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System simulation and synchronization for optimal evolutionary design of nonlinear controlled systems

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Abstract. Due to the influence of nonlinearity and time-variation, it is difficult to establish an accurate model of concrete frame structures that adopt active controllers. Fuzzy theory is a relatively appropriate method but susceptible to human subjective experience to decrease the performance. This paper proposes a novel artificial intelligence based EBA (Evolved Bat Algorithm) controller with machine learning matched membership functions in the complex nonlinear system. The proposed affine transformed membership functions are adopted and stabilization and performance criterion of the closed-loop fuzzy systems are obtained through a new parametrized linear matrix inequality which is rearranged by machine learning affine matched membership functions. The trajectory of the closed-loop dithered system and that of the closed-loop fuzzy relaxed system can be made as close as desired. This enables us to get a rigorous prediction of stability of the closed-loop dithered system by establishing that of the closed-loop fuzzy relaxed system.

Keywords: system simulation and synchronization; fuzzy relaxed system; intelligent algorithm

1. Introduction

The search for an effective management method to reduce structural responses to seismic operations and improve the structural capacity of earthquakes continues to be an important topic of construction research (Adeli and Jiang 2006, Zhou *et al*. 2015, Chen *et al*. 2019, Chen *et al*. 2020, Ying *et al.* 2019, Lee and Juang 2012, Battista *et al.* 2019). The scientific models of numerous physical and building frameworks are habitually of high measurement, or having intuitive unique marvels. The data preparing and prerequisites to explore different avenues regarding these models for control reasons for existing are typically over the top. Stability criteria so far has been drawn nearer in two primary ways as indicated by the reliance upon the measure of postponement. One basic property of control frameworks is steadiness and impressive reports have been issued in the writing on the soundness issue of unique frameworks (see Chen 2014a, b) and the references in that). Be that as it may, a writing overview shows that the soundness issue of frameworks with different time delays has not yet been settled. Therefore, according to general use, the stability criteria are based on the direct Lyapunov method to ensure stability.

State coupling makes the analysis of the system much more complex. Therefore, the problem of management response the system seems to be aggravated by a treatment inadequate due to the interaction of variables with different scale of time. (Sharkey and O'Reilly 1988, Razavi and Sarkar 2018, Eswaran and Reddy 2016, Hu *et al*. 2018, Berg *et al.* 2019). Because the design of a fuzzy controller is simpler than that of a conventional PID controller, fuzzy logic control has been suggested as an alternative approach to conventional control techniques for complex control systems. Hence, this study proposes a new approach that systematic method for adjusting the parameters of the fuzzy controller such that the closed-loop system is stable. Based on this approach, if the nonlinear singularly perturbed system cannot be stabilized, a dither, as an auxiliary of the controller, is injected into the nonlinear singularly perturbed system, and then the dither's parameters are regulated to make the nonlinear singularly perturbed system stable.

This paper is organized as follows: After the general introduction of Section I, preliminary notations throughout this study are defined in Section II. System description is represented in Section III. A novel approach to stabilize a nonlinear singularly perturbed system is proposed in Section IV. The design algorithm is shown in Section V. In Section VI, a numerical example is given to illustrate the feasibility of our approach. A conclusion is reached in Section VII.

2. Literature review

A few strategies for assessing solidness plans have been effectively connected, see Loria and Nesic (2003), Panteley and Loria (1998), Sontag and Wang (1995), and Sontag

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(1988). Computational insight methodologies, for example, neural systems and frameworks have additionally been utilized to show dynamics and applications in various regions. These apparatuses have turned out to be amazing and compelling. A few of the later works utilizing these methodologies can likewise be seen. Then again, swarm algorithms are likewise broadly used to build models of a framework or to find ideal solutions to issues in assembly, booking, and business coordination, background, and design. Panda *et al*. (2011) used CSO (see Chu and Tsai (2007)) to build a number of populace based learning rules for an interactive information retrieval (IIR) framework; Pardhan and Panda (2012) also used CSO to tackle numerous goal issues. In addition, Wang *et al*. (2012) utilized CSO to enhance data concealing outcomes. The IABC, which was proposed by Tsai *et al*. (2009), was effectively used to enhance the acknowledgment rate of the persistent confirmation framework (see Tsai *et al.* 2012, 2016) and to estimate the patterns outside the conversion scale; and EBA has been employed to provide the optimal recommended stock portfolio.

It has long been known that injecting a high frequency signal into a nonlinear system can improve performance. Better performance is considered to be less interference in the system output, better stability, and mitigation of limitations and jumping phenomena (Desoer and Shahruz 1985). Steinberg and Kadushin (1973) provided a precise analysis of stability in a general nonlinear system with control. It has been shown that the trajectory can be predicted by amplifying what corresponds to a model: a relaxed model.

Based on this approach, if the nonlinear singularly perturbed system cannot be stabilized, a dither, as an auxiliary of the controller, is injected into the nonlinear singularly perturbed system, and then the dither's parameters are regulated to make the nonlinear singularly perturbed system stable.

3. System description

3.1 Reduced model

Consider a nonlinear singularly perturbed system $S(0;\ 0)$ represented by the following equations

$$\dot{x} = f(x, y, u) \tag{1a}$$

$$\varepsilon\dot{y} = g(x, y, u)\,, \tag{1b}$$

where x is the state of the slow subsystem Eq. (1a), y is the state of the fast subsystem Eq. (1b) and u is the control vector. The nonlinear functions $f(.)$, $g(.)$ are continuously differentiable with respect to their arguments of any finite order and satisfy those assumptions of general continuity and boundedness given in Steinberg and Kadushin (1973). The parameter ε is a positive constant that represents the speed ratio of the slow versus the fast phenomena of the system.

By means of the singular perturbation approach, the reduced model of Eq. (1) is defined for $\varepsilon = 0$ as

$$\dot{x}_r = f(x_r, y_r, u_r) \tag{2a}$$

$$0 = g(x_r, y_r, u_r). \tag{2b}$$

From Eq. (2b), we obtain

$$y_r = J(x_r, u_r), \tag{3}$$

and substituted in Eq. (2a). Thus, the reduced model $\boldsymbol{S_r}(\mathbf{0};\ \mathbf{0})$ is described by

$$\dot{x}_r = \bar{f}(x_r, u_r). \tag{4}$$

The remainder of this section is divided into two parts: In Subsection 3.2, we establish the Takagi-Sugeno fuzzy model of the reduced model. In Subsection 3.3, the parallel-distributed-compensation scheme is used to design a fuzzy controller and a necessary and sufficient condition is given to determine whether the closed-loop fuzzy reduced system is stable.

3.2 Takagi-Sugeno (T-S) fuzzy model

The fuzzy dynamic TS model consists of IF-THEN rules that represent the local linear input / output relationship of a nonlinear system. This pattern is natural and simple. The structures of the system are taken with rules that characterize local pattern in the state space. The overall system pattern is obtained using a "combination" of linear system models. The *i*th rule of the fuzzy model of the reduced model $\boldsymbol{S_r}(\mathbf{0};\ \mathbf{0})$ is described in the following form:

Rule i:

$$\begin{array}{ll} \text{IF} & z_1(t) \text{ is } M_{i1} \text{ and } \cdots \text{ and } z_k(t) \text{ is } M_{ik} \\ \text{THEN} & \dot{x}_r(t) = A_i x_r(t) + B_i u_r(t)\,, \quad i = 1, 2, \cdots, g \end{array} \tag{5}$$

where

$$\begin{aligned} &x_r^T(t) = [x_{1r}(t), x_{2r}(t), \cdots, x_{nr}(t)] \\ &u_r^T(t) = [u_{1r}(t), u_{2r}(t), \cdots, u_{mr}(t)] \\ &x_r(t) \in R^n, \quad u_r(t) \in R^m, \quad A_i \in R^{n\times n}, \quad B_i \in R^{n\times m} \end{aligned}$$

g is the number of IF-THEN rules, M_{ij} $(j = 1, 2, \cdots, k)$ are fuzzy sets and $z_1(t) \sim z_k(t)$ are some measurable system variables, i.e., the premise variables. It is assumed in this study that the premise variables do not depend on the input variables $u(t)$. This assumption is employed to avoid a complicated defuzzification process of fuzzy controllers (Tanaka *et al.* 1996).

