHD free convection flow with fractional heat conduction law", *MHD*, **48**(4), 587-606. https://doi.org/10.22364/mhd

Ezzat, M.A. and El-Bary, A.A. (2016), "Modeling of fractional magneto-thermoelasticity for a perfect conducting materials", *Smart Struct. Syst*., *Int. J*., **18**(4), 707-731. https://doi.org/10.12989/sss.2016.18.4.707

Ezzat, M.A., Othman, M.I. and El-Karamany, A.S. (2001), "State space approach to generalized thermo-viscoelasticity with two relaxation times", *Int. J. Eng. Sci*., **40**(3), 283-302. https://doi.org/10.1016/S0020-7225(01)00045-3

Ezzat, M.A., Alsowayan, N.S., Al-Muhiameed, Z.I.A. and Ezzat, S.M. (2014), "Fractional modelling of Pennes' bioheat transfer equation", *Heat Mass Trans*., **50**(7), 907-914. https://doi.org/10.1007/s00231-014-1300-x

Fujita, Y. (1990), "Integrodifferential equation which interpolates the heat equation and wave equation (I)", *Osaka J. Math*., **27**(2), 309-321. https://doi.org/10.18910/4060

Gordon, J.P., Leite, R.C.C., Moore, R., Porto, S.P.S. and Whinnery, J.R. (1965), "Long-transient effects in lasers with inserted liquid samples", *J. Appl. Phys*., **36**(1), 3-12. https://doi.org/10.1063/1.1713919

Gross, B. (1953), *Mathematical Structure of the Theories of Viscoelasticity*, Hemann, Paris, France.

Gurtin, M.E. and Sternberg, E. (1962), "On the linear theory of viscoelasticity", *Arch. Rat. Mech. Anal.*, **11**(1), 182-191. https://doi.org/10.1007/BF00253942

Hobiny, A. and Abbas, I.A. (2018), "Analytical solutions of photo-thermo-elastic waves in a non-homogenous semiconducting material", *Res. Phys.*, **10**, 385-390. https://doi.org/10.1016/j.rinp.2018.06.035

Honig, G. and Hirdes, U. (1984), "A method for the numerical inversion of the Laplace transform", *J. Compos. Appl. Math.*, **10**(1), 113-132. https://doi.org/10.1016/0377-0427(84)90075-X

Ignaczak, J. (1989), *Generalized Thermoelasticity and Its Applications*. (Ed., Hetnarski, R.B.), Thermal Stresses III, Elsevier, New York, USA.

Ilioushin, A.A. (1968), "The approximation method of calculating the constructors by linear thermal viscoelastic theory", *Mekhanika. Polimerov*, *Riga*, **2**, 168-178.

Ilioushin, A.A. and Pobedria, B.E. (1970), *Mathematical Theory of Thermal Viscoelasticity*, Nauka, Moscow, Russia.

Kimmich, R. (2002), "Strange kinetics, porous media, and NMR", *Chem. Phys.*, **284**(1-2), 243-285. https://doi.org/10.1016/S0301-0104(02)00552-9

Kiriyakova, V. (1994), "Generalized fractional calculus and applications", In: *Pitman Research Notes in Mathematics Series*, Volume 301, Longman-Wiley, New York, USA.

Kumar, R. and Abbas, I.A. (2013), "Deformation due to thermal source in micropolar thermoelastic media with thermal and conductive temperatures", *J. Comput. Theor. Nanosci.*, **10**(9), 2241-2247. https://doi.org/10.1166/jctn.2013.3193

Kumar, R. and Devi, S. (2017), "Thermoelastic beam in modified couple stress thermoelasticity induced by laser pulse", *Comput. Concrete*, **19**(6), 701-709. https://doi.org/10.12989/cac.2017.19.6.701

Kumar, R., Sharma, N. and Lata, P. (2016a), "Thermomechanical interactions in a transversely isotropic magnetothermoelastic with and without energy dissipation with combined effects of rotation, vacuum and two temperatures", *Appl. Math. Model.*, **40**(13-14), 6560-6575. https://doi.org/10.1016/j.apm.2016.01.061

Kumar, R., Sharma, N. and Lata, P. (2016b), "Effects of Hall current in a transversely isotropic magnetothermoelastic two temperature medium with rotation and with and without energy dissipation due to normal force", *Struct. Eng. Mech., Int. J.*, **57**(1), 91-103. https://doi.org/10.12989/sem.2016.57.1.091

