**Advances in Nano Research**, *Vol. 7 No. 6 (2019) 391-403*
DOI: https://doi.org/10.12989/anr.2019.7.6.391


# Frequency response analysis of curved embedded magneto-electro-viscoelastic functionally graded nanobeams

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(*Received April 3, 2019, Revised May 16, 2019, Accepted June 8, 2019*)

**Abstract.** In this article the frequency response analysis of curved magneto-electro-viscoelastic functionally graded (CMEV-FG) nanobeams resting on viscoelastic foundation has been carried out. To this end, the study incorporates the Euler-Bernoulli beam model in association with Eringen's nonlocal theory to incorporate the size effects. The viscoelastic foundation in the current investigation is assumed to be the combination of Winkler-Pasternak layer and viscous layer of infinite parallel dashpots. The equations of motion are derived with the aid of Hamilton's principle and the solution to vibration problem of CMEV-FG nanobeams are obtained analytically. The material gradation is considered to follow Power-law rule. This study thoroughly investigates the influence of prominent parameters such as linear, shear and viscous layers of foundation, structural damping coefficient, opening angle, magneto-electrical field, nonlocal parameter, power-law exponent and slenderness ratio on the frequencies of FG nanobeams.

**Keywords:** curved nanobeaml free vibration; magneto-electro-viscoelastic materials; FGM; nonlocal elasticity

## 1. Introduction

The recent development in the field of engineering materials has disclosed the advantages associated with the smart/intelligent materials. Incorporation of these smart materials in various multifunctional structures has paved way for tremendous changes in different engineering fields. Among them, magneto-electro-elastic (MEE) materials are unique as a matter of fact that it exhibits triple energy conversion between elastic, electric and magnetic fields (Vinyas *et al.* 2018a). Therefore, it has become a potential candidate for sophisticated applications such as vibration control (Vinyas 2019, Vinyas and Kattimani 2019), energy harvesting, sensors and actuators etc. More recently, attempts were made to synthesize MEE structures through functionally graded (FG) materials and improvise the structural functionalities. Having realized that the smart structures made of magneto-electro-elastic functionally graded (MEE-FG) materials play a significant role in industrial fields many pioneers have devoted their research to assess the mechanical response in various working environments. Among them, Bhangale and Ganesan (2005) developed a finite element (FE) formulation and evaluated the vibration response of MEE-FG cylindrical shells. Pan and Han (2005) presented exact solution to assess the structural response of MEE-FG plates. Huang *et al.* (2007) proposed an analytical solution to deal with the vibration problem of MEE-FG beams. Through asymptotic approach Wu and Tsai (2007) investigated the bending response of MEE-FG shell. Analogously, Wu *et al.* (2010) exploited the Pagano method to demonstrate the static behaviour of MEE-FG plates. More recently, using FE methods, Vinyas and Kattimani contributed to the research community through their works which discuss the effect of various forms of thermal loading on the coupled static response of MEE-FG beams (Vinyas and Kattimani 2017) and MEE-FG plates (Vinyas and Kattimani 2017f). Extending their evaluation on the same grounds, the influence of moisture was also briefed out (Vinyas and Kattimani 2017g, Vinyas *et al.* 2018b). Nonlinear frequency response analysis of MEE-FG plates was performed by Kattimani and Ray (2015) using FE methods. In hygrothermal environment, the ability of higher order shear deformation theory (HSDT) to consider coupling effects to evaluate the natural frequencies of MEE-FG plates was thoroughly investigated by Vinyas and Kattimani (2018). In addition, the influence of carbon nano-tubes (CNT) on the frequency response of MEE plates with the aid of HSDT was assessed through a FE formulation by Vinyas (2019b). The free vibration behaviour of skew MEE plate was discussed by Vinyas *et al.* (2019) through HSDT. Sladek *et al.* (2015) studied the static response of MEE-FG circular plate incorporating meshless method. Bending behavior of layered FG neutral magneto-electro-elastic plates on elastic foundations is analyzed by Lezgy-Nazargah and Cheraghi (2015).

Unlike the macroscopic structures, nanostructures behave uniquely due to size effects. At nano range, significant influence of size effects is noticed on both physical as well as the mechanical properties. This phenomenon has motivated few researchers to divert their focus towards assessing the mechanical response of the nanostructures. The major limitation of the classical

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http://www.techno-press.org/?journal=journal=anr&subpage=5
ISSN: 2287-237X (Print), 2287-2388 (Online)

continuum mechanics is its inefficiency to model small size structures which paved way for the establishment of higher order continuum theories which incorporates the size-dependency of structure with ease. The Eringen's nonlocal elasticity theory (Eringen 1972, 1983) proved to be handy in employing the size-effects. In this regard, many articles have been published to make the best utilization of this theory in evaluating the size-dependent structural response (Aydogdu 2009, Thai 2012, Ebrahimi and Hosseini 2016a, Barati *et al*. 2016). The major outcome these researches indicate that with the higher value of nonlocal parameter, that nonlocal elastic models are efficient enough only to yield stiffness-softening effect. Incorporating the Eringen's nonlocal elasticity theory few researchers attempted to analyze the FGM nanostructures. Eltaher *et al*. (2012, 2013) demonstrated the procedure to evaluate the stability and frequency characteristics of nonlocal Euler-Bernoulli FG nanobeams by FE methods. Based on nonlocal Timoshenko beam model and nonlocal third-order shear deformable beam model, the stability characteristics of FG nanobeams was studied by Şimşek and Yurtcu (2013) and Rahmani and Jandaghian (2015), respectively. Ebrahimi *et al*. (2015a) proposed a semi-analytical formulation to solve the vibration problem of FG nanobeams. The influence of external thermal environment on the frequencies of temperature-dependent FGM nanobeams was studied thoroughly by Ebrahimi and Salari (2015a, b). Meanwhile, the literatures on the vibration (Beni 2016) and buckling response (Ebrahimi and Salari 2015a, 2016, Ebrahimi and Barati 2016a-f, Ebrahimi *et al*. 2015a, 2016, Ebrahimi and Hosseini 2016a-c) of analysis of piezoelectric FG nano beams also have significantly contributed to the research community to understand the benefit of nonlocal theories.

The results of the dynamic study of the nanostructures can be severely altered when the effect of damping is neglected. Therefore, it becomes very crucial to adapt damping mechanism to model the nano-electro-mechanical systems (NEMS) precisely. The main factors that contribute to the damping phenomenon include external forces and substrate interaction with the foundation. In any structural design, investigating the structure-foundation interaction is of much importance. More commonly, as witnessed in the literature review, the interaction of the beams resting on elastic foundation is considered as a conventional problem. However, for some applications it requires that the nanobeams are rooted in viscoelastic medium composed of an infinite set of dashpots and springs connected in parallel. In this regard, Lei *et al*. (2013) and Pouresmaeeli *et al*. (2013) investigated the influence of viscoelastic foundation on the frequency response of size-dependent Kelvin–Voigt viscoelastic damped Timoshenko nanobeams and viscoelastic orthotropic nanoscale plates, respectively. Similarly, Hashemi et al. (2015) investigated the natural frequency characteristics of double layered viscoelastic graphene sheets embedded in visco-Pasternak medium. Ananalytical solution was proposed by Hosseini and Jamalpoor (2015) to study the effects of thermal loads on the dynamic characteristics of double-viscoelastic FGM nanoplates.