Given a pair of $(x_r(t), u_r(t))$, the final output of the fuzzy reduced model $F_r(0;\ 0)$ is inferred as follows

$$\begin{aligned} \dot{x}_r(t) &= \frac{\sum_{i=1}^{g} w_i(z(t))\{A_i x_r(t) + B_i u_r(t)\}}{\sum_{i=1}^{g} w_i(z(t))} \\ &= \sum_{i=1}^{g} h_i(z(t))\{A_i x_r(t) + B_i u_r(t)\} \end{aligned} \tag{6}$$

Definition 3.1: If the pairs $(A_i \quad B_i)$, i = 1, 2, …, g are controllable, the fuzzy reduced model Eq. (6) is called locally controllable.

Here, it is assumed that the fuzzy reduced model Eq. (6) is locally controllable. The local state feedback controllers are designed based on the pairs $(A_i \quad B_i)$ in the following subsection.

3.3 Fuzzy controller

The concept of Parallel Distributed Compensation (PDC) is used to create a stable pattern. A single controller in each line shares the same sets with the pattern. Since each rule in the fuzzy model of TS is described as a linear input-output relationship, linear control methods can be used to design a complicated fit for each rule. The resulting controller, which is generally nonlinear (Wang *et al.* 1996), is also a mix of any linear controller.

The *i*th rule of the fuzzy controller is described as follows

Rule i:

$$\begin{array}{ll} \text{IF} & z_1(t) \text{ is } M_{i1} \text{ and } \cdots \text{ and } z_k(t) \text{ is } M_{ik} \\ \text{THEN} & u_r(t) = -F_i x_r(t) \quad i = 1,2,\dots,g. \end{array} \tag{7}$$

Hence, the final output of this fuzzy controller is

$$\begin{aligned} u_r(t) &= \frac{\sum_{i=1}^{g} w_i\big(z(t)\big)F_i x_r(t)}{\sum_{i=1}^{g} w_i\big(z(t)\big)} \\ &= -\sum_{i=1}^{g} h_i\big(z(t)\big)F_i x_r(t). \end{aligned} \tag{8}$$

Substituting Eq. (8) into Eq. (6), we can derive the closed-loop fuzzy reduced system $F_r(C;\ 0)$

$$\dot{x}_r(t) = \textstyle\sum_{i=1}^{g}\sum_{j=1}^{g} h_i\,(z(t))h_j(z(t))\{A_i - B_iF_j\}\,x_r(t). \tag{9}$$

Remark 3.1: In this study, the feedback gains $F_i^{'}s$ are obtained by solving Riccati equations for local optimal control.

Before proceeding to analyzing the stability of $F(C;\ 0)$, a robust stability condition that is presented by Tanaka *et al.* (1996) is recalled as follows.

Lemma 3.1 (Tanaka *et al.* 1996): A nonlinear system $\dot{x}(t) = \{A_0 + D\Delta(t)E\}x(t)$ (where $\Delta(t) = diag[\vartheta_1(t)\ \vartheta_2(t)\cdots\vartheta_q(t)]$ and $\sigma_i \le \vartheta_i(t) \le \delta_i$, i = 1, 2, …, q) is quadratically stable if and only if the following conditions hold

i) $A_0 + DOE$ is a stable matrix (10)

ii) $\|E(sI - A_0 - DOE)^{-1}DQ\|_\infty < 1,$ (11)

in which

$$\begin{aligned} O &\equiv diag[\frac{\delta_1+\sigma_1}{2} \quad \frac{\delta_2+\sigma_2}{2} \quad \cdots \quad \frac{\delta_q+\sigma_q}{2}] \\ Q &\equiv diag[\frac{\delta_1-\sigma_1}{2} \quad \frac{\delta_2-\sigma_2}{2} \quad \cdots \quad \frac{\delta_q-\sigma_q}{2}]. \end{aligned}$$

Let G be a matrix which has desired eigenvalues and it is independent of i, j and then Eq. (9) can be rewritten as follows

$$\begin{aligned} \dot{x}_r(t) &= \sum_{i=1}^{g}\sum_{j=1}^{g} h_i\big(z(t)\big)h_j\big\{A_i - B_iF_j - G + G\big\} \\ &= Gx_r(t) + \sum_{i=1}^{g}\sum_{j=1}^{g} h_i\big(z(t)\big)h_j\big(z(t)\big) \\ &\qquad \big\{A_i - B_iF_j - G\big\}x_r(t). \end{aligned} \tag{12}$$

Defining

$$\begin{aligned} &\Delta G_{ij} \equiv A_i - B_iF_j - G \\ &\Delta T_{ij} \equiv \Delta G_{ij} + \Delta G_{ji}, \qquad i<j \end{aligned}$$

the closed-loop fuzzy reduced system $F_r(C;\ 0)$ can be converted into the following form

$$\begin{aligned} \dot{x}_r(t) &= Gx_r(t) + \sum_{i=1}^{g}\sum_{j=1}^{g} h_i\big(z(t)\big)h_j\big(z(t)\big)\Delta G_{ij}x_r(t) \\ &= Gx_r(t) + \sum_{i=1}^{g} h_i\big(z(t)\big)h_j\big(z(t)\big)\Delta G_{ii}x_r(t) \\ &\quad + \sum_{i<j}^{g} h_i\big(z(t)\big)h_j\big(z(t)\big)\Delta T_{ii}x_r(t). \end{aligned} \tag{13}$$

In addition, G can be chosen to be

$$G = \frac{1}{g}\sum_{l=1}^{g}(A_l - B_lF_l). \tag{14}$$

Using the singular value decomposition technique, ΔG_{ii} and ΔT_{ij} can be decomposed as follows

$$\begin{aligned} &\Delta G_{ii} = U_{ii}S_{ii}V_{ii}^T \\ &\Delta T_{ij} = U_{ij}S_{ij}V_{ij}^T, \qquad i<j \end{aligned}$$

where U and V are unitary matrices. By introducing the U, S and V matrices, Eq. (13) can be represented in the following form

$$\dot{x}_r(t) = \{G + \bar{D}\bar{\Delta}(t)\bar{E}\}x_r(t) \tag{15}$$

where

$$\begin{aligned} &\bar{D} = \big[\bar{U}_1 \quad \bar{U}_2 \quad \cdots \quad \bar{U}_g\big] \\ &\bar{E} = \big[\bar{V}_1 \quad V_2 \quad \cdots \quad \bar{V}_g\big]^T \\ &\bar{\Delta}(t) = block(diag\big[\bar{S}_1^e\big(z(t)\big) \quad \bar{S}_2^e\big(z(t)\big) \quad \cdots \quad \bar{S}_g^e\big(z(t)\big)\big]) \end{aligned}$$

and

$$\begin{aligned} &\bar{U}_i = [U_{ii} \quad U_{ii+1} \quad \cdots \quad U_{ig}] \\ &\bar{V}_i = [V_{ii} \quad V_{ii+1} \quad \cdots \quad V_{ig}]^T \\ &\bar{S}_i^e(z(t)) = block(diag[e_{ii}\big(z(t)\big)S_{ii} \\ &\qquad\qquad e_{ii+1}\big(z(t)\big)S_{ii+1} \quad \cdots \quad e_{ig}(z(t))S_{ig}]) \\ &e_{ij}(z(t)) = h_i(z(t))h_j(z(t)). \end{aligned}$$

