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# Nonlocal strain gradient effects on forced vibrations of porous FG cylindrical nanoshells

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**Abstract.** The present paper explores forced vibrational properties of porosity-dependent functionally graded (FG) cylindrical nanoshells exposed to linear-type or triangular-type impulse load via classical shell theory (CST) and nonlocal strain gradient theory (NSGT). Employing such scale-dependent theory, two scale factors accounting for stiffness softening and hardening effects are incorporated in modeling of the nanoshell. Two sorts of porosity distributions called even and uneven have been taken into account. Governing equations obtained for porous nanoshell have been solved through inverse Laplace transforms technique to derive dynamical deflections. It is shown that transient responses of a nanoshell are affected by the form and position of impulse loading, amount of porosities, porosities dispensation, nonlocal and strain gradient factors.

**Keywords:** dynamic analysis; transient vibrations; nanoshell; porous FGMs; nonlocal strain gradient theory

## 1. Introduction

In a functional graded material, all material properties may change from one side to another side by means of a prescribed distribution. These two sides may be ceramic or metal. Mechanical characteristics of a FG material can be described based on the percentages of ceramic and metal phases (Tang and Yang 2018, Tang *et al.* 2019, She *et al.* 2018). The material distribution in FG materials may be characterized via a power-law function (Faleh *et al.* 2018a, b). FG materials are not always perfect because of porosity production in them (Shafiei *et al.* 2017, Mirjavadi *et al.* 2017a, b, 2018a,b, 2019a-f, Alasadi *et al.* 2019, Fenjan *et al.* 2019). Existence of porosities in the FG materials may significantly change their mechanical characteristics. For example, the elastic moduli of porous FG material is smaller than that of perfect FG material. Up to now, many authors focused on wave propagation, vibration and buckling analyzes of FG structures having porosities (Atmane *et al.* 2015). Also, there are several investigations concerning with the analysis of FG structures in thermal environments.

Recently, this kind of materials have found their applications in nano-scale structures (Azimi *et al.* 2017, 2018). Vibration behavior of a nano-scale plate is not the same as a macro-scale plate. This is because small-size effects are not present at macro scale. So, mathematical modeling of a nanoplate can be done with the use of nonlocal elasticity incorporating only one scale parameter (Ke *et al.* 2012, Eltaher *et al.* 2012, Aydogdu and Arda 2016, Zenkour 2016, Zhen *et al.* 2019, Uzun and Civalek 2019). Due to the ignorance of strain gradient effect in nonlocal elasticity theory, a more general theory will be required. Strain gradients at nano-scale are observed by many researchers (Lam *et al.* 2003). Thus, nonlocal-strain gradient theory was introduced as a general theory which contains an additional strain gradient parameter together with nonlocal parameter (Li and Hu 2015, Li *et al.* 2017, Houari *et al.* 2018, Kheroubi *et al.* 2016, Barati 2017, Ebrahimi and Haghi 2018, Bensaid *et al.* 2018). The scale parameters used in nonlocal strain gradient theory can be obtained by fitting obtained theoretical results with available experimental data and even molecular dynamic (MD) simulations (Mehralian *et al.* 2017, Mohammadi *et al.* 2018).

This paper uses classical shell formulation having three field variables and taking into account small scale impacts. Based upon Laplace transform approach and nonlocal strain gradient (NSGT) formulation, transient vibrational analysis of porous functionally graded (FG) nano-size shells under linear-type pulse load has been performed. The presented formulation incorporates two scale factors for examining vibrational behaviors of nano-dimension shells more accurately. The material properties for FG shell are porosity-dependent and defined employing a modified power-law form. It is supposed that the nano-sized shell is exposed to transverse shock loading. The governing equations achieved by Hamilton's principle are solved implementing Galerkin's method. Presented results indicate the prominence of pore variation, load factors, material gradient index, nonlocal coefficient, strain gradient coefficient and porosities on vibrational frequencies of FG nano-size shell.

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## 2. Theories and formulations

### 2.1 Porosity-dependent properties of FG nanoshell

The volume fraction of porosities which highlights their portion in the FG material can be denoted by ξ. With the use of power-law FG model with material gradation parameter *p*, all of material properties including elastic moduli ($E$) and mass density ($\rho$) can be defined as functions of the portion of porosity (Faleh *et al*. 2018a):

Even porosity distribution

$$E(z)=(E_c-E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p+E_m-\frac{\xi}{2}(E_c+E_m) \quad (1)$$

$$\rho(z)=(\rho_c-\rho_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p+\rho_m-\frac{\xi}{2}(\rho_c+\rho_m) \quad (2)$$

Uneven porosity distribution

$$E(z)=(E_c-E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p+E_m-\frac{\xi}{2}(E_c+E_m)\left(1-\frac{2|z|}{h}\right) \quad (3)$$

$$\rho(z)=(\rho_c-\rho_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p+\rho_m-\frac{\xi}{2}(\rho_c+\rho_m)\left(1-\frac{2|z|}{h}\right) \quad (4)$$

### 2.2 Classic shell model

As it is known in research community, classical shell theory (CST) is suitable for studying thin shells. However, the displacement field of the nanoshell ($u_1$, $u_2$, $u_3$) based on CST can be defined as function of axial ($u$), circumferential ($v$) and transverse ($w$) components in the following form

$$u_1(x,y,z)=u(x,y)-z\frac{\partial w}{\partial x}(x,y) \quad (5)$$

$$u_2(x,y,z)=v(x,y)-\frac{z}{R}\frac{\partial w}{\partial y}(x,y) \quad (6)$$

$$u_3(x,y,z)=w(x,y) \quad (7)$$

There are only three strains for the CST shells as follows

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}-z\frac{\partial^2 w}{\partial x^2}$$
$$\varepsilon_{yy}=\frac{\partial v}{\partial y}-\frac{w}{R}-z\frac{\partial^2 w}{\partial y^2} \quad (8)$$
$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}-2z\frac{\partial^2 w}{\partial x\partial y}$$