Following the development of FG nanobeams, much investigation on FG curved nanobeam was made considering its extensive applications in NEMS like nano-switches, nano-valves and nano-filters. In contrast to straight nanobeams, predominant characteristics such as bi-stability nature and efficient large stroke performance are exhibited by the curved beams. The frequencies of isotropic curved nanobeams and rings were studied by Assadi and Farshi (2011) incorporating the surface energies. Considering the surface effects, electromechanical response of curved piezoelectric nanobeams was investigated by Yan and Jiang (2011). The dynamic behaviour of curved nanobeams was studied by Kananipour *et al*. (2014) under the framework of nonlocal elasticity theory. On the basis of nonlocal Euler-Bernoulli beam model the stability analysis of embedded curved nanotubes subjected to thermal effects was probed by Setoodeh *et al*. (2015). The static behaviour of variable curvature nanobeams was demonstrated by Tufekci *et al*. (2016). Even though several articles have been reported on dynamic analysis of isotropic curved nanobeams, rings, and arches, very limited research has been done on frequency study of FGM curved nanobeams. In this regard, using nonlocal curved beam model a first attempt was made by Hosseini and Rahmani (2016) to analyze the frequencies of deep curved FG nanobeams. She *et al*. (2019a) estimated investigated the snap-buckling behaviour of porous functionally graded curved nano beam using nonlocal strain gradient theory. She and co-researchers (She *et al*. 2019b) extended their evaluation to probe the nonlinear bending behaviour of FG porous curved nano beam through non-local strain gradient theory.

In this article a first attempt has been made to evaluate the frequency response characteristics of curved magneto-electro-viscoelastic functionally graded (CMEV-FG) nanobeams through nonlocal elasticity theory. The influence of visco-Pasternak foundation composed of parallel springs and dashpots as well as a shear layer is also considered for evaluation. The equations of motion are derived with the aid of Hamilton's principle and the solution to vibration problem of CMEV-FG nanobeams are obtained analytically. The material gradation is considered to follow Power-law rule. This study thoroughly investigates the influence of prominent parameters such as linear, shear and viscous layers of foundation, structural damping coefficient, opening angle, magneto-electrical field, nonlocal parameter, power-law exponent and slenderness ratio on the frequencies of FG nanobeams.

## 2. Theory and formulation

### *2.1 The nonlocal elasticity model for magneto-electro-elastic nanobeams*

According to Eringen's nonlocal theory (Eringen 1972, 1983), the state of stress existing at an entity is dependent on the strains of the remaining points. In consideration with this theory, the relation pertaining to the nonlocal MEE structural entity can be shown as follows

$$\sigma_{ij}=\int_V \alpha(|x'-x|,\tau)\,[C_{ijkl}\varepsilon_{kl}(x') -e_{mij}E_m(x')-q_{nij}H_n(x')]dV(x') \quad (1)$$

$$D_i=\int_V \alpha(|x'-x|,\tau)\,[e_{ikl}\varepsilon_{kl}(x') +s_{im}E_m(x')+d_{in}H_n(x')]dV(x') \quad (2)$$

$$B_i=\int_V \alpha(|x'-x|,\tau)\,[q_{ikl}\varepsilon_{kl}(x') +d_{im}E_m(x')+\chi_{in}H_n(x')]dV(x') \quad (3)$$

in which, $\sigma_{ij}$, $\varepsilon_{ij}$, $D_i$, $E_i$, $B_i$ and $H_i$ represents the components of stress, strain, electric displacement, electric field components, magnetic induction and magnetic field, respectively; Similarly, the elastic constant is denoted by $C_{ijkl}$, $e_{mij}$ are piezoelectric constants, $s_{im}$ are dielectric constants, $q_{nij}$ are piezomagnetic constants. Further, $d_{ij}$ and $\chi_{ij}$ represents magneto-electric and magnetic constants, respectively. Meanwhile, the nonlocal kernel function is represented by the term $\alpha(|x'-x|,\tau)$ and $|x'-x|$ denotes the Euclidean distance. Adapting differential form of Eringen's nonlocal theory, the constitutive relations of a MEE solid can be explicitly represented as follows (Ebrahimi and Barati 2016a, b)

$$\sigma_{ij}-(e_0a)^2\nabla^2\sigma_{ij}=C_{ijkl}\varepsilon_{kl}-e_{mij}E_m-q_{nij}H_n \quad (4)$$

$$D_i-(e_0a)^2\nabla^2 D_i=e_{ikl}\varepsilon_{kl}+s_{im}E_m+d_{in}H_n \quad (5)$$

$$B_i-(e_0a)^2\nabla^2 B_i=q_{ikl}\varepsilon_{kl}+d_{im}E_m+\chi_{in}H_n \quad (6)$$

in the Eqs. (4)-(6), the Laplacian operator is represented as $\nabla^2$ and the small size effect is introduced through the nonlocal parameter $e_0a$.

### *2.2 Effective properties of P-FGM curved nanobeam*

The schematic representation of a curved MEE-FG nanobeam is depicted along with its coordinates in Fig. 1. The geometrical length and thickness are denoted by $L$ and $h$, respectively. The gradation of the material properties P($z$) across the thickness of curved MEE-FG is assumed to follow power law function Hence, the distribution of MEE material properties P($z$) can be represented by (Ebrahimi and Barati 2017)

$$P(z)=(P_u-P_l)\left(\frac{z}{h}+\frac{1}{2}\right)^p+P_l \quad (7)$$

where, $p$, $P_u$ and $P_l$ represents the power-law exponent, the material property at top (pure $BaTiO_3$) and bottom side (pure $CoFe_2O_4$), respectively, whose material properties are listed in Table 1.

### *2.3 Kinematic relations*

The displacement components of curved MEE-FG nanobeam is assumed to curved Euler–Bernoulli beam model, based on which the radial displacement $w_r$ and tangential displacement $u_\theta$ can be written as

$$u_\theta(\theta,r,t)=\left(1+\frac{z}{R}\right)u(\theta,t)+\frac{z}{R}\left(\frac{\partial w(\theta,t)}{\partial\theta}\right) \quad (8)$$

$$w_r(\theta,r,t)=-w(\theta,t) \quad (9)$$

where, $u$ and $w$ denote displacement components of the mid-surface in tangential and radial directions, respectively. Meanwhile, considering Maxwell's electromagnetic equations, the distributions of electric and magnetic field distributions can be approximated as follows (Ebrahimi and Barati 2016f)

Table 1 Magneto-electro -elastic coefficients of material properties

| | $BaTiO_3$ | $CoFe_2O_4$ |
|---|---|---|
| $c_{11}$ (GPa) | 166 | 286 |
| $e_{31}$ $(Cm^{-2})$ | -4.4 | 0 |
| $q_{31}$ (N/Am) | 0 | 580.3 |
| $s_{11}$ $(10^{-9}C^2m^{-2}N^{-1})$ | 11.2 | 0.08 |
| $s_{33}$ | 12.6 | 0.093 |
| $\chi_{11}(10^{-6}Ns^2C^{-2}/2)$ | 5 | -590 |
| $\chi_{33}$ | 10 | 157 |
| $d_{11}=d_{33}$ | 0 | 0 |
| $\rho$ $(kgm^{-3})$ | 5800 | 5300 |