On the other hand, from the matrix $\bar{\Delta}(t)$, if we choose $\sigma_i = 0$, then the matrices $\bar{O}$ and $\bar{Q}$ are described as follows

$$\bar{O} = \bar{Q} = block(diag[\bar{S}_1^d \quad \bar{S}_2^d \quad \cdots \quad \bar{S}_g^d])$$

where

$$\bar{S}_i^d = block\left(diag\left[\frac{d_{ii}}{2} S_{ii} \quad \frac{d_{ii+1}}{2} S_{ii+1} \quad \cdots \quad \frac{d_{ig}}{2} S_{ig}\right]\right)$$
$$d_{ij} = \max_{z(t)} h_i(z(t)) h_j(z(t)).$$

Lemma 3.2 (Tanaka *et al.* 1996): The closed-loop fuzzy reduced system $F_r(C;\ 0)$ is quadratically stable if and only if the following conditions hold

i) $G + \bar{D}\bar{O}\bar{E}$ is a stable matrix, (16)

ii) $\|\bar{E}(sI - G - \bar{D}\bar{O}\bar{E})^{-1}\bar{D}\bar{Q}\|_\infty \equiv \rho < 1.$ (17)

According to the stability conditions, which are addressed in Lemma 3.2, the closed-loop fuzzy reduced system $F_r(C;\ 0)$ is classified into two types as follows.

Type 1: $F_{1r}(C;\ 0)$. If ρ is less than unity and the matrix $G + \bar{D}\bar{O}\bar{E}$ is a stable matrix, the closed-loop fuzzy reduced system $F_{1r}(C;\ 0)$ is stable.

Type 2: $F_{2r}(C;\ 0)$. If ρ is not less than unity or if the matrix $G + \bar{D}\bar{O}\bar{E}$ is not a stable matrix, a dither, as an auxiliary of the fuzzy controller, is injected into the nonlinear singularly perturbed system $S(0;\ 0)$.

In the remainder of this study, attention is paid to the stability analysis of $F_{2r}(C;\ 0)$.

4. T-S fuzzy relaxed model and stability analysis

In this section, we address a new approach to make $F_{2r}(C;\ 0)$ stable without changing the feedback gain F_i of the fuzzy controller.

4.1 Reduced model and relaxed model

A nonlinear singularly perturbed system with an added dither is normally called a dithered system $S(0;\ d)$, and it is described as follows

$$\dot{x} = f(x, y, u, d) \tag{18a}$$

$$\varepsilon\dot{y} = g(x, y, u, d). \tag{18b}$$

Using the singular perturbation approach, the dithered reduced model of Eq. (18) is defined for $\varepsilon = 0$ as

$$\dot{x}_r = f(x_r, y_r, u_r, d) \tag{19a}$$

$$0 = g(x_r, y_r, u_r, d)\,. \tag{19b}$$

From Eq. (19b), we obtain

$$y_r = \hat{s}(x_r, u_r, d), \tag{20}$$

and substituted in Eq. (19a). Thus, the dithered reduced model is described by

$$\dot{x}_r = \hat{f}(x_r, u_r, d). \tag{21}$$

The algorithm for constructing a dither signal is given as follows (Steinberg and Kadushin 1973): The time interval $[0,\ \mathrm{T}]$ is divided into an arbitrary number ϕ of equal subintervals. The beginning of the first interval, the end of the first interval, the end of the second interval and the end of the ϕth interval are denoted by $t_0,\ t_1,\ t_2$ and t_ϕ respectively. Dividing every interval $[t_p,\ t_{p+1}]$ for $p = 0, 1,\ 2,\ \ldots,\ \phi - 1$ into τ subintervals, the length of the mth subinterval will be $\alpha_m(t_p)\ [t_{p+1} -\ t_p]$ for $m = 1,\ 2,\ \ldots,\ \tau$ and the control $\beta_m\ (t_p)$ is applied at the mth subinterval.

Hence, the repetition frequency, shape and amplitude of dither can be determined by regulating the parameters ϕ, $\alpha_m(t_p)$ and $\beta_m(t_p)$.

4.2 T-S fuzzy relaxed model

In this subsection, the T-S fuzzy model of the relaxed reduced model $S_{rR}(0;\ 0)$ is reconstructed. The ith rule of the fuzzy relaxed reduced model is described in the following form

Rule i:

$$\begin{aligned} &\text{IF} \quad z_{R1}(t) \text{ is } M_{Ri1}(\alpha_m, \beta_m) \text{ and ... and } z_{Rk}(t) \text{ is } M_{Rik}(\alpha_m, \beta_m) \\ &\text{THEN} \quad \dot{x}_{rR}(t) = A_i(\alpha_m, \beta_m) x_{rR}(t) + B_i(\alpha_m, \beta_m) u_{rR}(t), \quad i = 1, 2, \cdots, g. \end{aligned} \tag{22}$$

Similarly, the overall fuzzy relaxed reduced model $F_{rR}(0;\ 0)$ is described as follows

$$\dot{x}_{rR}(t) = \sum_{i=1}^{g} h_i(z_R(t), \alpha_m, \beta_m) \{A_i(\alpha_m, \beta_m) x_{rR}(t) + B_i(\alpha_m, \beta_m) u_{rR}(t)\}. \tag{23}$$

Using the PDC scheme allows us to derive the fuzzy controller to stabilize the fuzzy relaxed reduced model $F_{rR}(0;\ 0)$. The ith rule of fuzzy controller is described as follows

Rule i:

$$\begin{aligned} &\text{IF} \quad z_{R1}(t) \text{ is } M_{Ri1}(\alpha_m, \beta_m) \text{ and ... and } z_{Rk}(t) \text{ is } M_{Rik}(\alpha_m, \beta_m) \\ &\text{THEN} \quad u_{rR}(t) = -F_i x_{rR}(t), \quad i = 1, 2, \ldots, g. \end{aligned} \tag{24}$$

Hence, the final output of this fuzzy controller is

$$u_{rR}(t) = -\sum_{i=1}^{g} h_i(z_R(t), \alpha_m, \beta_m) F_i\, x_{rR}(t). \tag{25}$$

The feedback gain F_i of the ith rule of fuzzy controller is the same as that in Eq. (7).

Substituting Eq. (25) into Eq. (24), the closed-loop fuzzy relaxed reduced system $F_{rR}(C;\ 0)$ is represented as follows

$$\dot{x}_{rR}(t) = \sum_{i=1}^{g}\sum_{j=1}^{g} h_i(z_R(t), \alpha_m, \beta_m) h_j(z_R(t), \alpha_m, \beta_m) \{A_i(\alpha_m, \beta_m) - B_i(\alpha_m, \beta_m) F_j\} x_{rR}(t). \tag{26}$$

4.3 Stability criterion

Hereafter, we are concerned with stability of the closed-loop fuzzy relaxed reduced system $F_{rR}(C;\ 0)$ instead of discussing that of the closed-loop dithered system $S(C;\ d)$. A stability criterion of $F_{rR}(C;\ 0)$ is presented in the following theorem.

