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A two-stage Kalman filter for the identification of structural parameters with unknown loads

Jia He*, Xiaoxiong Zhang[a], Zhouquan Feng[b], Zhengqing Chen[c] and Zhang Cao[a]

College of Civil Engineering, Key laboratory of wind and bridge engineering of Hunan Province, Key Laboratory of Building Safety and Energy Efficiency of the Ministry of Education, Hunan University, China

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Abstract. The conventional Kalman Filter (KF) provides a promising way for structural state estimation. However, the physical parameters of structural systems or models should be available for the estimation. Moreover, it is not applicable when the loadings applied to the structures are unknown. To circumvent the aforementioned limitations, a two-stage KF with unknown input approach is proposed for the simultaneous identification of structural parameters and unknown loadings. In stage 1, a modified observation equation is employed. The structural state vector is estimated by KF on the basis of structural parameters identified at the previous time-step. Then, the unknown input is identified by Least Squares Estimation (LSE). In stage 2, based on the concept of sensitivity matrix, the structural parameters are updated at the current time-step by using the estimated structural states obtained from stage 1. The effectiveness of the proposed approach is numerically validated via a five-story shearing model under random and earthquake excitations. Shaking table tests on a five-story structure are also employed to demonstrate the performance of the proposed approach. It is demonstrated from numerical and experimental results that the proposed approach can be used for the identification of parameters of structure and the external force applied to it with acceptable accuracy.

Keywords: two-stage Kalman filter; parameter identification; unknown loading; modified observation equation; sensitivity matrix

1. Introduction

The estimation of structural states is important for evaluating the performance of in-service structure and guiding proper maintenance to ensure its safety and functionality (Li and Hao 2016, Xu and He 2017). The Kalman Filter (KF) is a well-known recursive algorithm that can provide unbiased and optimal state estimation. Due to its relative simplicity and robust nature, KF-based algorithms have been actively investigated and much progress has been made. This paper is not intended to provide a comprehensive review on the existing KF-based methods. Instead, a brief introduction of the KF technique for response estimation is given herein. For example, some researchers focus on data fusion of multi-type signals with different sampling frequencies by using KF. In this regard, Smyth and Wu (2007) proposed a multi-rate KF approach to cope with the problem of acceleration measurements with high sampling rates but displacement measurements with relatively low sampling rates. This multi-rate KF approach was extended by Kim *et al*. (2016, 2018) for dynamic displacement estimation. Based on multi-rate KF and unscented KF, Chatzi and Fuggini (2015) proposed a time-domain approach for the estimation of rotation responses.

Based on data fusion of different sampling rate measurements, Zheng *et al*. (2019) proposed a multi-rate KF approach for real-time displacement estimation. Besides the usage of KF technique for data fusion of multi-rate data, KF algorithm can also be employed for optimal sensor placement and response reconstruction. For example, by using limited observations, a KF-based approach for optimal sensor placement and multi-scale response reconstruction was proposed by Zhu *et al*. (2013) and validated via a long-span suspension bridge (Xu *et al*. 2016). This method was further extended by He *et al*. (2015) for active control system and Hu *et al*. (2018) for high-rise buildings. By using the delayed and periodic measurements, Lee *et al*. (2018) proposed a real-time approach for responses estimation with the aid of two KFs.

The aforementioned KF-based state estimation methods are performed upon two assumptions, saying the inputs applied to the system and the properties of the model are known. These assumptions may hinder the application of KF technique. To make it more practical and robust, various KF-based methods have been developed to circumvent the limitations mentioned above. Among them, two well-recognized families are Augmented Kalman Filter (AKF) and Extended Kalman Filter (EKF). In AKF technique, the input to be identified is added to the unknown state vector, forming the so-called augmented state-space model. Then, the unknown force can be estimated in conjunction with the

*Corresponding author, Ph.D., Associate Professor,
E-mail: jiahe@hnu.edu.cn
[a] Ph.D. Student
[b] Ph.D., Assistant Professor
[c] Ph.D., Professor

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states in a recursive manner. The comparison of AKF with other algorithms was conducted by Lourens *et al*. (2012). With the usage of AKF and some dummy-measurements, Naets *et al*. (2015) proposed an approach for identifying the unknown force stably. Based on AKF and modal expansion, Ren and Zhou (2017) proposed an approach for strain estimation of truss structures under uncertain loading conditions. With the combination of AKF and genetic algorithm, Saleem *et al*. (2019) proposed an approach for identifying the location and time history of impact force. The performance of AKF for load identification was discussed and compared with other methods by Cumbo *et al*. (2019). A key point of implementation of AKF technique is that a force model is required in the filter. However, the priori knowledge of the distribution of external loads in time domain is often a challenging task. Moreover, AKF technique focuses on joint state-input estimation and the structural parameters are assumed to be known.