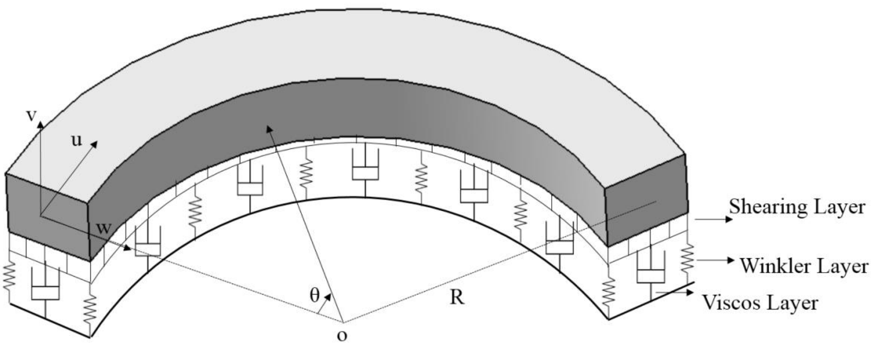


Fig. 1 Geometry and coordinates of curved FG nanobeam resting on viscoelastic medium

$$\Phi(x,z,t) = -\cos(\xi z)\phi(x,t) + \frac{2z}{h}V \tag{10}$$

$$\Upsilon(x,z,t) = -\cos(\xi z)\gamma(x,t) + \frac{2z}{h}\Omega \tag{11}$$

where $\xi = \pi/h$. The electric voltage and the corresponding electric potential are represented as $V$ and $\phi$, respectively. Analogously the magnetic field intensity and magnetic potential are denoted through $\Omega$ and $\gamma$, respectively. The nonzero normal strain is

$$\varepsilon = \varepsilon^0 + zk^0, \qquad \varepsilon^0 = \frac{1}{R}\left(-w + \frac{\partial u}{\partial \theta}\right), \qquad k^0 = \frac{1}{R^2}\left(\frac{\partial u}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2}\right) \tag{12}$$

In Eq. (12), the strains $\varepsilon^0$ and $k^0$ can be bifurcated into extensional and bending strains, respectively. The relationship between the magneto-electric potential and the non-zero components of electric and magnetic fields $(E_\theta, E_z, H_\theta, H_z)$ can be established through Eqs. (10) and (11), as follows

$$E_\theta = -\Phi_{,x} = \cos(\xi z)\frac{\partial \phi}{R\partial \theta}, \qquad E_z = -\Phi_{,z} = -\xi \sin(\xi z)\phi - \frac{2V}{h} \tag{13}$$

$$H_\theta = -\Upsilon_{,x} = \cos(\xi z)\frac{\partial \gamma}{R\partial \theta}, \qquad H_z = -\Upsilon_{,z} = -\xi \sin(\xi z)\gamma - \frac{2\Omega}{h} \tag{14}$$

Invoking Hamilton's principle

$$\int_0^t \delta(\Pi_S - \Pi_K + \Pi_W)dt = 0 \tag{15}$$

Here $\Pi_S$, $\Pi_K$ and $\Pi_W$ are strain energy, kinetic energy and external forces work, respectively. The strain energy can be written as

$$\delta\Pi_S = \int_v \sigma_{ij}\delta\varepsilon_{ij}dV = \int_v (\sigma_{\theta\theta}\delta\varepsilon_{\theta\theta} - D_\theta\delta E_\theta - D_z\delta E_z - B_\theta\delta H_\theta - B_z\delta H_z)dV \tag{16}$$

Inserting Eq. (14) into Eq. (11) gives

$$\delta\Pi_S = \int_0^\alpha \left(N(\delta\varepsilon^0) + M(\delta k^0)\right)Rd\theta + \int_0^\alpha \int_{-\frac{h}{2}}^{\frac{h}{2}} \Big[-\frac{D_\theta}{R}\cos(\beta z)\delta\left(\frac{\partial \phi}{\partial \theta}\right) + D_z\beta\sin(\beta z)\delta\phi - \frac{B_\theta}{R}\cos(\beta z)\delta\left(\frac{\partial \gamma}{\partial \theta}\right) + B_z\beta\sin(\beta z)\delta\gamma\Big]Rdzd\theta \tag{17}$$

in which $N$, $M$ respectively denote the axial force and bending moment. The available stress resultants in Eq. (18) are defined by

$$N = \int_A \sigma_{xx}\,dA, \qquad M = \int_A \sigma_{xx}z\,dA \tag{18}$$

The variation of kinetic energyis written as

$$\delta\Pi_K = \int_0^\alpha [I_0(\dot{u}\delta\dot{u} + \dot{w}\delta\dot{w}) + \frac{I_1}{R}\left(2\dot{u}\delta\dot{u} + \dot{u}\frac{\partial\delta\dot{w}}{\partial\theta} + \delta\dot{u}\frac{\partial\dot{w}}{\partial\theta}\right) + \frac{I_2}{R^2}\left(\dot{u}\delta\dot{u} + \dot{u}\frac{\partial\delta\dot{w}}{\partial\theta} + \delta\dot{u}\frac{\partial\dot{w}}{\partial\theta} + \frac{\partial\dot{w}}{\partial\theta}\frac{\partial\delta\dot{w}}{\partial\theta}\right]Rd\theta \tag{19}$$

where $(I_0, I_1, I_2)$ are the mass moment of inertias, defined as follows

$$(I_0, I_1, I_2) = \int_A \rho(z)(1, z, z^2)\,dA \tag{20}$$

Thus, the variationworks done by external loadscan be written as

$$\delta\Pi_W = \int_0^L \Big[\frac{(N^E + N^M)}{R^2}\left(\frac{\partial w}{\partial\theta}\frac{\partial\delta w}{\partial\theta}\right) + q\delta w\Big]Rd\theta \tag{21}$$

where $N^E$ and $N^M$ are applied electric and magnetic loads which is defined as

$$N^E = -\int_{-h/2}^{h/2} e_{31}\frac{2V}{h}dz \tag{22a}$$

$$N^M = -\int_{-h/2}^{h/2} q_{31}\frac{2\Omega}{h}dz \tag{22b}$$

The external transverse load $q$from viscoelastic medium are expressed by

$$q = -k_w w + \frac{k_p}{R^2}\frac{\partial^2 w}{\partial\theta^2} - c_d\frac{\partial w}{\partial t} \tag{23}$$