Theorem 4.1: The closed-loop fuzzy relaxed reduced system $F_{rR}(C;\ 0)$ is quadratically stable if and only if the following conditions hold

i) $G(\alpha_m,\beta_m)+\bar{D}(\alpha_m,\beta_m)\bar{O}(\alpha_m,\beta_m)\bar{E}(\alpha_m,\beta_m)$ is a stable matrix, (27)

ii) $$\begin{aligned}&\|\bar{E}(\alpha_m,\beta_m)(sI-G(\alpha_m,\beta_m)\\&-\bar{D}(\alpha_m,\beta_m)\bar{O}(\alpha_m,\beta_m)\bar{E}(\alpha_m,\beta_m)^{-1}\\&\bar{D}(\alpha_m,\beta_m)\bar{Q}(\alpha_m,\beta_m)\|_\infty\\&\equiv \rho(\alpha_m,\beta_m)<1.\end{aligned} \tag{28}$$

The proof of the above theorem can be similarly derived by following the same procedure as that in the proof of Theorem 4.1 of Tanaka *et al*. (1996) but with $\bar{G}$, $\bar{D}$, $\bar{E}$, $\bar{O}$ and $\bar{Q}$ being replaced by $\bar{G}(\alpha_m,\beta_m)$, $\bar{D}(\alpha_m,\beta_m)$, $\bar{E}(\alpha_m,\beta_m)$, $\bar{O}(\alpha_m,\beta_m)$ and $\bar{Q}(\alpha_m,\beta_m)$. This proof is lengthy, so it is not repeated here.

From the stability analysis above, the parameters α_m and β_m can be chosen to fulfill the requirements in Theorem 4.1. In other words, an appropriate dither may be chosen to guarantee that the closed-loop fuzzy relaxed reduced system $F_{rR}(C;\ 0)$ is stable.

5. System algorithm

To ensure the stability of the linear complexes of the solution, the T-S system models as well as the stability analysis are continued. For designing controls, complex systems represent T-S types . The term PDC described is used to determine the control structures associated with patterns in this section. At first, the i-th rule of the T–S fuzzy model is in the following:

Rule i:

IF $x_1(t)$ is M_{i1} and $\cdots$ and $x_p(t)$ *is* M_{ip}
THEN $\dot{X}(t)=A_iX(t)+B_iU(t)+E_i\varphi(t)$,

where $i=1,2,\dots,r$ and r is the rule number; $X(t)$ is the state vector; M_{ip} $(p=1,2,\dots,g)$ are the fuzzy sets and $x_1(t)-x_p(t)$ are the premise variables. By using the fuzzy inference method with a singleton fuzzifier, product inference, and center average defuzzifier, the dynamic fuzzy model can be expressed as follows

$$\begin{aligned}\dot{X}(t)&=\frac{\sum_{i=1}^r w_i(t)[A_iX(t)+B_iU(t)+E_i\varphi(t)]}{\sum_{i=1}^r w_i(t)}\\&=\sum_{i=1}^r h_i(t)[A_iX(t)+B_iU(t)]+E_i\varphi(t)\end{aligned}$$

with $w_i(t)=\prod_{p=1}^g M_{ip}[x_p(t)]$, $h_i(t)=\frac{w_i(t)}{\sum_{i=1}^r w_i(t)}$, $M_{ip}[x_p(t)]$ is the grade of membership of $x_p(t)$ in M_{ip}. It is assumed that $w_i(t)\geq 0$, $i=1,2,\dots,r$; $\sum_{i=1}^r w_i(t)>0$ for all t. Therefore, $h_i(t)\geq 0$ and $\sum_{i=1}^r h_i(t)=1$ for all t.

In order to design a global controller for the T–S fuzzy model, Parallel-distributed- compensation (PDC) technique is adopted in this paper. By the same premise, the i-th rule of the FLC can be obtained as follows.

Controller Rule i:

IF $x_1(t)$ is M_{i1} and ... and $x_g(t)$ is M_{ig}
THEN $U(t)=-K_iX(t)$, $i=1,2,\dots,r$

where K_i is the local feedback gain vector in the i-th subspace. The final model-based fuzzy controller is analytically represented by

$$U(t)=\frac{\sum_{i=1}^r w_i(t)K_iX(t)}{\sum_{i=1}^r w_i(t)}=-\sum_{i=1}^r h_i(t)K_iX(t).$$

The overall closed-loop controlled fuzzy complex system obtained as follows

$$\dot{X}(t)=\sum_{i=1}^r\sum_{j=1}^r h_i(t)h_j(t)[(A_i-B_iK_l)X(t)]+E_i\varphi(t).$$

A typical stability condition for fuzzy system is proposed. The equilibrium point of fuzzy control system is stable in the large if there exists a common positive definite matrix P and feedback gains K such that the following two inequalities are satisfied

$$(A_i-B_iK_i)^TP+P(A_i-B_iK_i)+\frac{1}{\eta^2}PE_iE_i^TP<0$$

and

$$\begin{aligned}&\left[\frac{(A_i-B_iK_l)+(A_l-B_lK_i)}{2}\right]^TP\\&+P\left[\frac{(A_i-B_iK_l)+(A_l-B_lK_i)}{2}\right]+\eta^2PE_iE_i^TP<0\end{aligned}$$

with $P=P^T>0$, for $i<l\leq r$ and $i=1,2,\dots,r$.

Proof: By using the Lyapunov function candidate for the fuzzy system

$$V=X^T(t)PX(t).$$

The time derivative of V is

$$\begin{aligned}\dot{v}&=\dot{X}^T(t)PX(t)+X^T(t)P\dot{X}(t)\\&=\left\{\sum_{i=1}^r\sum_{l=1}^r h_i(t)h_l(t)[(A_i-B_iK_l)X(t)]+E_i\varphi(t)\right\}^TPX(t)\\&+X^T(t)P\left\{\sum_{i=1}^r\sum_{l=1}^r h_i(t)h_l(t)[(A_i-B_iK_l)X(t)]+E_i\varphi(t)\right\}\\&=\sum_{i=1}^r\sum_{l=1}^r h_i(t)h_l(t)X^T(t)[(A_i-B_iK_l)^TP\\&+P(A_i-B_iK_l)]X(t)+\varphi^T(t)E_i^TPX(t)+X^T(t)PE_i\varphi(t)\\&-\left[\eta^2\varphi^T(t)\varphi(t)+\frac{1}{\eta^2}X^T(t)PE_iE_i^TPX(t)\right]^*\end{aligned} \tag{29}$$

$$
\begin{aligned}
&+\left[\eta^2\varphi^T(t)\varphi(t)+\frac{1}{\eta^2}X^T(t)PE_iE_i^TPX(t)\right]\\
&\leq \sum_{i=l=1}^{r} h_i^2(t)X^T(t)[(A_i-B_iK_i)^TP\\
&+P(A_i-B_iK_i)+\frac{1}{\eta^2}PE_iE_i^TP]X(t)\\
&+2\sum_{i<1}^{r} h_i(t)h_l(t)X^T(t)\left\{\left[\frac{(A_i-B_iK_l)+(A_i-B_lK_i)}{2}\right]^TP\right.\\
&\left.+P\left[\frac{(A_i-B_iK_l)+(A_l-B_lK_i)}{2}\right]+\frac{1}{\eta^2}PE_iE_i^TP\right\}X(t)\\
&+\eta^2\left\|\varphi_p(t)\right\|^2.
\end{aligned}
\tag{29}
$$

$$\left[\frac{1}{\eta}(PE_i)^TX(t)-\eta\varphi(t)\right]<0.$$

where "∗" can be represented as $-\left[\frac{1}{\eta}(PE_i)^TX(t)-\eta\varphi(t)\right]^T$

Evolved Bat Algorithm (EBA) is developed based on the bat echolocation fuzzy complex system in the natural world. Unlike the intelligence of the other algorithms, a strong EBA team is that it has only one variable called the mean, the need is to decide the procedure before hiring to solve the problem. The choice of different media determines the size of the research phase in the development process. In this study, we chose air as the medium because it is the original livelihood in the natural environment in which bats live. The activities of the EBA can be summarized in four phases:

Initialization: the artificial agents are spread into the solution space by randomly assigning coordinates to them.