Instead of adding force variables into the state vectors, the well-known EKF method is performed by introducing the unknown structural parameters into the state vectors. To date, a number of efficient approaches have been developed for parameter identification and damage detection by using EKF technique (e.g., Lei *et al*. 2014, Xu and He 2015, Zhang *et al*. 2017, Xin *et al*. 2019, Ding *et al*. 2019). However, these methods are performed based on the conventional EKF principle, and thus are not applicable for the case of unknown excitations. Therefore, much effort has been made for the development of EKF-based methods for identifying structural parameters with unknown inputs. For example, based on EKF and LSE, Lei *et al*. (2012a, b) proposed a time domain approach for damage detection with unknown inputs. Liu *et al*. (2016) further improved this method by using data fusion technique. Based on modal transformation, Liu *et al*. (2017) proposed a real-time approach for the simultaneous identification of structural parameters and unknown inputs without collocated acceleration measurements. To reduce the restrictions on sensor locations, Pan *et al*. (2016) proposed a general EKF approach for the identification of structural parameters and unknown inputs. Xiao *et al*. (2018) proposed an adaptive three-stage EKF approach for state and fault estimation in presence of unknown inputs. Hui *et al*. (2019) proposed an EKF-based approach for identifying the parameter of Single-Degree-Of-Freedom (SDOF) system and its aerodynamic force. Based on limited observations, He *et al*. (2019a) proposed an EKF-based approach for the identification of structural parameters and unknown inputs.

A basic concept of AKF or EKF methods is that the variables to be identified are considered as the additional components of structural states. Such state vector is then called augmented state vector or extended state vector. Hence, the sizes of the augmented or extended state vector are often large, and the corresponding state equation is highly nonlinear with respect to such state vector. This may lead to high computational cost or even the divergence of the identified results. To reduce the order of extended state vector, Lei *et al*. (2015) proposed a two-step KF approach for intelligent structural damage detection. However, in their approach, the external excitations are assumed to be available for the identification. More recently, the authors proposed an improved KF approach for joint estimation of structural states and unknown loadings (He *et al*. 2019b). This approach is, however, not applicable when the structural parameters are unknown. To circumvent the aforementioned limitations, a two-stage KF approach is then proposed in this paper for the simultaneous identification of structural parameters and unknown inputs. In stage 1, a modified observation equation is employed. The structural state vector including displacement and velocity is estimated by KF on the basis of structural parameters identified at the previous time-step. The unknown loading is then identified by LSE using the estimated structural states. In stage 2, based on sensitivity matrix, the structural parameters to be identified at the current time-step are updated by using the estimated structural states obtained from stage 1. The details are given in the following sections. Numerical examples and shaking table tests are used to demonstrate the effectiveness of the proposed approach.

2. Formulas of the proposed two-stage KF approach

In general, the equation of motion of a system with n DOFs can be given as

$$\begin{aligned} &M\ddot{x}(t) + F(x(t), \dot{x}(t), \theta) \\ &= \varphi^u f^u(t) \end{aligned} \tag{1}$$

in which M is a known mass matrix; $\ddot{x}(t)$, $\dot{x}(t)$ and $x(t)$ are the acceleration, velocity and displacement vectors, respectively; θ denotes the structural parameters including stiffness and damping coefficients; $F(\cdot)$ denotes structural restoring forces; f^u and φ^u are the unknown input and its influence matrix, respectively.

By introducing the state vector $Z = [x^T, \dot{x}^T]^T$, the corresponding state-space equation can be easily obtained as

$$\begin{aligned} \dot{Z}(t) &= \begin{bmatrix} \dot{x}(t) \\ M^{-1}[-F(x(t), \dot{x}(t), \theta) + \varphi^u f^u(t)] \end{bmatrix} \\ &= g(Z(t), f^u(t), \theta, t) + w(t) \end{aligned} \tag{2}$$

where $w(t)$ is process noise vector with zero mean and a covariance matrix $Q(t)$.

In this study, the acceleration responses are assumed to be measured. Then, the discretized observation equation can be found as

$$\begin{aligned} y_k &= M^{-1}[-F(x_k, \dot{x}_k, \theta_k) + \varphi^u f_k^u] + v_k \\ &= h(Z_k, \theta_k) - D f_k^u + v_k \end{aligned} \tag{3}$$

where $h(Z_k, \theta_k) = -M^{-1}F(x_k, \dot{x}_k, \theta_k)$; $D = -M^{-1}\varphi^u$; the subscript k denotes the k-th time step; v_k is the measurement noise vector with zero mean and a covariance matrix R_k. With the proper transformation, the following revised observation equation can be obtained

$$\Phi y_k = \Phi h(Z_k, \theta_k) + \Phi v_k \tag{4}$$

where $\Phi = I - D(D^T D)^{-1} D^T$. As compared with Eq. (3), it can be seen that the unknown input is not explicitly involved in the revised observation equation. Thus, the multiple regression problems shown in Eq. (3) can be converted into single regression problem, and then the principle of KF can be employed for the estimation. Some descriptions can also be found in He *et al*. (2019b).