Based upon Hamilton's principle and gathering the variable coefficients of displacements (δu, δv, δw) results in the below governing equations of shells

$$\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\frac{\partial^2 u}{\partial t^2}-I_1\frac{\partial^3 w}{\partial x\partial t^2} \quad (9)$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}=I_0\frac{\partial^2 v}{\partial t^2}-I_1\frac{\partial^3 w}{\partial y\partial t^2} \quad (10)$$

$$\frac{\partial^2 M_{xx}}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_{yy}}{\partial y^2}+\frac{N_{yy}}{R}=+I_0\frac{\partial^2 w}{\partial t^2}+I_1\frac{\partial^3 u}{\partial x\partial t^2}+I_1\frac{\partial^3 v}{\partial y\partial t^2}-I_2\left(\frac{\partial^4 w}{\partial x^2\partial t^2}+\frac{\partial^4 w}{\partial y^2\partial t^2}\right)+q_{dynamic} \quad (11)$$

in which $q_{dynamic}$ is applied load $q_{dynamic}=f_0\delta(x-x_0)(1-t/t_0)(H(t)-H(t-t_0))$ in which $f_0$ is load amplitude and $x_0$ is load location and

$$(I_0,I_1,I_2)=\int_{-h/2}^{h/2}(1,z,z^2)\rho(z)dz \quad (12)$$

Also, in-plane normal $N_{ij}$ forces and bending moment $M_{ij}$ can be defined as

$$\{N_{xx},N_{yy},N_{xy}\}=\int_{-h/2}^{h/2}\{\sigma_{xx},\sigma_{yy},\sigma_{xy}\}dz \quad (13)$$

$$\{M_{xx},M_{yy},M_{xy}\}=\int_{-h/2}^{h/2}\{\sigma_{xx},\sigma_{yy},\sigma_{xy}\}zdz \quad (14)$$

Here, $\sigma_{ij}$ denotes the stresses.

Based on NSGT with nonlocality coefficient *ea* and strain gradient coefficient *l*, the constitutive equation of a nano-scale shell may be introduced as (Faleh *et al*. 2018a)

$$[1-(ea)^2\nabla^2]\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}=[1-l^2\nabla^2]\frac{E(z)}{1-v^2}\begin{pmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} \quad (15)$$

Here, strain gradient coefficient *l* is used to consider non-uniform strain field observed at small scales and also nonlocal coefficient *ea* is used to consider the nonlocality of stress field due to atomic interaction.

By integrating Eq. (15) over the thickness, one may derive following relations based on NSGT formulation of porous nanoshells

$$(1-(ea)^2\nabla^2)N_{xx}=(1-l^2\nabla^2)\left[A_{11}\frac{\partial u}{\partial x}-B_{11}\frac{\partial^2 w}{\partial x^2}+A_{12}\left(\frac{\partial v}{\partial y}-\frac{w}{R}\right)-B_{12}\frac{\partial^2 w}{\partial y^2}\right)\right] \quad (16)$$

$$(1-(ea)^2\nabla^2)M_{xx}=(1-l^2\nabla^2)\left[B_{11}\frac{\partial u}{\partial x}-D_{11}\frac{\partial^2 w}{\partial x^2}\right. \quad (17)$$

$$+B_{12}\left(\frac{\partial v}{\partial y}-\frac{w}{R}\right)-D_{12}\frac{\partial^2 w}{\partial y^2}\Bigg] \tag{17}$$

$$\begin{aligned}&(1-(ea)^2\nabla^2)N_{yy}\\&=(1-l^2\nabla^2)\left[A_{12}\frac{\partial u}{\partial x}-B_{12}\frac{\partial^2 w}{\partial x^2}\right.\\&\left.+A_{11}\left(\frac{\partial v}{\partial y}-\frac{w}{R}\right)-B_{11}\frac{\partial^2 w}{\partial y^2}\right]\end{aligned} \tag{18}$$

$$\begin{aligned}&(1-(ea)^2\nabla^2)M_{yy}\\&=(1-l^2\nabla^2)\left[B_{12}\frac{\partial u}{\partial x}-D_{12}\frac{\partial^2 w}{\partial x^2}\right.\\&\left.+B_{11}\left(\frac{\partial v}{\partial y}-\frac{w}{R}\right)-D_{11}\frac{\partial^2 w}{\partial y^2}\right]\end{aligned} \tag{19}$$

$$\begin{aligned}&(1-(ea)^2\nabla^2)N_{xy}\\&=(1-l^2\nabla^2)\left[A_{66}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-2B_{66}\frac{\partial^2 w}{\partial x\partial y}\right]\end{aligned} \tag{20}$$

$$\begin{aligned}&(1-(ea)^2\nabla^2)M_{xy}\\&=(1-l^2\nabla^2)\left[B_{66}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-2D_{66}\frac{\partial^2 w}{\partial x\partial y}\right]\end{aligned} \tag{21}$$

in which

$$\begin{aligned}A_{11}&=\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{E(z)}{1-v^2}dz,\\B_{11}&=\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{E(z)}{1-v^2}z\,dz,\\D_{11}&=\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{E(z)z^2}{1-v^2}dz,\\A_{12}&=\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{vE(z)}{1-v^2}dz,\\B_{12}&=\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{vE(z)}{1-v^2}z\,dz,\\D_{12}&=\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{vE(z)z^2}{1-v^2}dz,\\A_{66}&=\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{E(z)}{2(1+v)}dz,\\B_{66}&=\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{E(z)}{2(1+v)}zdz,\\D_{66}&=\int_{-h/2}^{h/2}\frac{E(z)}{2(1+v)}z^2\,dz\end{aligned} \tag{22}$$