Movement: the artificial agents are moved. A random number is generated and then it is checked whether it is larger than the fixed pulse emission rate. If the result is positive, the artificial agent is moved using the random walk process.

$$x_i^T = x_i^{t-1} + D,$$

where x_i^t indicates the coordinate of the i-th artificial agent at the t-th iteration, x_i^{t-1} represents the coordinate of the i-th artificial agent at the last iteration, and D is the moving distance that the artificial agent goes in this iteration.

$$D = \gamma \cdot \Delta T$$

where γ is a constant corresponding to the medium chosen in the experiment, and $\Delta T \in [-1,1]$ is a random number. $\gamma = 0.17$ is used in our experiment because the chosen medium is air.

$$x_i^{t_R} = \beta(x_{\text{best}} - x_i^t), \qquad \beta \in [0,1]$$

where β is a random number; x_{best} indicates the coordinate of the near best solution found so far throughout all artificial agents; and $x_i^{t_R}$ represents the new coordinates of the artificial agent after the operation of the random walk process.

Evaluation: the fitness of the artificial agents is calculated by the user defined fitness function and updated to the stored near best solution.

Termination: the termination conditions are checked in order to decide whether to go back to Step 2 or terminate the program and output the near best solution.

The evaluation criterion for determining the fitness of a bat is based on a user defined fitness function. A fitness function is employed in the paper to find the common symmetric positive definite matrix and the control force of the controller.

6. A numerical example

The algorithm discussed in the preceding section is illustrated below.

Example: Consider the nonlinear singularly perturbed system $S(0;\ 0)$ represented by the following equation

$$\ddot{\bar{x}} = 0.2\bar{x} + 1.2\dot{\bar{x}} + \bar{y} \tag{30a}$$

$$\varepsilon\dot{\bar{y}} = 0.3\bar{x} - \bar{y} + \bar{x}^3u. \tag{30b}$$

The unforced system can be easily demonstrated to be unstable, at least in the neighborhood of the origin. This may be explained by considering the open-loop linear part that contains three poles in the right-half plane of the complex plane. The purpose of this example is to synthesize a fuzzy controller such that the closed-loop nonlinear singularly perturbed system $S(C;\ 0)$ is stable. If the nonlinear singularly perturbed system cannot be stabilized by the designed fuzzy controller, a dither, as an auxiliary of the controller, is injected into $S(0;\ 0)$. Then the dither's parameters are regulated to make the closed-loop dithered system $S(C;\ d)$ stable.

Solution: We can solve this problem according to the following steps.

Step 1: Using the singular perturbation approach, the reduced model $S_r(0;\ 0)$ of Eq. (30) is defined for $\varepsilon = 0$ as

$$\ddot{\bar{x}}_r = 0.2\bar{x}_r + 1.2\dot{\bar{x}}_r + \bar{y}_r \tag{31a}$$

$$0 = 0.3\bar{x}_r - \bar{y}_r + \bar{x}_r{}^3u_r. \tag{31b}$$

From Eq. (31b), we have

$$\bar{y}_r = 0.3\bar{x}_r + \bar{x}_r{}^3u_r. \tag{32}$$

Substituting Eq. (32) into Eq. (31a), the reduced model $S_r(0;\ 0)$ is described by

$$\ddot{\bar{x}}_r = 0.5\bar{x}_r + 1.2\dot{\bar{x}}_r + \bar{x}_r{}^3u_r. \tag{33}$$

The nonlinear term of the reduced model is $\bar{x}_r{}^3u_r$, which satisfies the following condition for $\bar{x}_r \in [-3\ \ 3]$

$$-27u_r \leq \bar{x}_r{}^3u_r \leq 27u_r. \tag{34}$$

The above equation proves that the nonlinear term can be confined by an upper bound and a lower bound. According to the interpolation algorithm, the nonlinear term can be represented by the following equation

$$\bar{x}_r^{\,3}(t)u_r = \frac{[M_1(\bar{x}_r(t)) \times (-27) + M_2(\bar{x}_r(t)) \times (27)]}{M_1(\bar{x}_r(t)) + M_2(\bar{x}_r(t))} u_r. \quad (35)$$

Assume that

$$M_1(\bar{x}(t)) + M_2(\bar{x}(t)) = 1 \quad (36)$$

and

$$M_1(\bar{x}_r(t)) \in [0 \quad 1], \qquad M_2(\bar{x}_r(t)) \in [0 \quad 1].$$

Then, by solving Eqs. (35) and (36), $M_1(\bar{x}_r(t))$ and $M_2(\bar{x}_r(t))$ are obtained as follows

$$M_1(\bar{x}_r(t)) = 0.5 - \frac{\bar{x}_r^{\,3}(t)}{54}, \quad M_2(\bar{x}_r(t)) = 0.5 + \frac{\bar{x}_r^{\,3}(t)}{54}. \quad (37)$$

Step 2: From Eqs. (4) and (5), the reduced model $S_r(0;\ 0)$ can be described by the following fuzzy reduced model:

Rule 1:

IF $\bar{x}_r$ is about M_1
THEN $\ddot{\bar{x}}_r = 0.5\bar{x}_r + 1.2\dot{\bar{x}}_r - 27u_r$, (38a)

Rule 2:

IF $\bar{x}_r$ is about M_2
THEN $\ddot{\bar{x}}_r = 0.5\bar{x}_r + 1.2\dot{\bar{x}}_r + 27u_r$, (38b)

By introducing matrix representation, the fuzzy reduced model can be rewritten as follows:

Rule 1:

IF $\bar{x}_r(t)$ is about M_1
THEN $\dot{x}_r(t) = A_1 x_r(t) + B_1 u_r(t)$, (39a)

Rule 2:

IF $\bar{x}_r(t)$ is about M_2
THEN $\dot{x}_r(t) = A_2 x_r(t) + B_2 u_r(t)$, (39b)

where

$$x_r(t) = [\dot{\bar{x}}_r(t) \quad \bar{x}_r(t)]^T, \quad A_1 = \begin{bmatrix} 1.2 & 0.5 \\ 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -27 \\ 0 \end{bmatrix}, \quad (39c)$$

$$A_2 = \begin{bmatrix} 1.2 & 0.5 \\ 1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 27 \\ 0 \end{bmatrix}. \quad (39d)$$

Hence, the overall fuzzy reduced model is as follows

$$\begin{aligned} \dot{x}_r(t) &= \sum_{i=1}^{2} M_i(\bar{x}_r(t))\{A_i x_r(t) + B_1 u_r(t)\} \\ &= M_1(\bar{x}_r(t))\{A_1 x_r(t) + B_1 u_r(t)\} \\ &\quad + M_2(\bar{x}_r(t))\{A_2 x_r(t) + B_2 u_r(t)\} \end{aligned} \quad (40)$$

The fuzzy sets $M_1(\bar{x}_r(t))$ and $M_2(\bar{x}_r(t))$ are illustrated in Fig. 1.