Kumar, R., Sharma, N. and Lata, P. (2018), "Effects of Hall current and two temperatures in transversely isotropic magnetothermoelastic with and without energy dissipation due to Ramp type heat", *Mech. Adv. Mater. Struct.*, **24**(8), 625-635. https://doi.org/10.1080/15376494.2016.1196769

Kreuzer, L.B. (1971), "Ultralow gas concentration infrared absorption spectroscopy", *J. Appl. Phys.*, **42**(7), 2934-2943. https://doi.org/10.1063/1.1660651

Lata, P. (2018), "Effect of energy dissipation on plane waves in sandwiched layered thermoelastic medium", *Steel Compos. Struct., Int. J.*, **27**(4), 439-451. https://doi.org/10.12989/scs.2018.27.4.439

Lata, P., Kumar, R. and Sharma, N. (2016), "Plane waves in an anisotropic thermoelastic", *Steel Compos. Struct., Int. J.*, **22**(3), 567-587. https://doi.org/10.12989/scs.2016.22.3.567

Lord, H. and Shulman, Y. (1967), "A generalized dynamical theory of thermoelasticity", *J. Mech. Phys. Solids*, **15**(5), 299-309. https://doi.org/10.1016/0022-5096(67)90024-5

Lotfy, K., Kumar, R., Hassan, W. and Gabr, M. (2018), "Thermomagnetic effect with microtemperature in a semiconducting photothermal excitation medium", *Appl. Math. Mech.*, **39**(5),783-796. https://doi.org/10.1007/s10483-018-2339-9

Mainardi, F. and Gorenflo, R. (2000), "On Mittag-Lettler-type function in fractional evolution processes", *J. Comput. Appl. Math.*, **118**(1-2), 283-299. https://doi.org/10.1016/S0377-0427(00)00294-6

Itu, C., Ochsner, A., Vlase, S. and Marin, M.I. (2019), "Improved rigidity of composite circular plates through radial ribs", *Proceedings of the Institution of Mechanical Engineers*, *Part L*: *Journal of Materials*: *Design and Applications*, **233**(8), 1585-1593.

Meyers, M.A. and Chawla, K.K. (1999), *Mechanical Behavior of Materials*. Prentice-Hall, NJ, USA, Volume 98, pp. 103.

Mukhopadhyay, S. and Kumar, R. (2009), "Thermoelastic interactions on two-temperature generalized thermoelasticity in an infinite medium with a cylindrical cavity", *J. Therm. Stress.*, **32**(4)**,** 341-360. https://doi.org/10.1080/01495730802637183

Opsal, J. and Rosencwaig, A. (1985), "Thermal and plasma wave depth profiling in silicon", *Appl. Phys. Lett.*, **47**(5), 498-500. https://doi.org/10.1063/1.96105

Othman, M.I., Fekry, M. and Marin, M. (2020), "Plane waves in generalized magneto-thermo-viscoelastic medium with voids under the effect of initial stress and laser pulse heating", *Struct. Eng. Mech., Int. J.*, **73**(6), 621-629. https://doi.org/10.12989/sem.2020.73.6.621

Podlubny, I. (1999), *Fractional Differential Equations*, Academic: New York, USA.

Povstenko, Y.Z. (2005), "Fractional heat conduction equation and associated thermal stress", *J. Therm. Stress.*, **28**(1), 83-102. https://doi.org/10.1080/014957390523741

Press, W.H., Flannery, B.P., Teukolsky, S.A. and Vetterling, W.T. (1992), *Numerical Recipes*, (2nd ed.), Cambridge University Press, Cambridge, UK.

Sharma, S. and Sharma, K. (2014), "Influence of heat sources and relaxation time on temperature distribution in tissues", *Int. J. Appl. Mech. Eng.*, **19**(2), 427-433. https://doi.org/10.2478/ijame-2014-0029

Sharma, S., Sharma, K. and Bhargava, R. (2013), "Effect of viscosity on wave propagation in anisotropic thermoelastic with Green-Naghdi theory type-II and type-III", *Mat. Phys. Mech.*, **16**,144-158.

Sherief, H.H. and Abd El-Latief, M. (2016), "Modeling of variable Lamé's Modulii for a FGM generalized thermoelastic half space", *Lat. Am. J. Solids Struct.*, **13**(4), 715-730.