Next, governing equations of the FGM nanoshell in the framework of NSGT might be established as follows by substituting Eqs. (19)-(26), into Eqs. (9)-(13)

$$\begin{aligned}&A_{11}\left(\frac{\partial^2 u}{\partial x^2}\right)-B_{11}\frac{\partial^3 w}{\partial x^3}+A_{12}\left(\frac{\partial^2 v}{\partial x\partial y}-\frac{1}{R}\frac{\partial w}{\partial x}\right)\\&-B_{12}\frac{\partial^3 w}{\partial x\partial y^2}+A_{66}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\right)\\&-2B_{66}\frac{\partial^3 w}{\partial x\partial y^2}+A_{31}^e\frac{\partial\phi}{\partial x}+A_{31}^m\frac{\partial\gamma}{\partial x}\\&=(1-(ea)^2\nabla^2)\left[I_0\frac{\partial^2 u}{\partial t^2}-I_1\frac{\partial^3 w}{\partial x\partial t^2}\right]\end{aligned} \tag{23}$$

$$\begin{aligned}&A_{66}\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial x^2}\right)-2B_{66}\frac{\partial^3 w}{\partial x^2\partial y}+A_{12}\left(\frac{\partial^2 u}{\partial x\partial y}\right)\\&-B_{12}\frac{\partial^3 w}{\partial x^2\partial y}+A_{11}\left(\frac{\partial^2 v}{\partial y^2}-\frac{1}{R}\frac{\partial w}{\partial y}\right)\\&-B_{11}\frac{\partial^3 w}{\partial y^3}+A_{31}^e\frac{\partial\phi}{\partial y}+A_{31}^m\frac{\partial\gamma}{\partial y}\\&=(1-(ea)^2\nabla^2)\left[I_0\frac{\partial^2 v}{\partial t^2}-I_1\frac{\partial^3 w}{\partial y\partial t^2}\right]\end{aligned} \tag{24}$$

$$\begin{aligned}&B_{11}\left(\frac{\partial^3 u}{\partial x^3}\right)-D_{11}\frac{\partial^4 w}{\partial x^4}-2D_{12}\frac{\partial^4 w}{\partial x^2\partial y^2}\\&-4D_{66}\frac{\partial^4 w}{\partial x^2\partial y^2}-D_{11}\frac{\partial^4 w}{\partial y^4}+B_{12}\left(\frac{\partial^3 v}{\partial x^2\partial y}-\frac{1}{R}\frac{\partial^2 w}{\partial x^2}\right)\\&+2B_{66}\left(\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial^3 v}{\partial x^2\partial y}\right)+B_{12}\left(\frac{\partial^3 u}{\partial x\partial y^2}\right)\\&+B_{11}\left(\frac{\partial^3 v}{\partial y^3}-\frac{1}{R}\frac{\partial^2 w}{\partial y^2}\right)+\frac{A_{12}}{R}\left(\frac{\partial u}{\partial x}\right)-\frac{B_{12}}{R}\frac{\partial^2 w}{\partial x^2}\\&+\frac{A_{11}}{R}\left(\frac{\partial v}{\partial y}-\frac{w}{R}\right)-\frac{B_{11}}{R}\frac{\partial^2 w}{\partial y^2}+E_{31}^e\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)\\&+E_{31}^m\left(\frac{\partial^2\gamma}{\partial x^2}+\frac{\partial^2\gamma}{\partial y^2}\right)+\frac{A_{31}^e}{R}\phi+\frac{A_{31}^m}{R}\gamma\\&+(1-(ea)^2\nabla^2)\left[-(q_{dynamic})(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2})\right]\\&=(1-(ea)^2\nabla^2)\left[+I_0\frac{\partial^2 w}{\partial t^2}+I_1\frac{\partial^3 u}{\partial x\partial t^2}\right.\\&\left.+I_1\frac{\partial^3 v}{\partial y\partial t^2}-I_2\left(\frac{\partial^4 w}{\partial x^2\partial t^2}+\frac{\partial^4 w}{\partial y^2\partial t^2}\right)\right]\end{aligned} \tag{25}$$

## 3. Solution technique

Since the transient vibration of the nanoshell is a time-dependent problem, the governing equations will be solved in Laplace domain. However, in the first step the equations have been discretized with the help of Galerkin's method based on the following displacement assumptions

$$u=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U_{mn}(t)\frac{\partial F_m(x)}{\partial x}cos(n\theta) \tag{26}$$

$$v=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V_{mn}(t)F_m(x)\,sin(n\theta) \tag{27}$$

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}(t)F_m(x)\,cos(n\theta) \tag{28}$$

where $U_{mn}$ , $V_{mn}$ , $W_{mn}$ , $\Phi_{mn}$ and $\Theta_{mn}$ are oscillation amplitude. Also, the function $F_m(x) = \sin\left(\frac{m\pi}{L}x\right)$ is an admissible function to satisfy below boundary conditions based on one term (m = 1)

$$w = \frac{\partial^2 w}{\partial x^2} = \frac{\partial^4 w}{\partial x^4} = 0 \tag{29}$$