2.1 Stage 1: Estimation of structural states and unknown inputs

Let $\hat{Z}_{k|k}$, $\hat{f}_k^u$ and $\hat{\theta}_k$ be the estimates of Z_k, f_k and θ_k at time $t = k \times \Delta t$ with Δt being time interval, respectively. In this stage, the structural parameters identified at the k-th time step are directly employed for the estimation of structural states at the (k+1)-th time step. Then, the priori state estimate $\hat{Z}_{k+1|k}$ can be calculated as

$$\hat{Z}_{k+1|k} = \hat{Z}_{k|k} + \int_{t_k}^{t_{k+1}} g(\hat{Z}_{k|k}, \hat{f}_k^u, \hat{\theta}_k, t_k)\, dt \tag{5}$$

The priori estimate error covariance matrix is determined as

$$\begin{aligned} P_{k+1|k}^Z &= E\big((Z_{k+1} - \hat{Z}_{k+1|k})(Z_{k+1} - \hat{Z}_{k+1|k})^T\big) \\ &= \Gamma_1 P_{k|k}^Z \Gamma_1^T + \Gamma_2 R_k \Gamma_2^T + \Delta t^2 Q_k \end{aligned} \tag{6}$$

in which

$$\begin{aligned} \Gamma_1 &= I + \Delta t \frac{\partial g(Z_k, f_k^u, \theta_k, t_k)}{\partial Z_k}\Big|_{\substack{Z_k = \hat{Z}_{k|k} \\ f_k^u = \hat{f}_k^u \\ \theta_k = \hat{\theta}_k}} + \Delta t \begin{bmatrix} 0 \\ -H_{k|k}^Z \end{bmatrix} \\ \Gamma_2 &= \Delta t \begin{bmatrix} 0 \\ -I \end{bmatrix} \\ H_{k|k}^Z &= \frac{\partial h(Z_k, \theta_k)}{\partial Z_k}\Big|_{\substack{Z_k = \hat{Z}_{k|k} \\ \theta_k = \hat{\theta}_k}} \end{aligned} \tag{7}$$

Based on Eqs. (4)-(5) and the principle of KF, the posteriori state estimate can be computed as

$$\hat{Z}_{k+1|k+1} = \hat{Z}_{k+1|k} + G_{k+1}^Z\big[\Phi y_{k+1} - \Phi h(\hat{Z}_{k+1|k}, \hat{\theta}_k)\big] \tag{8}$$

in which G_{k+1}^Z is the gain matrix for structural state estimation. It can be determined by the following equation

$$\begin{aligned} G_{k+1}^Z &= P_{k+1|k}^Z (H_{k+1|k}^Z)^T \Phi^T \\ &\quad * \big[\Phi\big(H_{k+1|k}^Z P_{k+1|k}^Z (H_{k+1|k}^Z)^T + R_{k+1}\big)\Phi^T\big]^{-1} \end{aligned} \tag{9}$$

where $H_{k+1|k}^Z$ can be obtained according to Eq. (7) while $Z_k = \hat{Z}_{k+1|k}$.

The posteriori estimate error covariance matrix can be computed as

$$\begin{aligned} P_{k+1|k+1}^Z &= \big(I - G_{k+1}^Z \Phi H_{k+1|k}^Z\big) P_{k+1|k}^Z \big(I - G_{k+1}^Z \Phi H_{k+1|k}^Z\big)^T \\ &\quad + G_{k+1}^Z \Phi R_{k+1} (G_{k+1}^Z \Phi)^T \end{aligned} \tag{10}$$

With the usage of the posteriori state estimate, the unknown external excitations can be determined as

$$\hat{f}_{k+1}^u = (D^T D)^{-1} D^T \big(h(\hat{Z}_{k+1|k+1}, \hat{\theta}_k) - y_{k+1}\big) \tag{11}$$

In summary, the estimation of structural state vector and unknown loading are derived based on the improved KF approach (He *et al*. 2019b) under the condition that structural parameters identified at the previous time step are employed.