Sherief, H.H. and Ezzat, M.A. (1994), "Solution of the generalized problem of thermoelasticity in the form of series of functions", *J. Therm. Stress.*, **17**(1), 75-95. https://doi.org/10.1080/01495739408946247

Sherief, H.H. and Hussein, E.M. (2018), "Contour integration solution for a thermoelastic problem of a spherical cavity", *Appl. Math. Comput.*, **320**, 557-571. https://doi.org/10.1016/j.amc.2017.10.024

Sherief, H.H., El-Said, A. and Abd El-Latief, A. (2010), "Fractional order theory of thermoelasticity", *Int. J. Solid Struct.*, **47**(2), 269-275. https://doi.org/10.1016/j.ijsolstr.2009.09.034

Song, Y., Todorovic, D.M., Cretin, B. and Vairac, P. (2010), "Study on the generalized thermoelastic vibration of the optically excited semiconducting microcantilevers", *Int. J. Solids Struct.*, **47**(14-15), 1871-1875. https://doi.org/10.1016/j.ijsolstr.2010.03.020

Song, Y., Bai, J. and Ren, Z. (2012), "Reflection of plane waves in a semiconducting medium under photothermal theory", *Int. J. Thermophys.*, **33**(7), 1270-1287. https://doi.org/10.1007/s10765-012-1239-4

Sternberg, E. (1963), "On the analysis of thermal stresses in viscoelastic solids", *Brown University Providence RI DIV of Applied Mathematics*, **19**, 213- 219.

Tam, A.C. (1983), *Ultrasensitive Laser Spectroscopy* (Academic, New York), pp. 1-108.

Tam, A.C. (1989), *Photothermal Investigations in Solids and Fluids* (Academic, Boston), pp. 1-33

Todorović, D. (2003), "Plasma, thermal, and elastic waves in semiconductors", *Rev. Sci. Instrum*, **74**(1), 582-585. https://doi.org/10.1063/1.1523133

Youssef, H. (2010), "Theory of fractional order generalized thermoelasticity", *J. Heat Transf.*, **132**(1), 1-7. https://doi.org/10.1115/1.4000705

Yu, Y-J., Tian, X-G. and Tian, J-L. (2013), "Fractional order generalized electro-magnetothermo-elasticity", *Eur. J. Mech., A/Solids*, **42**,188-202. https://doi.org/10.1016/j.euromechsol.2013.05.006

Yu, Y-J., Hu, W. and Tian, X-G. (2014), "A novel generalized thermoelasticity model based on memory-dependent derivative", *Int. J. Eng. Sci.*, **81**, 123-134. https://doi.org/10.1016/j.ijengsci.2014.04.014.

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Nomenclature

x	Cartesian coordinates (x, y, z)
$\boldsymbol{\lambda, \mu}$	Lame's constants
ρ	density
t	time
E	excitation energy
E_g	energy gap of the semiconductor
N	$= n - n_o,\ n_o$ is the equilibrium carrier concentration
C_E	specific heat at constant strain
ζ	$= \frac{\partial n_o}{\partial \vartheta}$, coupling parameter of thermal activation
D_e	carrier diffusion coefficient
K	$= \lambda + (2/3)\mu$, bulk modulus
k	thermal conductivity
T	temperature
$\boldsymbol{S_{ij}}$	components of stress deviator tensor
σ_{ij}	components of stress tensor
e_{ij}	components of strain deviator tensor
ε_{ij}	components of strain tensor
u_i	components of displacement vector
α_T	coefficient of linear thermal expansion
α_N	coefficient of electronic deformation
c_o	$= [(\lambda + 2\mu)/\rho]^{1/2}$, speed of propagation of isothermal elastic waves
η_o	$= \rho C_E/k$
γ_T	$= 3K\alpha_T$
γ_N	$= 3K\alpha_N$
θ	$= T - T_0$, such that $\lvert\theta/T_0\rvert << 1$,
e	$= \varepsilon_{ii}$, dilatation
τ_o	thermal relaxation
α	fractional order
T_o	$= \rho c_o^2/\gamma_T$, reference temperature
ε	$= \gamma_T/\rho C_E$, thermoelastic coupling parameter
$R(t)$	relaxation function
A, β, a^*	empirical constants