Next, a compact form of governing equations may be obtained by Replacing Eqs. (26)-(28) into Eqs. (23)-(25)

$$[K]\begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{Bmatrix} + [M]\begin{Bmatrix} \ddot{U}_{mn} \\ \ddot{V}_{mn} \\ \ddot{W}_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F(t) \end{Bmatrix} \tag{30}$$

where the coefficients of [K] and [M] are calculated based on Galerkin's method. Then, the time-dependent Eq. (30) is transformed into Laplace domain with initial zero condition as follows

$$[K]\{L[D_{mn}]\} + S^2[M]\{L[D_{mn}]\} = F(S) = \begin{Bmatrix} 0 \\ 0 \\ L[F(t)] \end{Bmatrix} \tag{31}$$

where $D_{mn}$ is the amplitude vector $D_{mn} = \{U_{mn}, V_{mn}, W_{mn}\}$ and S is Laplace operator. Thus, the governing equations may be in Laplace domain. In order to obtain Eq. (31) from Eq. (30), some properties of Laplace transform technique must presented

$$L[\ddot{D}_{mn}] = S^2 L[D_{mn}] - SD_{mn}(0) - \dot{D}_{mn}(0) \tag{32}$$

Next, implementation of inverse Laplace transform in Eq. (32) yields the time-dependent governing equation as

$$D_{mn}(t) = L^{-1}[F(S)([K] + S^2\{M])^{-1}] \tag{33}$$

After solving the problem, the results will be presented based on normalized parameters as

$$t^* = \frac{t}{t_0}, \quad \bar{W} = W\frac{100E_c I}{f_0 L^3}, \quad \lambda = \frac{l}{L}, \quad \mu = \frac{ea}{L} \tag{34}$$

## 4. Numerical results and discussions

The present section focuses on transient vibration behavior of a nanoshell with graded porous material properties exposed to linear type of impulse loadings based upon NSGT. Graphical results are presented for time response of the nanoshell for different porosity distributions, impulse loadings, load location, two small scale coefficients, and also radius-to-thickness ratio of the nanoshell. In order to validate the present formulation based on NSGT, vibration frequency of a nano-scale shell is compared with the data provided by Mehralian *et al*. (2017) implementing analytical method and molecular dynamic simulation (MD). Results for this verification are presented in Table 2 for different values of nanoshell length based on obtained values of *ea* = 3.3-3.5 $nm^2$ and *l* = 0.1-0.4 $nm^2$ for nonlocal and strain gradient coefficients. In the following

Table 1 Material properties for a FG material

| Properties | Steel | Alumina ($Al_2O_3$) |
|---|---|---|
| E | 210 (GPa) | 390 (GPa) |
| ρ | 7800 ($kg/m^3$) | 3960 ($kg/m^3$) |
| ν | 0.3 | 0.24 |

Table 2 Verification of vibrational frequencies of a nano-scale shell

| L/2R | MD (Mehralian *et al.* 2017) | NSGT (Mehralian *et al.* 2017) | Present |
|---|---|---|---|
| 4.86 | 1.138 | 1.209 | 1.211 |
| 8.47 | 0.466 | 0.448 | 0.451 |
| 13.89 | 0.190 | 0.192 | 0.198 |
| 17.47 | 0.122 | 0.126 | 0.129 |

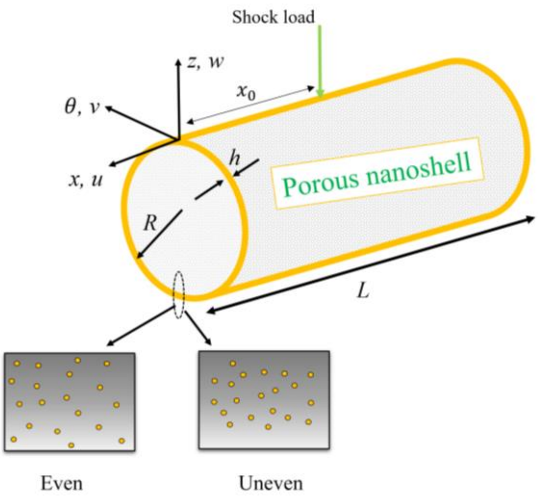


Fig. 1 A cylindrical shell under shock loading

sentences, obtained new results and complete discussion related to transient vibration of a FG nanoshell will be presented. The geometry of FG nanoshell is shown in Fig. 1.

Impacts of nonlocality and strains gradient factors on dynamical response of a functional graded nano-size shell exposed to triangular-type shock load have been plotted in Fig. 2. The ratio of radius to thickness has been selected by *R* = 5h and power-law exponent is selected by *p* = 1. Based on the range 0 < *t** < 1 for normalized vibrations time, a functional graded nano-size shell exhibits transient vibrations due to applied shock load. After passing t* = 1, transient vibrations have been eliminated. In other words, after this time the nano-size shell is freely oscillating. The present graph also demonstrate that the free/transient vibration of functional graded nano-size shell has considerable dependency on nonlocality and strains gradient factors. It can be understand from Fig. 2 that dynamic deflection of system will rise with nonlocal coefficient and will reduce with strain gradient coefficient. This observation is valid for all ranges of applied load. So, vibration behavior of the nanoshell system is dependent on

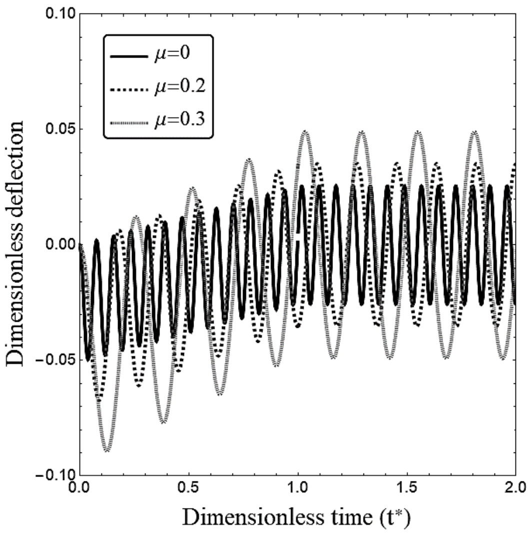


Fig. 2 Time response variation of nanoshell based upon various nonlocal factors (R/h = 5, L/h = 50, ξ = 0, λ = 0.2, p = 1)

both scale effects. An important finding is that the transient regions and deflection magnitudes are outstandingly affected by the values of nonlocal strain gradient coefficients.