The nonlocal constitutive relations (8) and (9) may be rewritten for a curved Magneto-electro-elasticEuler-Bernoulli nanobeam as

$$\sigma_{\theta\theta} - \mu\frac{\partial^2\sigma_{\theta\theta}}{R^2\partial\theta^2} = c_{11}\varepsilon_{\theta\theta} - e_{31}E_z - q_{31}H_z \tag{24}$$

$$D_\theta - \mu\frac{\partial^2 D_\theta}{R^2\partial\theta^2} = s_{11}E_\theta + d_{11}H_\theta \tag{25}$$

$$D_z - \mu\frac{\partial^2 D_z}{R^2\partial\theta^2} = e_{31}\varepsilon_{\theta\theta} + s_{33}E_z + d_{33}H_z \tag{26}$$

$$B_\theta - \mu\frac{\partial^2 B_\theta}{R^2\partial\theta^2} = d_{11}E_\theta + \chi_{11}H_\theta \tag{27}$$

$$B_z - \mu\frac{\partial^2 B_z}{R^2\partial\theta^2} = q_{31}\varepsilon_{\theta\theta} + d_{33}E_z + \chi_{33}H_z \tag{28}$$

where $\mu = ea^2$. The following governing equations are obtained by inserting Eqs. (17)-(21) in Eq. (15) when the coefficients of $\delta u, \delta w$ and $\delta\phi$ are equal to zero

$$-\frac{\partial N}{\partial \theta}-\frac{1}{R}\frac{\partial M}{\partial \theta}=-RI_0\ddot{u}-I_1\left(2\ddot{u}+\frac{\partial \ddot{w}}{\partial \theta}\right)-\frac{I_2}{R}\left(\ddot{u}+\frac{\partial \ddot{w}}{\partial \theta}\right) \quad (29)$$

$$\frac{1}{R}\frac{\partial^2 M}{\partial \theta^2}-N-\frac{(N^E+N^M)}{R}\frac{\partial^2 w}{\partial \theta^2}=-RI_0\ddot{w}+I_1\frac{\partial \ddot{u}}{\partial \theta}+\frac{I_2}{R}\left(\frac{\partial^2 \ddot{w}}{\partial \theta^2}+\frac{\partial \ddot{u}}{\partial \theta}\right)+k_wRw-\frac{k_p}{R}\frac{\partial^2 w}{\partial \theta^2}+c_dR\dot{w} \quad (30)$$

$$\int_{-h/2}^{h/2}\left(cos(\beta z)\frac{1}{R}\frac{\partial D_\theta}{\partial \theta}+\beta\, sin(\beta z)D_z\right)dz=0 \quad (31)$$

$$\int_{-h/2}^{h/2}\left(cos(\beta z)\frac{1}{R}\frac{\partial B_\theta}{\partial \theta}+\beta\, sin(\beta z)B_z\right)dz=0 \quad (32)$$

Under the following boundary conditions

$$N+\frac{M}{R}=0 \text{ or } u=0 \text{ at } \theta=0 \text{ and } \theta=\alpha \quad (33)$$

$$\frac{\partial M}{R\partial x}+I_1\ddot{u}+\frac{I_2}{R}\left(\ddot{u}+\frac{\partial \ddot{w}}{\partial x}\right)=0 \text{ or } w=0 \quad at\ \ \theta=0 \text{ and } \theta=\alpha \quad (34)$$

$$M=0 \text{ or } \frac{\partial w}{\partial \theta}=0 \quad at\ \ \theta=0 \text{ and } \theta=\alpha \quad (35)$$

$$\int_A D_\theta\, cos(\beta z)\, dA=0 \text{ or } \phi=0 \quad \text{at } \theta=0 \text{ and } \theta=\alpha \quad (36)$$

$$\int_A B_\theta\, cos(\beta z)\, dA=0 \text{ or } \gamma=0 \quad \text{at } \theta=0 \text{ and } \theta=\alpha \quad (37)$$

The nonlocal relations related to the magneto-electro-viscoelastic nanobeam model can be attained by integrating Eqs. (24)-(28) by incorporating the Kelvin's model on elastic materials with viscoelastic structural damping coefficient ($g$) as follows

$$N-\mu\frac{\partial^2 N}{R^2\partial \theta^2}=\left(1+g\frac{\partial}{\partial t}\right)\left[\frac{A_{11}}{R}\left(-w+\frac{\partial u}{\partial \theta}\right)+\frac{B_{11}}{R^2}\left(\frac{\partial u}{\partial \theta}+\frac{\partial^2 w}{\partial \theta^2}\right)\right]+A_{31}^e\phi+A_{31}^m\gamma-N_x^E-N_x^M \quad (38)$$

$$M-\mu\frac{\partial^2 M}{R^2\partial \theta^2}=\left(1+g\frac{\partial}{\partial t}\right)\left[\frac{B_{11}}{R}\left(-w+\frac{\partial u}{\partial \theta}\right)+\frac{D_{11}}{R^2}\left(\frac{\partial u}{\partial \theta}+\frac{\partial^2 w}{\partial \theta^2}\right)\right]+E_{31}^e\phi+E_{31}^m\gamma-M_x^E-M_x^M \quad (39)$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}}\left\{D_\theta-\frac{\mu}{R^2}\frac{\partial^2 D_\theta}{\partial \theta^2}\right\}cos(\xi z)dz=+\frac{F_{11}^e}{R}\frac{\partial \phi}{\partial \theta}+\frac{F_{11}^m}{R}\frac{\partial \gamma}{\partial \theta} \quad (40)$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}}\left\{D_z-\frac{\mu}{R^2}\frac{\partial^2 D_z}{\partial \theta^2}\right\}\xi\, sin(\xi z)dz=\frac{A_{31}^e}{R}\left(-w+\frac{\partial u}{\partial \theta}\right)+\frac{E_{31}^e}{R^2}\left(\frac{\partial u}{\partial \theta}+\frac{\partial^2 w}{\partial \theta^2}\right)-F_{33}^e\phi-F_{33}^m\gamma \quad (41)$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}}\left\{B_\theta-\frac{\mu}{R^2}\frac{\partial^2 B_\theta}{\partial \theta^2}\right\}cos(\xi z)dz=+\frac{F_{11}^m}{R}\frac{\partial \phi}{\partial \theta}+\frac{X_{11}^m}{R}\frac{\partial \gamma}{\partial \theta} \quad (42)$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}}\left\{B_z-\frac{\mu}{R^2}\frac{\partial^2 B_z}{\partial \theta^2}\right\}\xi\, sin(\xi z)dz=\frac{A_{31}^m}{R}\left(-w+\frac{\partial u}{\partial \theta}\right)+\frac{E_{31}^m}{R^2}\left(\frac{\partial u}{\partial \theta}+\frac{\partial^2 w}{\partial \theta^2}\right)-F_{33}^m\phi-X_{33}^m\gamma \quad (43)$$

The cross-sectional rigidities are defined as follows

$$(A_{11},B_{11},D_{11})=\int_{-h/2}^{h/2}c_{11}(z)(1,z,z^2)\,dz \quad (44)$$

$$\{A_{31}^e,E_{31}^e\}=\int_{-h/2}^{h/2}e_{31}\{\xi\, sin(\xi z),z\xi\, sin(\xi z)\}\,dz \quad (45)$$

$$\{A_{31}^m,E_{31}^m\}=\int_{-h/2}^{h/2}q_{31}\{\xi\, sin(\xi z),z\xi\, sin(\xi z)\}\,dz \quad (46)$$

$$\{F_{11}^e,F_{33}^e\}=\int_{-h/2}^{h/2}\{s_{11}\,cos^2(\xi z),s_{33}\xi^2\,sin^2(\xi z)\}\,dz \quad (47)$$

$$\{F_{11}^m,F_{33}^m\}=\int_{-h/2}^{h/2}\{d_{11}\,cos^2(\xi z),d_{33}\xi^2\,sin^2(\xi z)\}\,dz \quad (48)$$

$$\{X_{11}^m,X_{33}^m\}=\int_{-h/2}^{h/2}\{\chi_{11}\,cos^2(\xi z),\chi_{33}\xi^2\,sin^2(\xi z)\}\,dz \quad (49)$$