2.2 Stage 2: Identification of structural parameters

To assure the reliability of the identified results, the structural parameters at the current time step should be updated accordingly. As shown in Eq. (5) and Eq. (8), the structural state can be treated as an implicit function of the structural parameters θ. Based on Eq. (3), the discretized observation equation at time instant $t = (k + 1) \times \Delta t$ can be also re-arranged as

$$y_{k+1} = \bar{h}(Z_{k+1}, \theta_{k+1}, f_{k+1}^u) + v_{k+1} \tag{12}$$

where $\bar{h}(Z_{k+1}, \theta_{k+1}, f_{k+1}^u) = M^{-1}[-F(x_{k+1}, \dot{x}_{k+1}, \theta_{k+1}) + \varphi^u f_{k+1}^u]$.

Using Taylor expansion, the following linearized expression can be found

$$\begin{aligned} &\bar{h}(Z_{k+1}, \theta_{k+1}, f_{k+1}^u) \\ &= \bar{h}(\hat{Z}_{k+1|k+1}, \hat{\theta}_k, \hat{f}_{k+1}^u) + H_{k+1}^\theta(\theta_{k+1} - \hat{\theta}_k) \end{aligned} \tag{13}$$

where the derivative matrix H_{k+1}^θ is determined according to the chain rule of partial differentiation with respect to the parametric vector θ as follows

$$H_{k+1}^\theta = H_{k+1}^{h\theta} + H_{k+1}^{hZ} H_k^{Z\theta} \tag{14}$$

where

$$\begin{aligned} H_{k+1}^{h\theta} &= \frac{\partial \bar{h}(Z, \theta, f^u)}{\partial \theta}\Big|_{\substack{\theta = \hat{\theta}_k \\ Z = \hat{Z}_{k+1|k+1} \\ f^u = \hat{f}_{k+1}^u}} \\ H_{k+1}^{hZ} &= \frac{\partial \bar{h}(Z, \theta, f^u)}{\partial Z}\Big|_{\substack{\theta = \hat{\theta}_k \\ Z = \hat{Z}_{k+1|k+1} \\ f^u = \hat{f}_{k+1}^u}}, \quad H_k^{Z\theta} = \frac{\partial Z}{\partial \theta}\Big|_{\theta = \hat{\theta}_k} \end{aligned} \tag{15}$$

The matrix $H_k^{Z\theta}$ in Eq. (15) is referred to as sensitivity matrix. Based on such sensitivity matrix, another state space equation regarding the parametric vector θ can be obtained as (Yang *et al.* 2009)

$$\bar{g}\big(H^{Z\theta}, \theta, f^u, t\big) = \frac{dH^{Z\theta}}{dt} \tag{16}$$

Then, $H_{k+1}^{Z\theta}$ can be calculated as

$$H_{k+1}^{Z\theta} = H_k^{Z\theta} + \int_{t_k}^{t_{k+1}} \bar{g}(H_{t|k}^{Z\theta}, \hat{\theta}_k, \hat{f}_k) dt \tag{17}$$

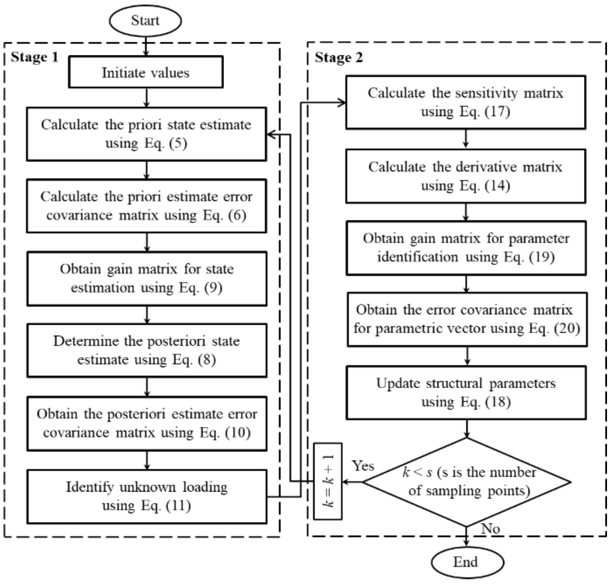


Fig. 1 Flowchart of the proposed two-stage KF approach

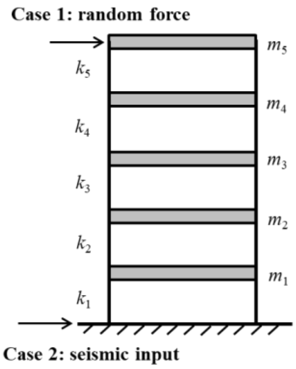


Fig. 2 The five-story shear building structure

Table 1 Structural parameter identification under random excitation

Parameters	Actual	Identified	Error (%)
k_1 (kN/m)	96	95.57	-0.45
k_2 (kN/m)	120	118.76	-1.03
k_3 (kN/m)	120	119.32	-0.57
k_4 (kN/m)	120	119.91	-0.08
k_5 (kN/m)	120	118.28	-1.43
α	0.5461	0.5481	0.36
β ($\times 10^{-3}$)	1.2475	1.2133	-2.74