Illustrated in Fig. 3 is the influence of pore volume fraction on time responses of the FG nanoshell with and without porosities subjected to linear type of impulse loadings when $R/h = 5$, μ = 0.2 and λ = 0.1. This figure focuses on FG nanoshells having even porosity distribution. Based on rectangular impulse loading, a monotonic procedure may be seen during transient zone and when the load is abruptly deleted at $t^* = 1$, free vibration happens. Also, by increase of t* based on triangular impulsive load, the transient region linearly (not abruptly) reaches to free vibration region. Thus, greater values of porosity amount lead to larger deflection during both transient and free oscillations. Such observation is due to the fact that the total stiffness of a nanoshell diminishes with increase of porosity amount. Consequently, porosity content in the material structure should be controlled for a reliable design of FG structures subjected to dynamical loadings.

Depicted in Fig. 4 is the impact of pore distribution type on transient/free vibrations of porous FG nanoshell subjected to a linearly varying impulse load. This figure focuses on FG nanoshells accounting for various material gradation indices (*p*) as well as pore distributions having the parameter $\xi = 0.2$. When p = 0, the results become related to uniform material. For the case of un-even type, pores have been produced in middle zone of the cross section, but they may vanish at corners for the case of even type. This issue highlights that the elastic moduli of a nanoshell with un-even pore type are smaller than that of even type. Hence,

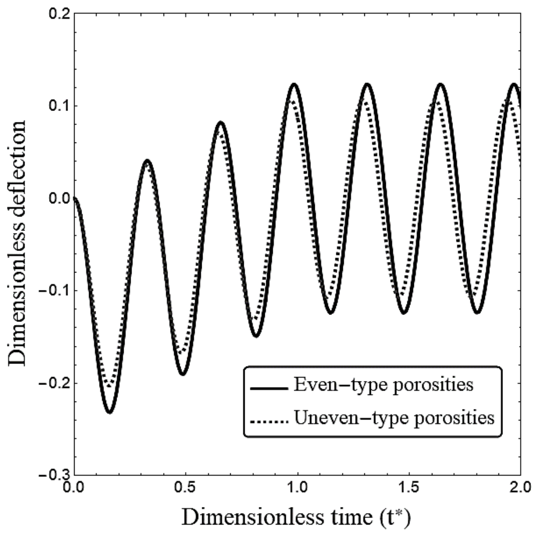


Fig. 4 Time response variation of the nanoshell based upon various porosity dispersions (R/h = 5, L/h = 50, μ = 0.2, λ = 0.1, ξ = 0.2, p = 2)

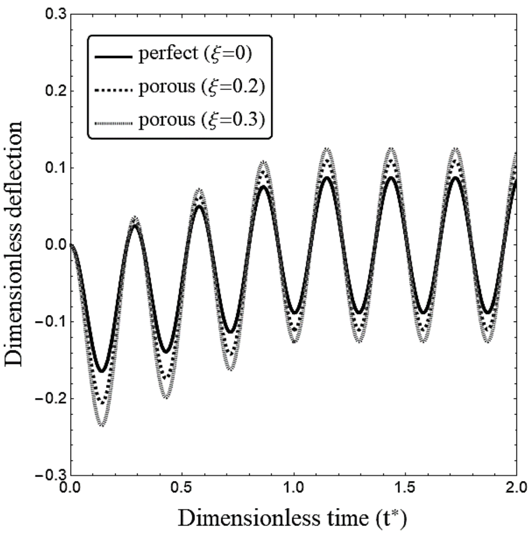


Fig. 3 Time response variation of the nanoshell based upon various porosity factors (p = 1, R/h = 5, L/h = 50, μ = 0.2, λ = 0.1)

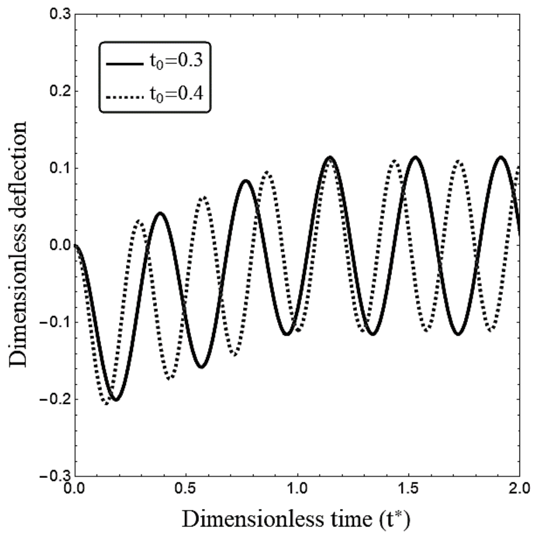


Fig. 5 Time response variation of the nanoshell based upon various shock times (R/h = 5, L/h = 50, μ = 0.2, λ = 0.1, p = 1, ξ = 0.2)