And

$$N_x^E=-\int_{-\frac{h}{2}}^{\frac{h}{2}}e_{31}\frac{2V}{h}dz,\quad N_x^M=-\int_{-h/2}^{h/2}q_{31}\frac{2\Omega}{h}dz \quad (50)$$

$$M_x^E=-\int_{-\frac{h}{2}}^{\frac{h}{2}}e_{31}\frac{2V}{h}zdz,\quad M_x^M=-\int_{-h/2}^{h/2}q_{31}\frac{2\Omega}{h}zdz \quad (51)$$

The governing equations of a CMEV-FG can be obtained inserting Eqs. (38)-(43), respectively, into Eqs. (29)-(32) as

$$\left(1+g\frac{\partial}{\partial t}\right)\left[\frac{A_{11}}{R}\left(-\frac{\partial w}{\partial\theta}+\frac{\partial^2 u}{\partial\theta^2}\right)+\frac{B_{11}}{R^2}\left(-\frac{\partial w}{\partial\theta}+2\frac{\partial^2 u}{\partial\theta^2}+\frac{\partial^3 w}{\partial\theta^3}\right)+\frac{D_{11}}{R^3}\left(\frac{\partial^2 u}{\partial\theta^2}+\frac{\partial^3 w}{\partial\theta^3}\right)\right]$$
$$+A_{31}^e\frac{\partial\phi}{\partial\theta}+\frac{E_{31}^e}{R}\frac{\partial\phi}{\partial\theta}+A_{31}^m\frac{\partial\gamma}{\partial\theta}+\frac{E_{31}^m}{R}\frac{\partial\gamma}{\partial\theta}$$
$$-RI_0\ddot{u}-I_1\left(2\ddot{u}+\frac{\partial\ddot{w}}{\partial\theta}\right)-\frac{I_2}{R}\left(\ddot{u}+\frac{\partial\ddot{w}}{\partial\theta}\right)\tag{52}$$
$$+\frac{\mu}{R^2}\left(+RI_0\frac{\partial^2\ddot{u}}{\partial\theta^2}+I_1\left(2\frac{\partial^2\ddot{u}}{\partial\theta^2}+\frac{\partial^3\ddot{w}}{\partial\theta^3}\right)+\frac{I_2}{R}\left(\frac{\partial^2\ddot{u}}{\partial\theta^2}+\frac{\partial^3\ddot{w}}{\partial\theta^3}\right)\right)=0$$

$$\left(1+g\frac{\partial}{\partial t}\right)\left[\frac{A_{11}}{R}\left(-w+\frac{\partial u}{\partial\theta}\right)+\frac{B_{11}}{R^2}\left(\frac{\partial u}{\partial\theta}+2\frac{\partial^2 w}{\partial\theta^2}-\frac{\partial^3 u}{\partial\theta^3}\right)-\frac{D_{11}}{R^3}\left(\frac{\partial^3 u}{\partial\theta^3}+\frac{\partial^4 w}{\partial\theta^4}\right)\right]-\frac{E_{31}^e}{R}\frac{\partial^2\phi}{\partial\theta^2}+A_{31}^e\phi$$
$$-\frac{E_{31}^m}{R}\frac{\partial^2\gamma}{\partial\theta^2}+A_{31}^m\gamma+k_wRw-\frac{k_p}{R}\frac{\partial^2 w}{\partial\theta^2}$$
$$+c_dR\dot{w}-RI_0\ddot{w}+I_1\frac{\partial\ddot{u}}{\partial\theta}+\frac{I_2}{R}\left(\frac{\partial^2\ddot{w}}{\partial\theta^2}+\frac{\partial\ddot{u}}{\partial\theta}\right)\tag{53}$$
$$+\left(\frac{N^E+N^M}{R}\right)\frac{\partial^2 w}{\partial\theta^2}+\frac{\mu}{R^2}\left(+RI_0\ddot{w}-I_1\frac{\partial\ddot{u}}{\partial\theta}-\frac{I_2}{R}\left(\frac{\partial^2\ddot{w}}{\partial\theta^2}+\frac{\partial\ddot{u}}{\partial\theta}\right)-\left(\frac{N^E+N^M}{R}\right)\frac{\partial^4 w}{\partial\theta^4}\right.$$
$$\left.-k_wR\frac{\partial^2 w}{\partial\theta^2}+\frac{k_p}{R}\frac{\partial^4 w}{\partial\theta^4}-c_dR\frac{\partial^2\dot{w}}{\partial\theta^2}\right)=0$$

$$\frac{F_{11}^e}{R}\frac{\partial^2\phi}{\partial\theta^2}+\frac{F_{11}^m}{R}\frac{\partial^2\gamma}{\partial\theta^2}+A_{31}^e\left(-w+\frac{\partial u}{\partial\theta}\right)+\left(\frac{\partial u}{\partial\theta}+\frac{\partial^2 w}{\partial\theta^2}\right)-RF_{33}^e\phi-RF_{33}^m\gamma=0\tag{54}$$

$$\frac{F_{11}^m}{R}\frac{\partial^2\phi}{\partial\theta^2}+\frac{X_{11}^m}{R}\frac{\partial^2\gamma}{\partial\theta^2}+A_{31}^m\left(-w+\frac{\partial u}{\partial\theta}\right)+\frac{E_{31}^m}{R}\left(\frac{\partial u}{\partial\theta}+\frac{\partial^2 w}{\partial\theta^2}\right)-RF_{33}^m\phi-RX_{33}^m\gamma=0\tag{55}$$

## 3. Solution procedure

The nonlocal governing equations derived previously are solved analytically. To satisfy the simply-supported boundary conditions, the following solution for displacement variables is employed

$$u(\theta,t)=\sum_{n=1}^{\infty}U_n\,cos[\frac{n\pi}{\theta}\alpha]\,e^{i\omega_n t}\tag{56}$$

$$w(\theta,t)=\sum_{n=1}^{\infty}W_n\,sin[\frac{n\pi}{\theta}\alpha]\,e^{i\omega_n t}\tag{57}$$

$$\phi(\theta,t)=\sum_{n=1}^{\infty}\Phi_n\,sin[\frac{n\pi}{\theta}\alpha]\,e^{i\omega_n t}\tag{58}$$

$$\gamma(\theta,t)=\sum_{n=1}^{\infty}\Upsilon_n\,sin[\frac{n\pi}{\theta}\alpha]\,e^{i\omega_n t}\tag{59}$$

in which $(U_n, W_n, \Phi_n, \Upsilon_n)$ are the unknown Fourier coefficients. Inserting Eqs. (56) and (59) into Eqs. (52) to (55) respectively, leads to

$$\{[K]+[C]\omega+[M]\omega^2\}\begin{Bmatrix}U_n\\W_n\\\Phi_n\\\Upsilon_n\end{Bmatrix}=0\tag{60}$$

where [K], [C] and [M] are the stiffness, damping, and mass matrixes for FG nanobeam, respectively. Also, for nontrivial solution of Eq. (60), the determinant of $\{[K]+[C]\omega+[M]\omega^2\}$ should be zero to obtain natural frequencies.