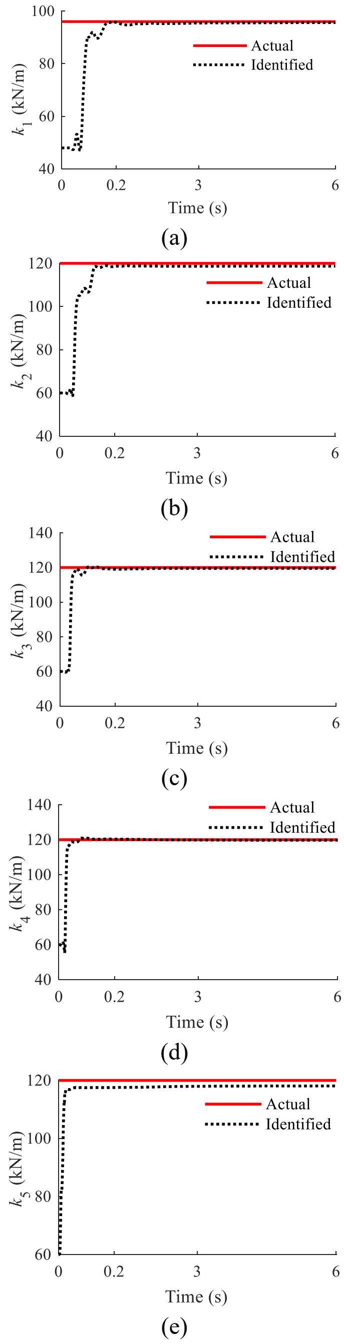


Fig. 3 Comparison of the identified stiffness under random excitation

where $H_{t|k}^{Z\theta}$ is the solution of Eq. (16) in $k\Delta tb \le t \le (k+1)\Delta t$ with the initial condition $H_k^{Z\theta}$. Based on the principle of KF, the recursive solution for $\hat{\theta}_{k+1}$ can be obtained as follows

$$\hat{\theta}_{k+1} = \hat{\theta}_k + G_{k+1}^{\theta}\left[y_{k+1} - \bar{h}(\hat{Z}_{k+1|k+1}, \hat{\theta}_k, \hat{f}_{k+1}^{u})\right] \tag{18}$$

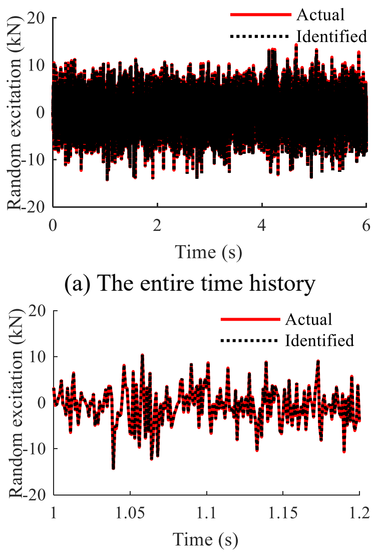
in which G_{k+1}^{θ} is the gain matrix for structural parameter identification

$$G_{k+1}^{\theta} = P_k^{\theta}(H_{k+1}^{\theta})^T\left[H_{k+1}^{\theta}P_k^{\theta}(H_{k+1}^{\theta})^T + R_{k+1}\right]^{-1} \tag{19}$$

where P_k^{θ} is the error covariance matrix for parametric vector and can be determined in a recursive manner as

$$P_{k+1}^{\theta} = \left[I - G_{k+1}^{\theta}H_{k+1}^{\theta}\right]P_k^{\theta} \tag{20}$$

It can be found that the structural state and external excitation estimated in stage 1 are employed to update the structural parameter in stage 2. For ease of understanding,



(a) The entire time history

(b) Time segment from 1 s to 1.2 s

Fig. 4 Comparison of the identified random excitation

the flowchart of the proposed two-stage KF approach is plotted in Fig. 1. Numerical and experimental examples are given in the following sections.

3. Numerical verification

In this section, a five-story building model under two loading cases, i.e., random and seismic input, as shown in Fig. 2 is considered. The physical parameters of the structure are set to be m_i = 60 kg and k_i = 120 kN/m (i = 1, 2, …, 5). Rayleigh damping assumption is adopted and the damping coefficients for mass matrix and stiffness matrix are α = 0.5461, β = 1.2475 × 10^{-3}, respectively. In order to mimic structural damage, 20% degradation of stiffness is considered on the first floor. The structural responses are calculated by state-space method with time interval of 0.001 s. In this study, the acceleration responses are assumed to be measured for the identification. All the measured signals are superimposed into 5% noise-to-signal ratio in terms of root-mean-square to mimic measurements pollution.