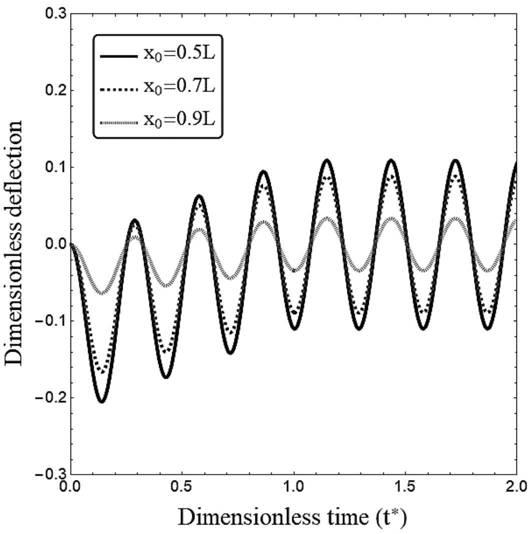


Fig. 6 Time response variation of the nanoshell based upon various load position (R/h = 5, L/h = 50, μ = 0.2, λ = 0.1, ξ = 0.1)

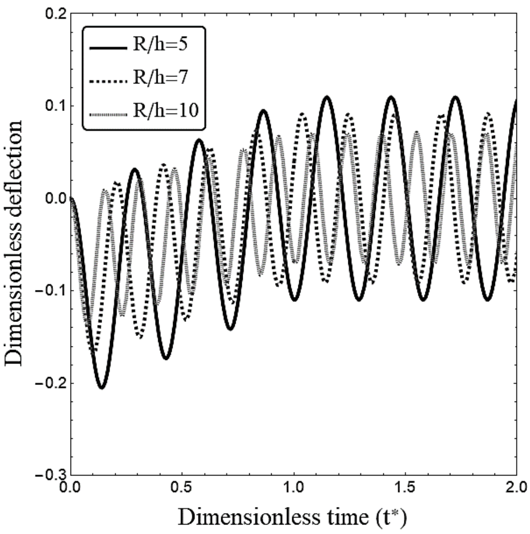


Fig. 7 Time response variation of the nanoshell based upon various radius-to-thickness ratio (L/h = 50, μ = 0.2, λ = 0.1, p = 1, ξ = 0.2)

one can see from the figure that un-even type results in lower deflections than even type. Regardless of the type of porosity distribution, increasing in the values of material gradient index results in larger dynamic deflection in both transient and free vibration zones. Based on the values amounts of pores and p, it is possible to make the structure stiffer with more capability of tolerating pulse loads.

Plotted in Figs. 5 and 6 are respectively the influences of loading duration time ($t_0$) and load location ($x_0$) on transient/free vibrations of porous FG nanoshells subjected to linear impulse load. These figures deal with FGM nanoshells based on the material gradation $p$ = 1 and also even porosity distribution. It is obvious that the vibrations number in transient region becomes higher as the magnitude of loading time becomes greater. As a deduction, it is possible to express that as loading time is lower the nanoshells pass from transient zone with fewer vibrations number. Another observation is that as the load moves away from the center point of the nanoshell, the dynamical deflections in transient region become lower. Thus, the maximum deflections occur when the impulse load is embedded at the center point of the nanoshell. Moreover, vibrations number in the transient zone remains constant with changing of loading position.

Fig. 7 illustrates the influence of radius-to-thickness ratio ($R/h$) on time response of a porous FG nanoshell exposed to various kinds of impulsive loads. In this figure, the nanoshell contains even pores based on the volume fraction of $\xi$ = 0.2. One may observe that as the radius-to-thickness ratio growths, the dynamical deflection in both transient and free vibration regions becomes smaller. This is because of the increase in the stiffness of nanoshell. Thus, the geometry of nanoshell has a great affection on its transient response.

## 5. Conclusions

The presented article focused on transient vibration behavior of a nanoshell with graded porous material properties exposed to linear typesof impulse loading based upon nonlocal strain gradient theory. Based on NSGT, two scale parameters were included into the formulation in order to capture small size effects. The problem was solved based on inverse Laplace transform method to find dynamic deflections for three types of impulse loadings. It was reported that greater values for nonlocal parameter were corresponding to lager dynamic deflections due to stiffness-softening mechanism presented by nonlocal effects. Also, stiffness-hardening mechanism due to strain gradients led to smaller deflections. It was also concluded that greater values of porosity volume fraction led to greater deflections during both transient and free oscillations. But, the magnitude of dynamic deflection or oscillation amplitudes dependent on the type of porosity distribution. Even porosities led to larger deflections than uneven porosities. Moreover, the vibrations number in the transient zone remained constant with changing of loading position.

## References


Alasadi, A.A., Ahmed, R.A. and Faleh, N.M. (2019), “Analyzing nonlinear vibrations of metal foam nanobeams with symmetric and non-symmetric porosities”, *Adv. Aircraft Spacecraft Sci., Int. J.*, **6**(4), 273-282. https://doi.org/10.12989/aas.2019.6.4.273

Atmane, H.A., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2015), “A computational shear displacement model for vibrational analysis of functionally graded beams with porosities”, *Steel Compos. Struct., Int. J.*, **19**(2), 369-384. https://doi.org/10.12989/scs.2015.19.2.369

Aydogdu, M. and Arda, M. (2016), “Forced vibration of nanorods using nonlocal elasticity”, *Adv. Nano Res., Int. J.*, **4**(4), 265-279. https://doi.org/10.12989/anr.2016.4.4.265

Azimi, M., Mirjavadi, S.S., Shafiei, N. and Hamouda, A.M.S. (2017), “Thermo-mechanical vibration of rotating axially functionally graded nonlocal Timoshenko beam”, *Appl. Phys. A*, **123**(1), 104. https://doi.org/10.1007/s00339-016-0712-5