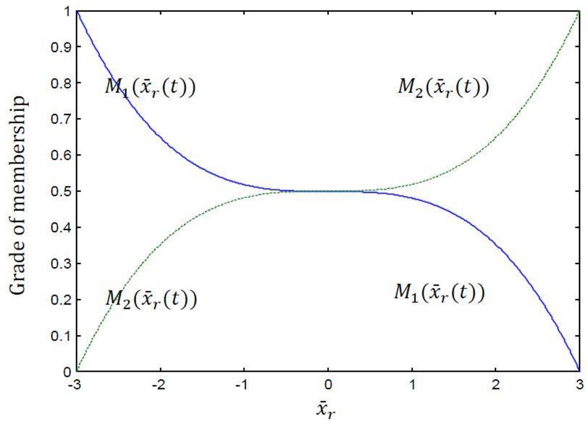

Fig. 1 Fuzzy sets $M_1(\bar{x}_r(t))$ and $M_2(\bar{x}_r(t))$

Step 3: According to Eqs. (39c) and (39d), we know that the pairs $(A_1 \quad B_1)$ and $(A_2 \quad B_2)$ are controllable. Hence, two local fuzzy controllers are designed to stabilize Eqs. (39a) and (39b), respectively. From Eqs. (7) and (8), the fuzzy controller is designed via the PDC scheme to decide two feedback gains in its consequent parts. The feedback gains F_1 and F_2 can be obtained by the linear quadratic optimal control technique as follows

$$F_1 = [-1.0824 \quad -1.0187], \quad F_2 = [0.7428 \quad 0.9316]. \quad (41)$$

The Riccati equation for linear continuous systems is used to design these feedback gains because each consequent part is represented by a linear state equation. According to the conditions above, we can obtain the matrix G from Eq. (14) as follows

$$G = \frac{1}{2}\sum_{l=1}^{2} A_l - B_l F_l = \begin{bmatrix} -23.4402 & -25.8290 \\ 1 & 0 \end{bmatrix}. \quad (42)$$

According to Eq. (8), the final output of the fuzzy controller is

$$\begin{aligned} u_r(t) &= -\sum_{i=1}^{2} M_i(\bar{x}_r(t)) F_i x_r(t) \\ &= -[M_1(\bar{x}_r(t))F_1 + M_2(\bar{x}_r(t))F_2] x_r(t). \end{aligned} \quad (43)$$

Substituting Eq. (43) into Eq. (40), we can derive the closed-loop fuzzy reduced system $F_r(C;\ 0)$.

$$\begin{aligned} \dot{x}_r(t) &= \sum_{i=1}^{2}\sum_{j=1}^{2} M_i(\bar{x}_r(t)) M_j(\bar{x}_r(t)) \\ &= \{M_1(\bar{x}_r(t))M_1(\bar{x}_r(t))[A_1 - B_1F_1] + \\ &\quad + M_1(\bar{x}_r(t))M_2(\bar{x}_r(t))[A_1 - B_1F_2] \\ &\quad + M_2(\bar{x}_r(t))M_1(\bar{x}_r(t))[A_2 - B_2F_1] \\ &\quad + M_2(\bar{x}_r(t))M_2(\bar{x}_r(t))A_2 - B_2F_2\} x_r(t). \end{aligned} \quad (44)$$

Step 4: From Eqs. (12)-(17), we have

$$G + \bar{D}\bar{O}\bar{E} = \begin{bmatrix} -11.1201 & -38.9936 \\ 1 & 0 \end{bmatrix}, \quad \|\bar{E}(sI - G - \bar{D}\bar{O}\bar{E})^{-1}\bar{D}\bar{Q}\|_\infty = \rho = 2.8932 > 1. \quad (45)$$

Hence, according to Lemma 3.2, the closed-loop fuzzy reduced system Eq. (44) is not quadratically stable. This implies that the fuzzy controller Eq. (43) cannot stabilize the fuzzy reduced model $F_r(0;\ 0)$. Simulation results of the closed-loop nonlinear singularly perturbed system $S(C;\ 0)$ with $\varepsilon = 0.01$ are shown in Fig. 2.

Subsequently, we attempt to improve the stability of the closed-loop nonlinear singularly perturbed system $S(C;\ 0)$ by injecting a periodic symmetrical square-wave dither $d(t)$ with sufficiently high frequency. Notice that the dither signal is injected just in front of the nonlinearity. The dithered system $S(0;\ d)$ is described as follows

$$\ddot{\bar{x}} = 0.2\bar{x} + 1.2\dot{\bar{x}} + \bar{y} \tag{46a}$$

$$\varepsilon\dot{\bar{y}} = 0.3\bar{x} - \bar{y} + (d + \bar{x})^3 u. \tag{46b}$$

Step 5: Using the singular perturbation approach, the dithered reduced model $S_r(0;\ d)$ of Eq. (46) is defined for $\varepsilon = 0$ as

$$\ddot{\bar{x}}_r = 0.2\bar{x}_r + 1.2\dot{\bar{x}}_r + \bar{y}_r \tag{47a}$$

$$0 = 0.3\bar{x}_r - \bar{y}_r + (d + \bar{x}_r)^3 u_r. \tag{47b}$$

From Eq. (47b), we have

$$\bar{y}_r = 0.3\bar{x}_r + (d + \bar{x}_r)^3 u_r. \tag{48}$$

Substituting Eq. (48) into Eq. (47a), the dithered reduced model $S_r(0;\ d)$ is described by

$$\ddot{\bar{x}}_r = 0.5\bar{x}_r + 1.2\dot{\bar{x}}_r + (d + \bar{x}_r)^3 u_r. \tag{49}$$

Step 6: To analyze the dithered reduced model Eq. (49), the corresponding relaxed reduced

$$\begin{aligned}\ddot{\bar{x}}_R = 0.5\bar{x}_R + 1.2\dot{\bar{x}}_R + [\alpha_1(\beta_1 + \bar{x}_R)^3 \\ + \alpha_2(\beta_2 + \bar{x}_R)^3]u_R\end{aligned} \tag{50}$$

with

$$\alpha_1 = 0.5, \qquad \alpha_2 = 1 - \alpha_1 = 0.5$$

and

$$\beta_1 = -\beta_2 = \xi,$$

where α_i and β_i are the dither's parameters and ξ is a real constant number. Substituting $\alpha_1, \alpha_2, \beta_1$ and β_2 into Eq. (50) allows us to rewrite the relaxed reduced model $S_{rR}(0;\ 0)$ as follows

$$\ddot{\bar{x}}_{rR} = 0.5\bar{x}_{rR} + 1.2\dot{\bar{x}}_{rR} + (3\xi^2\bar{x}_{rR} + \bar{x}_{rR}^3)u_{rR}. \tag{51}$$

The nonlinear term of this system is $(3\xi^2\bar{x}_{rR} + \bar{x}_{rR}^3)u_{rR}$, which satisfies the following condition for $\bar{x}_{rR} \in [-3 \quad 3]$

$$\begin{aligned}(-9\xi^2 - 27)u_{rR} \le (3\xi^2\bar{x}_{rR} + \bar{x}_{rR}^3)u_{rR} \\ \le (9\xi^2 + 27)u_{rR}.\end{aligned}$$