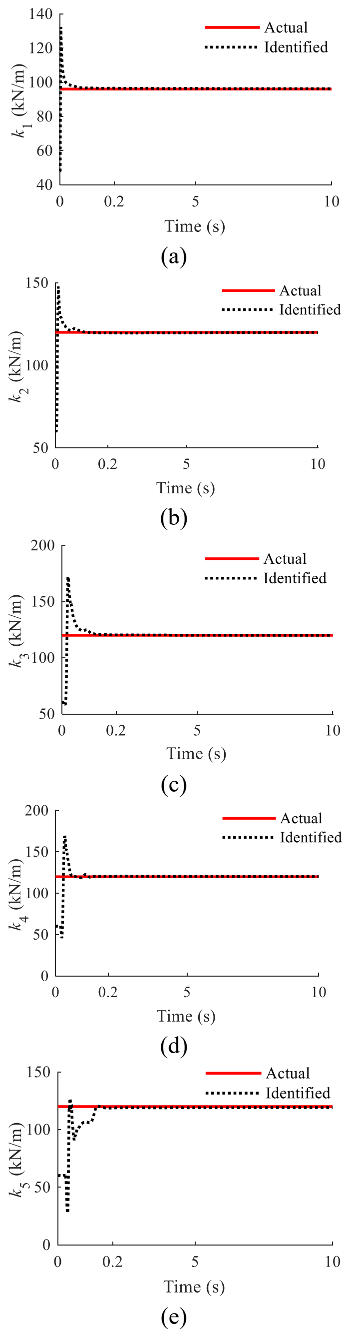
3.1 Case 1: Random excitation

In this case, a random force is assumed to be applied to the top floor of the structure. The acceleration responses are used for the identification. The initial values of the stiffness and damping coefficients are assumed to be 50% of the actual ones. The covariance matrices Q and R are set to be 10^{-6} × I and I, respectively, where I is an identity matrix with proper dimension. The initial guess of sensitivity matrix $H^{Z\theta}$ is a zero matrix with proper dimension.

Based on the information mentioned above, the structural stiffness and damping coefficients are identified as listed in Table 1. It can be seen that a good agreement between the identified results and actual ones can be found. For ease of comparison, Fig. 3 gives the convergence processes of the identified stiffness coefficients. The

Table 2 Structural parameter identification under seismic input

Parameters	Actual	Identified	Error (%)
k_1 (kN/m)	96	96.08	0.08
k_2 (kN/m)	120	120.16	0.13
k_3 (kN/m)	120	120.06	0.05
k_4 (kN/m)	120	120.11	0.09
k_5 (kN/m)	120	119.70	-0.25
α	0.5461	0.5594	2.44
β (× 10^{-3})	1.2475	1.2155	-2.57



(a)

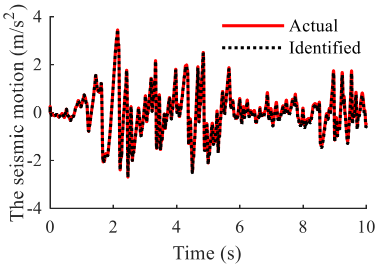
(b)

(c)

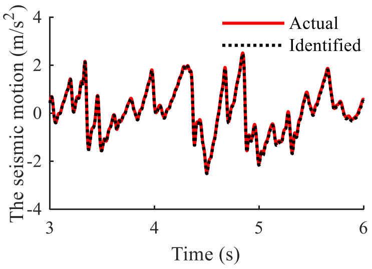
(d)

(e)

Fig. 5 Comparison of the identified stiffness under seismic input



(a) The entire time history



(b) Time segment from 3 s to 6 s

Fig. 6 Comparison of the identified seismic input

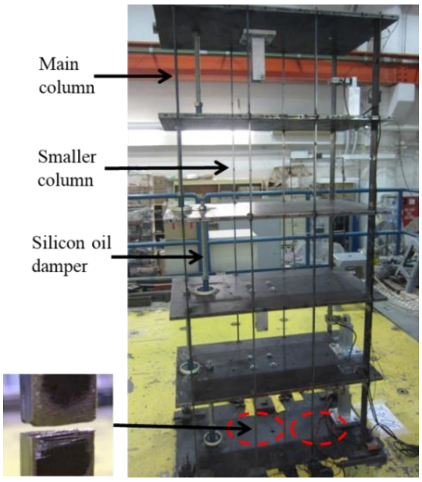


Fig. 7 Five-story building structure

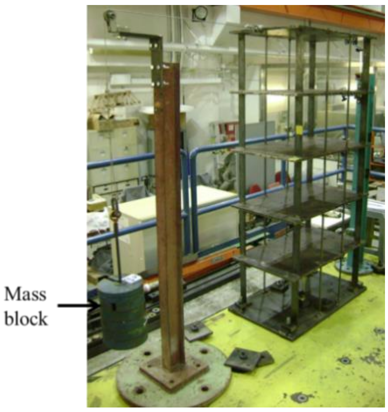


Fig. 8 The static tests

identified results are plotted by dashed lines, whereas the actual ones are represented by solid lines. In order to show the convergence process more clearly, the *x*-axis is plotted in irregular scale. It can be seen that the accurate and stable identification results can be obtained.