Azimi, M., Mirjavadi, S.S., Shafiei, N., Hamouda, A.M.S. and Davari, E. (2018), "Vibration of rotating functionally graded Timoshenko nano-beams with nonlinear thermal distribution", *Mech. Adv. Mater. Struct.*, **25**(6), 467-480. https://doi.org/10.1080/15376494.2017.1285455

Barati, M.R. (2017), "Nonlocal-strain gradient forced vibration analysis of metal foam nanoplates with uniform and graded porosities", *Adv. Nano Res., Int. J.*, **5**(4), 393-414. https://doi.org/10.12989/anr.2017.5.4.393

Bensaid, I., Bekhadda, A. and Kerboua, B. (2018), "Dynamic analysis of higher order shear-deformable nanobeams resting on elastic foundation based on nonlocal strain gradient theory", *Adv. Nano Res., Int. J.*, **6**(3), 279-298. https://doi.org/10.12989/anr.2018.6.3.279

Ebrahimi, F. and Haghi, P. (2018), "Wave dispersion analysis of rotating heterogeneous nanobeams in thermal environment", *Adv. Nano Res., Int. J.*, **6**(1), 21-37. https://doi.org/10.12989/anr.2018.6.1.021

Eltaher, M.A., Emam, S.A. and Mahmoud, F.F. (2012), "Free vibration analysis of functionally graded size-dependent nanobeams", *Appl. Mathe. Computat.*, **218**(14), 7406-7420. https://doi.org/10.1016/j.amc.2011.12.090

Faleh, N.M., Ahmed, R.A. and Fenjan, R.M. (2018a), "On vibrations of porous FG nanoshells", *Int. J. Eng. Sci.*, **133**, 1-14. https://doi.org/10.1016/j.ijengsci.2018.08.007

Faleh, N.M., Fenjan, R.M. and Ahmed, R.A. (2018b), "Dynamic analysis of graded small-scale shells with porosity distributions under transverse dynamic loads", *Eur. Phys. J. Plus*, **133**(9), 348. https://doi.org/10.1140/epjp/i2018-12152-5

Fenjan, R.M., Ahmed, R.A. and Faleh, N.M. (2019), "Investigating dynamic stability of metal foam nanoplates under periodic in-plane loads via a three-unknown plate theory", *Adv. Aircraft Spacecraft Sci., Int. J.*, **6**(4), 297-314. https://doi.org/10.12989/aas.2019.6.4.297

Houari, M.S.A., Bessaim, A., Bernard, F., Tounsi, A. and Mahmoud, S.R. (2018), "Buckling analysis of new quasi-3D FG nanobeams based on nonlocal strain gradient elasticity theory and variable length scale parameter", *Steel Compos. Struct., Int. J.*, **28**(1), 13-24. https://doi.org/10.12989/scs.2018.28.1.013

Karami, B., Janghorban, M. and Tounsi, A. (2018), "Variational approach for wave dispersion in anisotropic doubly-curved nanoshells based on a new nonlocal strain gradient higher order shell theory", *Thin-Wall. Struct.*, **129**, 251-264. https://doi.org/10.1016/j.tws.2018.02.025

Ke, L.L., Wang, Y.S. and Wang, Z.D. (2012), "Nonlinear vibration of the piezoelectric nanobeams based on the nonlocal theory", *Compos. Struct.*, **94**(6), 2038-2047. https://doi.org/10.1016/j.compstruct.2012.01.023

Kheroubi, B., Benzair, A., Tounsi, A. and Semmah, A. (2016), "A new refined nonlocal beam theory accounting for effect of thickness stretching in nanoscale beams", *Adv. Nano Res., Int. J.*, **4**(4), 251-264. https://doi.org/10.12989/anr.2016.4.4.251

Lam, D.C., Yang, F., Chong, A.C.M., Wang, J. and Tong, P. (2003), "Experiments and theory in strain gradient elasticity", *J. Mech. Phys. Solids*, **51**(8), 1477-1508. https://doi.org/10.1016/S0022-5096(03)00053-X

Lee, Y.S. and Lee, K.D. (1997), "On the dynamic response of laminated circular cylindrical shells under impulse loads", *Comput. Struct.*, **63**(1), 149-157. https://doi.org/10.1016/S0045-7949(96)00312-4

Li, L. and Hu, Y. (2015), "Buckling analysis of size-dependent nonlinear beams based on a nonlocal strain gradient theory", *Int. J. Eng. Sci.*, **97**, 84-94. https://doi.org/10.1016/j.ijengsci.2015.08.013

Li, X., Li, L., Hu, Y., Ding, Z. and Deng, W. (2017), "Bending, buckling and vibration of axially functionally graded beams based on nonlocal strain gradient theory", *Compos. Struct.*, **165**, 250-265. https://doi.org/10.1016/j.compstruct.2017.01.032

Li, L., Tang, H. and Hu, Y. (2018), "Size-dependent nonlinear vibration of beam-type porous materials with an initial geometrical curvature", *Compos. Struct.*, **184**, 1177-1188. https://doi.org/10.1016/j.compstruct.2017.10.052

Mehralian, F., Beni, Y.T. and Zeverdejani, M.K. (2017), "Nonlocal strain gradient theory calibration using molecular dynamics simulation based on small scale vibration of nanotubes", *Physica B: Condensed Matter*, **514**, 61-69. https://doi.org/10.1016/j.physb.2017.03.030

Mirjavadi, S.S., Rabby, S., Shafiei, N., Afshari, B.M. and Kazemi, M. (2017a), "On size-dependent free vibration and thermal buckling of axially functionally graded nanobeams in thermal environment", *Appl. Phys. A*, **123**(5), 315. https://doi.org/10.1007/s00339-017-0918-1