## 4. Numerical results and discussions

This section addresses the frequency response of nonlocal CMEV-FG nanobeams resting on visco-Pasternak medium. This study considers the length of curved nanobeams ($L$) to be 10 nm. The verification of the proposed model is carried out by comparing the frequency results of curved FG nanobeams reported by Hosseini and Rahmani (2016). A similar geometric conditions and materials properties as that of Hosseini and Rahmani (2016) are incorporated in this verification study as well. It can be witnessed from the results of Table 2 that the proposed model correlates well with the results presented in Hosseini and Rahmani (2016). Therefore, it is justified that the results presented in this article is accurate to predict the frequency response of nonlocal curved magneto-electro-viscoelastic FG nanobeams. Further, the relations established to calculate the non dimensional natural frequencies can be expressed as follows

$$\tilde{\omega}=\omega L^2\sqrt{\frac{\rho_u A}{c_{11}^u I}},\quad K_w=k_w\frac{L^4}{c_{11}^u I},$$
$$K_p=k_p\frac{L^2}{c_{11}^u I},\quad C=c_d\frac{L^2}{\sqrt{c_{11}^u I\rho_u A}},\quad G=\frac{g}{L^2}\sqrt{\frac{c_{11}^u I}{\rho_u A}}\tag{61}$$

The influence of damping coefficient (C) on the dimensionless frequency CMEV-FG nanobeams with different opening angles is illustrated in Fig. 2. It can be observed that irrespective of the opening angle, ‘C’ has a detrimental effect on the frequencies of CMEV-FG nanobeams. Further, higher value of C shows a predominant effect on the reduction of dimensionless frequency. Meanwhile, it can also be seen that a higher value of opening angle results in lesser frequency. The discrepancies in frequencies are predominant with respect to the opening angles a fixed electric voltage and magnetic potential. To elaborate, the frequency difference existing between α = π/4 and π/3 in minimal in contrast to that existing between α = π/2 and 2π/3. Therefore, it suggests that the opening angle has a significant contribution in assessing the effect of ‘C’

Table 2 Comparison of dimensionless frequency of curved FG nanobeam for various opening angles and nonlocal parameters (L/h = 50)

| μ | $\alpha = \pi/3$, p = 0 | | $\alpha = \pi/4$, p = 0.5 | | $\alpha = \pi/2$, p = 1 | | $\alpha = 2\pi/3$, p = 5 | |
|---|---|---|---|---|---|---|---|---|
| | Hosseini and Rahmani (2016) | Present | Hosseini and Rahmani (2016) | Present | Hosseini and Rahmani (2016) | Present | Hosseini and Rahmani (2016) | Present |
| 0 | 8.31770 | 8.32132 | 7.30721 | 7.31014 | 4.72079 | 4.72275 | 2.63872 | 2.64019 |
| 1 | 7.93532 | 7.93877 | 6.97129 | 6.97408 | 4.50376 | 4.50563 | 2.51741 | 2.51881 |
| 2 | 7.60125 | 7.60455 | 6.67780 | 6.68048 | 4.31416 | 4.31595 | 2.41143 | 2.41277 |
| 3 | 7.30611 | 7.30928 | 6.41852 | 6.42108 | 4.14665 | 4.14837 | 2.3178 | 2.31909 |

on the frequency response of CMEV-FG nanobeams.

The effect of nonlocal parameters ($\mu$) associated with different damping coefficient on the vibration behaviour the smart CMEV-FG nanobeams is depicted in Fig. 3. From this figure, it can be noticed that at a fixed value of ‘C’, higher value of $\mu$ yields lower frequency of CMEV-FG nanobeams. This may be due to the fact that CMEV-FG nanobeams become less stiff as its size reduces. Further, it also implies that the CMEV-FG nanobeams exhibit a stiffness-softening effect with an improvement in the value

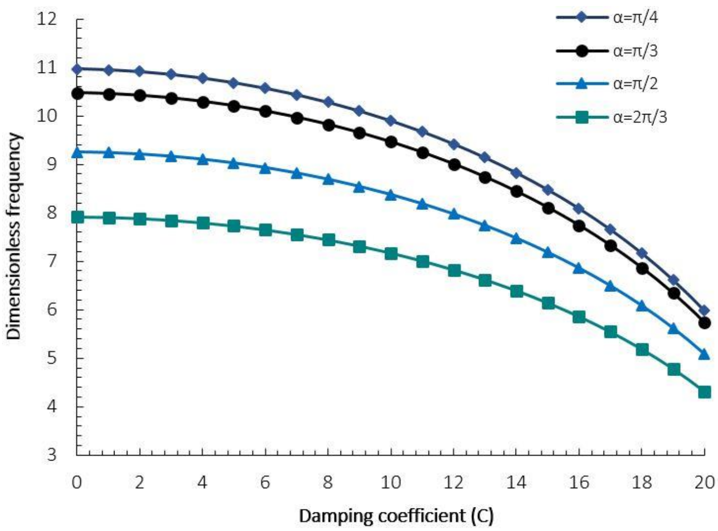


Fig. 2 Effect of damping coefficient on the frequency response of CMEV-FG nanobeams with different opening angles (L/h = 20, $\mu$ = 1 nm$^2$, V = Ω = 0, p = 1, $K_w$ = 25, $K_p$ = 5, G = 0.01)

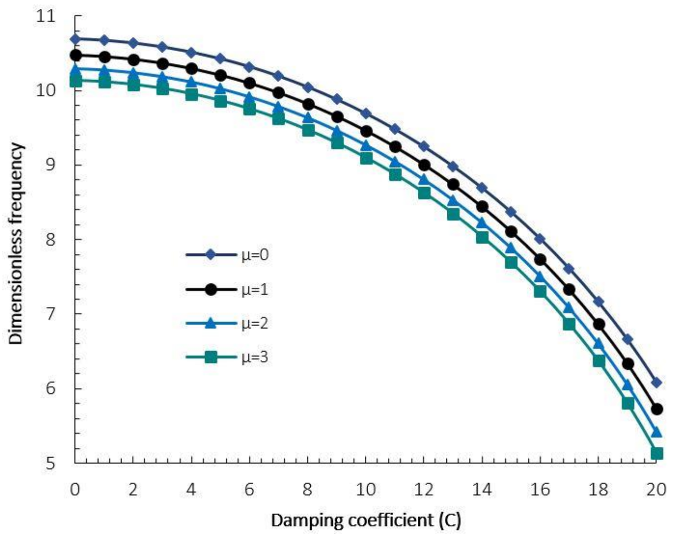


Fig. 3 Effect of damping coefficient on the frequency response of CMEV-FG nanobeams with different nonlocal parameters (L/h = 20, V = Ω = 0, p = 1, $K_w$ = 25, $K_p$ = 5, G = 0.01, α = π/3)

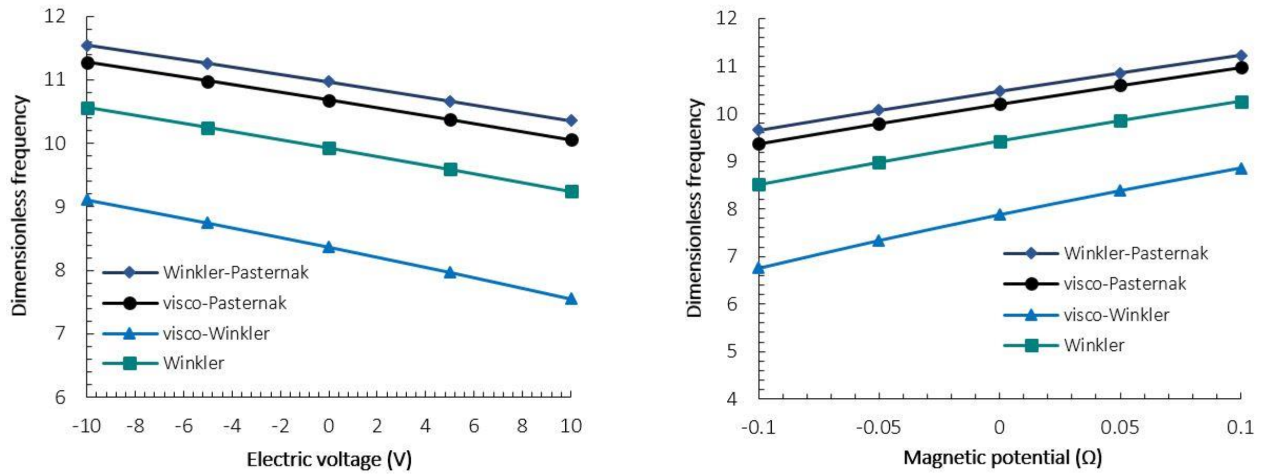


Fig. 4 Effect of electric voltage and magnetic potential on the frequency response of CMEV-FG nanobeams resting on various types of foundation (L/h = 20, μ = 1, p = 1, G = 0.01, α = π/3)