By using the proposed approach, the unknown external excitation can be identified as well (see Fig. 4). The time segment from 1 s to 1.2 s is given in Fig. 4(b) for clarity of comparison. It is obvious that the identified inputs are close to the actual ones.

3.2 Case 2: Seismic input

In this case, the El-Centro earthquake with a Peak Ground Acceleration (PGA) of 0.34 g is considered. Similarly, the acceleration responses with 5% noise are used for the simultaneous identification of structural parameters and ground motion. The initial values of the structural coefficients are assumed to be 50% of the actual ones. The initial guess of sensitivity matrix is a zero matrix. The covariance matrices of process noise and measurement noise are $10^{-6} \times I$ and I, respectively.

Using the proposed approach, the structural parameters and the unknown seismic input can be identified. The identified structural parameters are listed in Table 2. The maximum error is only 2.57%, indicating that the identified results are reliable. Fig. 5 gives the convergence processes of the identified stiffness coefficients. It can be found from Fig. 5 that the identified coefficients are able to promptly and stably converge to the actual ones. The unknown ground motion can be also identified as shown in Fig. 6. The time segment from 3 s to 6 s is shown in Fig. 6(b). It is clear that the identified seismic input has a good agreement with the actual one.

4. Experimental validation

To further verify the effectiveness of the proposed approach, a five-story building structure as shown in Fig. 7 is employed. The height of the structure is 1750 mm and planar size is 850 mm × 500 mm. The thickness of the rigid plate is 16 mm. The cross section of the main columns and smaller columns are 50 mm × 6 mm and 10 mm × 6 mm, respectively. In order to increase structural damping, a silicon oil damper is installed on each floor of the building. The lumped mass of the structure from 1st floor to 5th floor are 67.43 kg, 61.65 kg, 56.54 kg, 62.82 kg and 59.66 kg, respectively. More details can be found in He *et al*. (2019b).

To mimic structural damage, the smaller columns on the 1st floor were cut off in this study. Firstly, the values of structural stiffness were determined via some static tests as shown in Fig. 8. The weight of each mass block was

Table 3 The identified structural stiffness in the experiment

Parameters	Measured	Identified	Error (%)
k_1 (kN/m)	218.1	215.6	-1.14
k_2 (kN/m)	257.6	245.8	-4.58
k_3 (kN/m)	261.9	255.1	-2.59
k_4 (kN/m)	263.9	269.8	2.24
k_5 (kN/m)	271.3	281.1	3.61