Mirjavadi, S.S., Afshari, B.M., Shafiei, N., Hamouda, A.M.S. and Kazemi, M. (2017b), "Thermal vibration of two-dimensional functionally graded (2D-FG) porous Timoshenko nanobeams", *Steel Compos. Struct., Int. J.*, **25**(4), 415-426. https://doi.org/10.12989/scs.2017.25.4.415

Mirjavadi, S.S., Afshari, B.M., Barati, M.R. and Hamouda, A.M. S. (2018a), "Strain gradient based dynamic response analysis of heterogeneous cylindrical microshells with porosities under a moving load", *Mater. Res. Express*, **6**(3), 035029. https://doi.org/10.1088/2053-1591/aaf5a2

Mirjavadi, S.S., Afshari, B.M., Khezel, M., Shafiei, N., Rabby, S. and Kordnejad, M. (2018b), "Nonlinear vibration and buckling of functionally graded porous nanoscaled beams", *J. Brazil. Soc. Mech. Sci. Eng.*, **40**(7), 352. https://doi.org/10.1007/s40430-018-1272-8

Mirjavadi, S.S., Forsat, M., Hamouda, A.M.S. and Barati, M.R. (2019a), "Dynamic response of functionally graded graphene nanoplatelet reinforced shells with porosity distributions under transverse dynamic loads", *Mater. Res. Express*, **6**(7), 075045. https://doi.org/10.1088/2053-1591/ab1552

Mirjavadi, S.S., Forsat, M., Nikookar, M., Barati, M.R. and Hamouda, A.M.S. (2019b), "Nonlinear forced vibrations of sandwich smart nanobeams with two-phase piezo-magnetic face sheets", *Eur. Phys. J. Plus*, **134**(10), 508. https://doi.org/10.1140/epjp/i2019-12806-8

Mirjavadi, S.S., Afshari, B.M., Barati, M.R. and Hamouda, A.M.S. (2019c), "Transient response of porous FG nanoplates subjected to various pulse loads based on nonlocal stress-strain gradient theory", *Eur. J. Mech.-A/Solids*, **74**, 210-220. https://doi.org/10.1016/j.euromechsol.2018.11.004

Mirjavadi, S.S., Afshari, B.M., Barati, M.R. and Hamouda, A.M. S. (2019d), "Nonlinear free and forced vibrations of graphene nanoplatelet reinforced microbeams with geometrical imperfection", *Microsyst. Technol.*, **25**, 3137-3150. https://doi.org/10.1007/s00542-018-4277-4

Mirjavadi, S.S., Forsat, M., Barati, M.R., Abdella, G.M., Hamouda, A.M.S., Afshari, B.M. and Rabby, S. (2019e), "Post-buckling analysis of piezo-magnetic nanobeams with geometrical imperfection and different piezoelectric contents", *Microsyst. Technol.*, **25**(9), 3477-3488. https://doi.org/10.1007/s00542-018-4241-3

Mirjavadi, S.S., Forsat, M., Barati, M.R., Abdella, G.M., Afshari, B.M., Hamouda, A.M.S. and Rabby, S. (2019f), "Dynamic response of metal foam FG porous cylindrical micro-shells due to moving loads with strain gradient size-dependency", *Eur. Phys. J. Plus*, **134**(5), 214. https://doi.org/10.1140/epjp/i2019-12540-3

Mohammadi, K., Mahinzare, M., Ghorbani, K. and Ghadiri, M. (2018), "Cylindrical functionally graded shell model based on the first order shear deformation nonlocal strain gradient elasticity theory", *Microsyst. Technol.*, **24**(2), 1133-1146. https://doi.org/10.1007/s00542-017-3476-8

Shafiei, N., Mirjavadi, S.S., Afshari, B.M., Rabby, S. and Hamouda, A.M.S. (2017), "Nonlinear thermal buckling of axially functionally graded micro and nanobeams", *Compos. Struct.*, **168**, 428-439. https://doi.org/10.1016/j.compstruct.2017.02.048

She, G.L., Yuan, F.G., Ren, Y.R., Liu, H.B. and Xiao, W.S. (2018), "Nonlinear bending and vibration analysis of functionally graded porous tubes via a nonlocal strain gradient theory", *Compos. Struct.*, **203**, 614-623. https://doi.org/10.1016/j.compstruct.2018.07.063

Tang, Y. and Yang, T. (2018), "Post-buckling behavior and nonlinear vibration analysis of a fluid-conveying pipe composed of functionally graded material", *Compos. Struct.*, **185**, 393-400. https://doi.org/10.1016/j.compstruct.2017.11.032

Tang, Y., Lv, X. and Yang, T. (2019), "Bi-directional functionally graded beams: asymmetric modes and nonlinear free vibration", *Compos. Part B: Eng.*, **156**, 319-331. https://doi.org/10.1016/j.compositesb.2018.08.140

Uzun, B. and Civalek, O. (2019), "Free vibration analysis Silicon nanowires surrounded by elastic matrix by nonlocal finite element method", *Adv. Nano Res., Int. J.*, **7**(2), 99-108. https://doi.org/10.12989/anr.2019.7.2.099

Zenkour, A.M. (2016), "Buckling of a single-layered graphene sheet embedded in visco-Pasternak's medium via nonlocal first-order theory", *Adv. Nano Res., Int. J.*, **4**(4), 309-326. https://doi.org/10.12989/anr.2016.4.4.309

Zhen, Y.X., Wen, S.L. and Tang, Y. (2019), "Free vibration analysis of viscoelastic nanotubes under longitudinal magnetic field based on nonlocal strain gradient Timoshenko beam model", *Physica E: Low-dimens. Syst. Nanostruct.*, **105**, 116-124. https://doi.org/10.1016/j.physe.2018.09.005

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