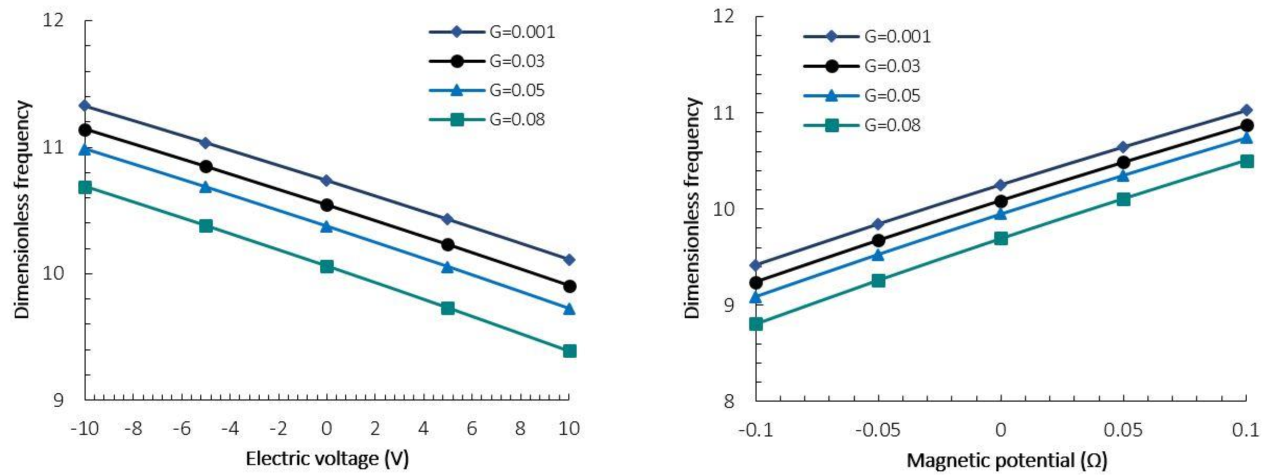


Fig. 5 Effect of electric voltage and magnetic potential on the frequency response of CMEV-FG nanobeams with different internal damping constant (L/h = 20, μ = 1, p = 1, $K_w$ = 25, $K_p$ = 5, C = 5, α = π/3)

of $\mu$. Hence, it can be concluded that a predominant influence of $\mu$ prevails on the damping vibration of CMEV-FG nanobeams.

The effect of externally applied electric voltage ($V$) and magnetic potential (Ω) on the vibration characteristics of CMEV-FG nanobeams are illustrated in Fig. 4. In addition, influence on different forms of elastic foundations viz. Winkler-Pasternak ($K_w$ = 25, $K_p$ = 5, C = 0), visco-Pasternak ($K_w$ = 25, $K_p$ = 5, C = 5), visco-Winkler ($K_w$ = 25, $K_p$ = 0, C = 5) and Winkler foundations are also considered for evaluation. The results reveal that with the increase in the positive electric voltage, the frequency of CMEV-FG nanobeams reduces. However, the natural frequencies improve with the increase in negative electric voltage. On the other hand, a reverse trend is noticed with respect to the magnetic potential. Meanwhile, at a fixed value of electric voltage and magnetic potential, the effect of elastic foundations on the frequency response of CMEV-FG is in the following order: Winkler-Pasternak medium > Visco-Pasternak > Winkler medium > Visco-Winkler medium.

The variation of dimensionless frequencies of CMEV-FG nanobeams with different values of internal damping constant (G) against externally applied $V$ and Ω is depicted in Fig. 5. As seen from this figure, it can be inferred that at a constant value of $V$ and Ω, incrementing the value of 'G' reduces the frequency. It implies that viscoelastic material properties play a prominent role in deciding the stiffness characteristics of CMEV-FG nanobeams. A previously discussed, a similar trend of variation with respect to $V$ and Ω is noticed here as well. It may be attributed to the fact that the positive and negative values of $V$ results in development of the compressive and tensile forces in CMEV-FG nanobeams, respectively. Analogously, magnetic field exhibits a reverse trend on the vibration frequencies.

The effect of externally applied $V$ and Ω associated with frequencies of CMEV-FG nanobeams with various opening angle is demonstrated in Fig. 6. The results from this figure suggest that a prominent influence of opening angle prevails on frequency response of CMEV-FG nanobeams. Vibration behavior of curved magneto-electro-elastic FG nanobeams is dependent on the value of opening angle. Further an

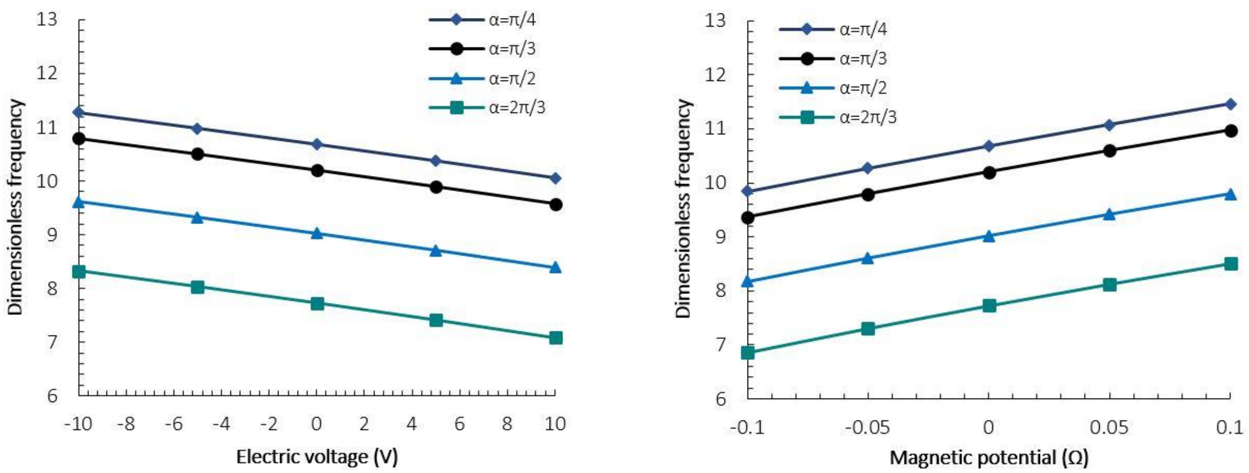


Fig. 6 Effect of electric voltage and magnetic potential on the frequency response of CMEV-FG nanobeams with different values of opening angle (L/h = 20, μ = 1, p = 1, $K_w$ = 25, $K_p$ = 5, C = 5)