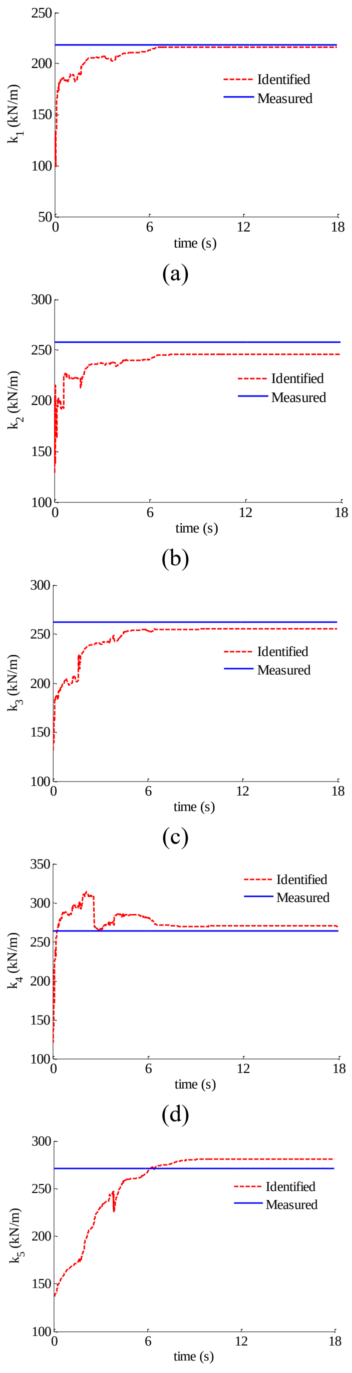


Fig. 9 Comparison of the identified stiffness in the experiment

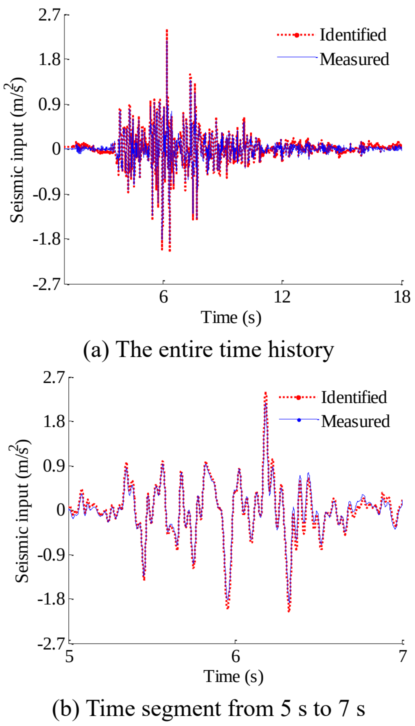


Fig. 10 Comparison of the identified seismic input in the experiment

measured by electronic balance before the test. As the number of mass blocks increased one by one, the displacement of each floor measured by the dial gauge increased accordingly. Then, the structural stiffness can be determined based on the measured weight and inter-story shift. The structural stiffness from 1st floor to 5th floor can be found as 218.1 kN/m, 257.6 kN/m, 261.9 kN/m, 263.9 kN/m and 271.3 kN/m, respectively. Moreover, modal tests were carried out to obtain other structural properties including natural frequencies and damping ratios. Based on the analyses of frequency response functions, the first two natural frequencies and damping ratios were found as 2.845 Hz, 8.408 Hz, 0.89% and 1.08%, respectively.

The Northridge earthquake with the PGA scaled to 0.217 g was selected as seismic input. The acceleration responses were measured and the sampling frequency is 1000 Hz. The cut-off frequencies of a band pass filter were chosen as 1 Hz and 30 Hz for the purpose of de-noise. The structural parameters including structural stiffness and Rayleigh damping coefficients, and the ground motion are the unknown quantities to be identified. The initial guesses of the unknowns are set to be 50% of the measured ones. The initial guess of sensitivity matrix is a zero matrix. The covariance matrices of measurement noise and process noise are assumed to be I and $10^{-4} \times I$, respectively.

Based on the aforementioned information, the proposed two-stage KF approach is then used for identifying the parameters of building structure and the seismic input applied to it. The identified structural stiffness is listed in Table 3. The maximum relative error is only 4.58% indicating that the identified results are relatively reliable. For ease of comparison, these identified parameters are also plotted in Fig. 9. It is clear that stable results can be obtained after few seconds.

The Rayleigh damping coefficients can be identified at the same time. However, these two coefficients are generally small, and it is often difficult to assess them in a straightforward manner. Thus, the damping ratios are used herein for the purpose of ease of comparison. With the usage of the identified Rayleigh damping coefficients, the first two damping ratios of the structure are found as 1.63% and 2.01%. Although the identified values are relatively larger than those determined by hammer tests, it is still acceptable, to some extent, because of the complexity of structural damping.

Here, the unknown seismic input can also be simultaneously identified as shown in Fig. 10. Similarly, the time segment from 5 s to 7 s is given in Fig. 10(b). It is obvious that the identified ground acceleration is close to the measure one.

5. Conclusions

In the classic KF algorithm, the structural parameters and external excitations are required for the state estimation. Although various KF-based methods have been developed, such as the aforementioned AKF and EKF, they are basically performed under the condition that an augmented or extended state vector are used. The dimension of the augmented or extended state vector is often large, and this may lead to high computational cost or even the divergence of the identified results. Thus, a two-stage KF approach is proposed in this paper for the simultaneous identification of structural parameters and unknown inputs without defining the augmented or extended state vector. In stage 1, a modified observation equation is employed. The structural state vector and unknown loading are first estimated based on KF and LSE techniques. In stage 2, with the usage of sensitivity matrix, the structural parameters to be identified are then updated. Numerical examples and shaking table tests are used to demonstrate the effectiveness of the proposed approach. The results show that the proposed approach is capable of identifying structural parameters and unknown inputs with acceptable accuracy.

It should be noted that the complete acceleration responses are used for the identification in this study. When partial measurements are employed, an influence matrix associated with the location of limited sensors should be added in the observation equation (i.e., Eq. (3)). The revised observation equation (i.e., Eq. (4)) and the follow-up formulas in the proposed approach should be changed accordingly. Since the location of sensors is assumed to be available, such influence matrix is known as well. Thus, the proposed approach can be still used for the identification. However, it should be noted that the number and location of sensors would influence the accuracy of the identification results. The optimal sensor placement is another interesting research topic but not discussed herein. The research on the effect of type, number and location of sensors for the identification will be conducted in the near future.

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