Similarly, the nonlinear term can be represented as follows

$$\begin{aligned}&(3\xi^2\bar{x}_{rR} + \bar{x}_{rR}^3)u_{rR} \\ &= \left[\frac{M_{R1} \times (-9\xi^2 - 27) + M_{R2} \times (9\xi^2 + 27)}{M_{R1} + M_{R2}}\right]u_{rR},\end{aligned} \tag{52}$$

where

$$M_{R1} + M_{R2} = 1 \tag{53}$$

and

$$M_{R1} \in [0 \quad 1], \qquad M_{R2} \in [0 \quad 1].$$

By solving Eqs. (52) and (53), M_{R1} and M_{R2} are obtained as follows

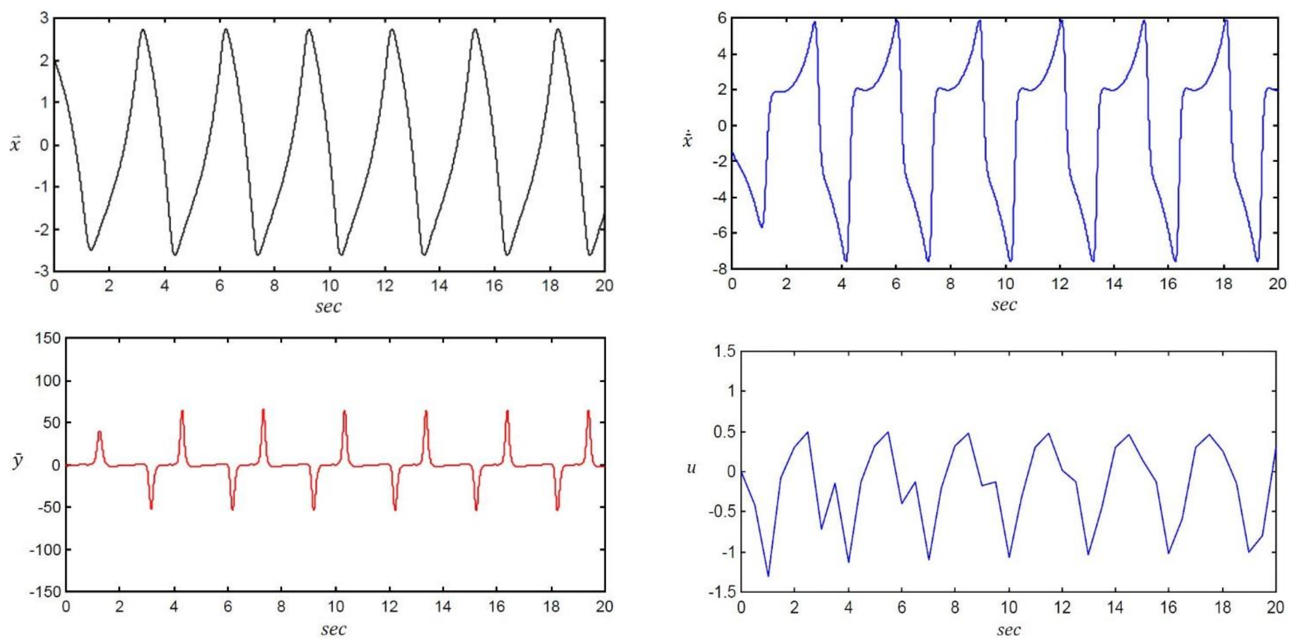

Fig. 2 Simulation results of the closed-loop nonlinear singularly perturbed system $S(C;\ 0)$

$$M_{R1}\big(\bar{x}_{rR}(t)\big) = 0.5 - \frac{3\xi^2\bar{x}_{rR} + \bar{x}_{rR}{}^3}{2(9\xi^2+27)},$$
$$M_{R2}(\bar{x}_{rR}(t)) = 0.5 + \frac{3\xi^2\bar{x}_{rR} + \bar{x}_{rR}{}^3}{2(9\xi^2+27)}. \quad (54)$$

Step 7: By using M_{R1} and M_{R2}, the relaxed reduced model $S_{rR}(0;\ 0)$ can be represented by the following fuzzy relaxed reduced model:

Rule 1:

$$\begin{aligned}&\text{IF} \quad \bar{x}_{rR}(t) \text{ is about } M_{R1}\\ &\text{THEN} \quad \ddot{\bar{x}}_{rR} = 0.5\bar{x}_{rR} + 1.2\dot{\bar{x}}_{rR} - (9\xi^2+27)u_{rR},\end{aligned} \quad (55a)$$

Rule 2:

$$\begin{aligned}&\text{IF} \quad \bar{x}_{rR}(t) \text{ is about } M_{R2}\\ &\text{THEN} \quad \ddot{\bar{x}}_{rR} = 0.5\bar{x}_{rR} + 1.2\dot{\bar{x}}_{rR} + (9\xi^2+27)u_{rR}\end{aligned} \quad (55b)$$

By introducing matrix representation, Eq. (55a) can be also rewritten as follows:

Rule 1:

$$\begin{aligned}&\text{IF} \quad \bar{x}_{rR}(t) \text{ is about } M_{R1}\\ &\text{THEN} \quad \dot{x}_{rR}(t) = A_1x_{rR}(t) + B_1u_{rR}(t),\end{aligned} \quad (56a)$$

Rule 2:

$$\begin{aligned}&\text{IF} \quad \bar{x}_{rR}(t) \text{ is about } M_{R2}\\ &\text{THEN} \quad \dot{x}_{rR}(t) = A_2x_{rR}(t) + B_2u_{rR}(t),\end{aligned} \quad (56b)$$

where

$$x_{rR}(t) = [\dot{\bar{x}}_{rR}(t) \quad \bar{x}_{rR}(t)]^T,$$
$$A_1 = \begin{bmatrix} 1.2 & 0.5 \\ 1 & 0 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -9\xi^2-27 \\ 0 \end{bmatrix}, \quad (56c)$$

$$A_2 = \begin{bmatrix} 1.2 & 0.5 \\ 1 & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 9\xi^2+27 \\ 0 \end{bmatrix}. \quad (56d)$$

The final output of the fuzzy relaxed reduced model $F_{rR}(0;\ 0)$ is inferred as follows

$$\begin{aligned}\dot{x}_{rR}(t) &= \sum_{j=1}^{2} M_{Ri}\big(\bar{x}_{rR}(t)\big)\{A_ix_{rR}(t) + B_iu_{rR}(t)\}\\ &= M_{R1}\big(\bar{x}_{rR}(t)\big)\{A_1x_{rR}(t) + B_1u_{rR}(t)\}\\ &\quad + M_{R2}\big(\bar{x}_{rR}(t)\big)\{A_2x_{rR}(t) + B_2u_{rR}(t)\}.\end{aligned} \quad (57)$$

Based on Eq. (25), we can obtain the corresponding fuzzy controller

$$\begin{aligned}u_{rR}(t) &= -\sum_{j=1}^{2} M_{Ri}(\bar{x}_{rR}(t),\xi)F_ix_{rR}(t)\\ &= -[M_{R1}(\bar{x}_{rR}(t),\xi)F_i + M_{R2}(\bar{x}_{rR}(t),\xi)F_2]x_{rR}(t)\end{aligned} \quad (58)$$

Step 8: Substituting Eq. (58) into Eq. (57), the closed-loop fuzzy relaxed reduced system $F_{rR}(C;\ 0)$ is obtained as follows

$$\begin{aligned}\dot{x}_{rR}(t) &= \sum_{i=1}^{2}\sum_{j=1}^{2} M_{Ri}(\bar{x}_{rR}(t),\xi)M_{Rj}(\bar{x}_{rR}(t),\xi)\\ &\qquad \{A_i(\xi) - B_i(\xi)F_j\}x_{rR}(t)\\ &= \{M_{R1}(\bar{x}_{rR}(t),\xi)M_{R1}(\bar{x}_{rR}(t),\xi)[A_1(\xi) - B_1(\xi)F_1]\\ &\quad + M_{R1}(\bar{x}_{rR}(t),\xi)M_{R2}(\bar{x}_{rR}(t),\xi[A_1(\xi) - B_1(\xi)F_2]\\ &\quad + M_{R2}(\bar{x}_{rR}(t),\xi)M_{R1}(\bar{x}_{rR}(t),\xi)[A_2(\xi) - B_2(\xi)F_1]\\ &\quad + M_{R2}(\bar{x}_{rR}(t),\xi)M_{R2}(\bar{x}_{rR}(t),\xi)\\ &\qquad [A_2(\xi) - B_2(\xi)F_2]\}x_{rR}(t).\end{aligned} \quad (59)$$

Theorem 4.1 is employed to check whether the closed-loop fuzzy relaxed reduced system $F_{rR}(C;\ 0)$ is stable. First, solving the inequality Eq. (28) allows us to derive the curve of ρ with respect to ξ (Fig. 3). According to this figure, when the dither's amplitude ξ is greater than 8.56, the values of $\rho(\xi)$ are less than unity. Moreover, as the dither's amplitude ξ is greater than 8.56, the matrix $G(\xi) + \bar{D}(\xi)\bar{O}(\xi)\bar{E}(\xi)$ is a stable matrix. From the analysis above, the closed-loop fuzzy relaxed reduced system $F_{rR}(C;\ 0)$ is quadratically stable if the injected dither's amplitude is greater than 8.56. Here, we choose dither's amplitude ξ to be 9, and the feedback gains of the fuzzy controller are shown in Eq. (41). Simulation results of the closed-loop fuzzy relaxed reduced system and the closed-loop dithered system ($\varepsilon = 0.01$, $\omega = 200$ rad/s and $\omega = 2000$ rad/s) with the initial states ($\bar{x}_R(0) = -1.5$, $\dot{\bar{x}}_R(0) = 2$) are shown in Fig. 4. Obviously, the closed-loop dithered system is approximated by its corresponding closed-loop fuzzy relaxed reduced system and the approximation improves as the frequency of dither increases.

7. Conclusions

This study combines fuzzy theory and the parallel-distributed-compensation scheme to design a fuzzy controller to stabilize the nonlinear singularly perturbed system. If the nonlinear singularly perturbed system cannot be stabilized by the designed fuzzy controller, a dither is injected into the nonlinear singularly perturbed system to improve the stability of the closed-loop nonlinear singularly perturbed system. Simulation results display that the fuzzy controller can stabilize the nonlinear singularly perturbed system by choosing appropriate parameters of dither.

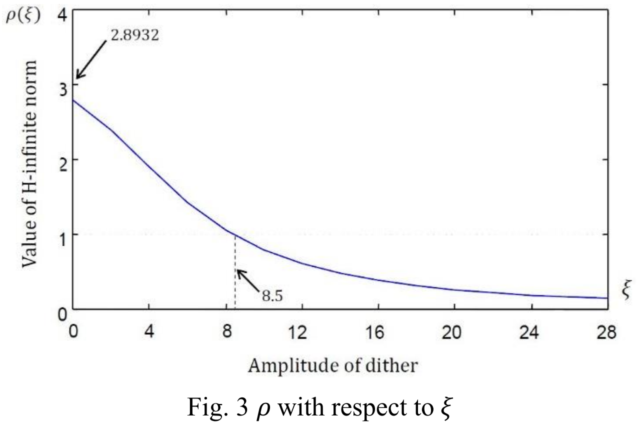

Fig. 3 ρ with respect to ξ

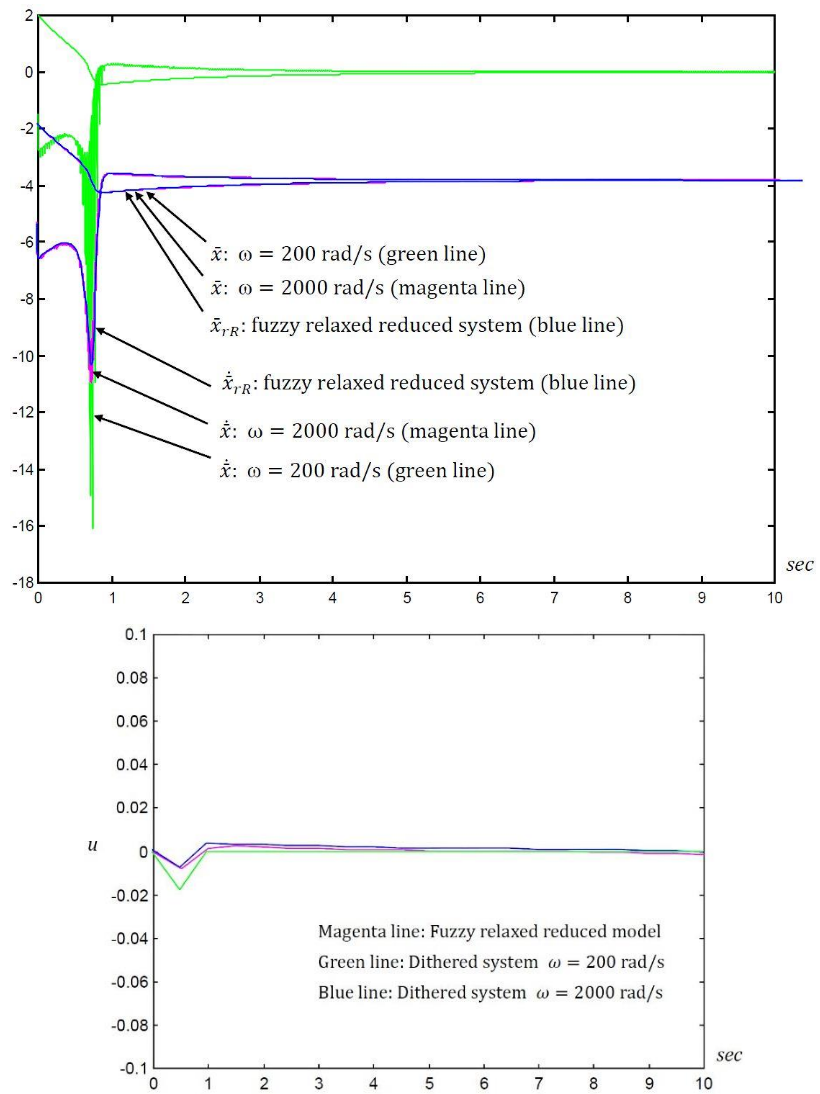

Fig. 4 Simulation results of the closed-loop fuzzy relaxed reduced system $F_{rR}(C;\ 0)$ and the closed-loop dithered system $S(C;\ d)$ ($\varepsilon = 0.01,\ \xi = 9,\ \omega = 200$ and 2000 rad/s)

Compliance with Ethical Standards

The authors declare that there are no conflicts of interest regarding the publication of this paper. All analyzed data during this study are included in this article. This article does not contain an11y studies with human participants performed by any of the authors.

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