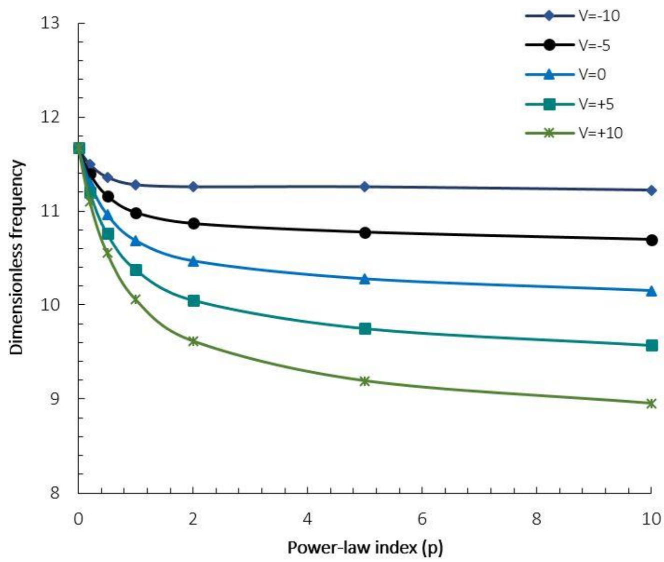


Fig. 7 Effect of power-law index on the frequency response of CMEV-FG nanobeams with various magnitudes of electric voltages (L/h = 20, μ = 1, $K_w$ = 25, $K_p$ = 5, C = 5, α = π/3)

increase in opening angle leads to reduced frequencies. A predominant influence of opening angles is noticed at its value improves. The vibration behaviour of CMEV-FG nanobeams with different power-law index (p) subjected to various magnitude of external electric voltage (V = -10, -5, 0, +5, +10) is displayed in Fig. 7. It is noticeable that at a constant power-law index, negative electric voltage yields a higher frequency. In addition, the frequencies of CMEV-FG nanobeam decreases as the power law index increases. This can be attributed to the fact that the stiffness of the FG nanobeam decreases as the portion of $BaTiO_3$ phase which has a lesser elastic stiffness in the overall material composition increases. A significant influence of power law index on the frequency prevails and it is dependent on the sign and value of electric voltage.

The analysis is further extended to evaluate the influence of power-law index associated with different magnitudes of magnetic potential. The results presented in Fig. 8 illustrates that at a given value of magnetic potential, with a small increment in the power-law index, the dimensionless frequency drastically reduces. The reduction is predominant for small values of power-law index in contrast to larger power-law index. Further, unlike electric voltage noticed higher frequencies are noticed for positive magnetic potentials. For different magnitudes of externally applied magnetic potential, the effect of slenderness ratio (*L/h*) on the vibrations of simply supported CMEV-FG nanobeams with various damping coefficient (C) and internal damping (G) are studied in Fig. 10. It can be noticed that at the lesser *L/h* ratio the discrepancies of natural frequencies are minimal. However, at the larger values of *L/h* ratio a significant effect can be witnessed. It implies that compared to thin CMEV-FG nanobeams, a significant influence of magnetic potentials exists on thick CMEV-FG nanobeams. In addition, the negative magnetic potential deteriorates the natural frequency whereas the positive magnetic potential enhances the vibration frequencies of CMEV-FG nanobeam. Meanwhile, $\Omega = 0$,

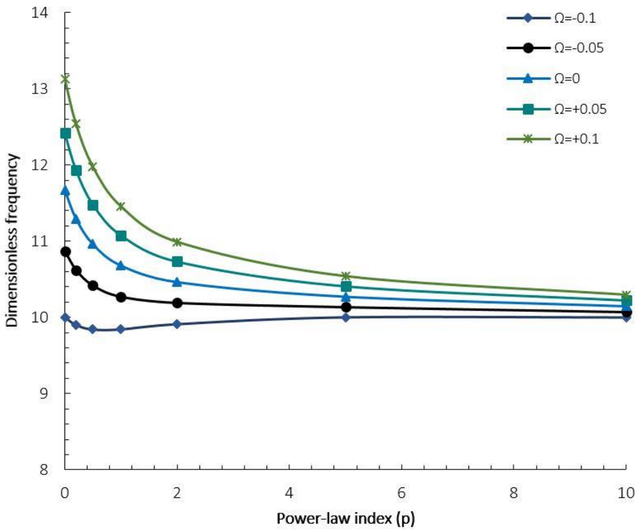


Fig. 8 Effect of power-law index on the frequency response of CMEV-FG nanobeams with various magnitudes of magnetic potentials (L/h = 20, μ = 1, $K_w$ = 25, $K_p$ = 5, C = 5, α = π/3)

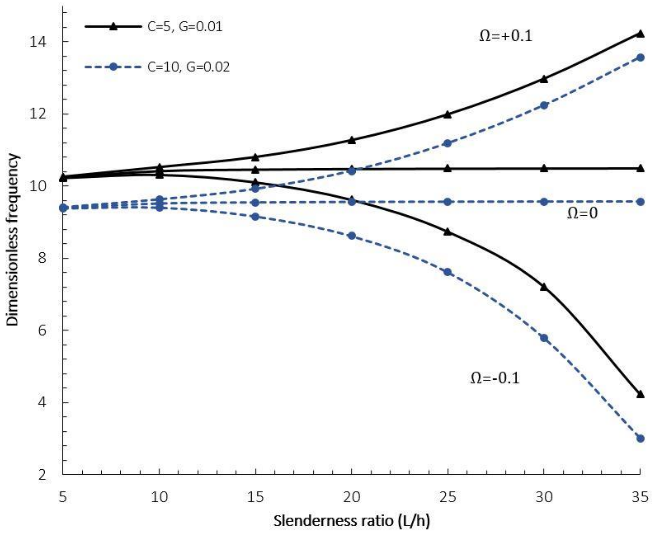


Fig. 9 Variation of dimensionless frequency of curved FG nanobeam versus slenderness ratio for various magnetic potentials (p = 1, μ = 2, V = 0, $K_w$ = 25, $K_p$ = 5, α = π/4)

does not influence the frequency variation in any means even with the change in *L/h* ratio. These phenomena associated the positive/negative magnetic potentials is due to the development of tensile and compressive forces developed, respectively. The improvement in the value of 'C' and 'G' yields lesser frequency of CMEV-FG nanobeam irrespective of applied magnetic potential and *L/h* ratio.

## 4. Conclusions

In this article a first attempt has been made to assess the frequency response of CMEV-FG nanobeam under the framework of Euler-Bernoulli beam theory. In addition, emphasize has been made on investigating the influence of three-parameter viscoelastic medium of the natural frequencies of CMEV-FG nanobeam. The material graation is assumed to follow Power-law distribution. The equations of motion are solved by incorporating The Navier solution. The numerical results reveal that higher opening angles have a deteriorating effect on the frequencies of CMEV-FG nanobeam. The dimensionless frequencies of CMEV-FG nanobeam are observed to reduce with the inclusion of nonlocal parameter. Meanwhile, the influence of externally applied electric and magnetic fields are found to be predominant on thin CMEV-FG nanobeams (larger slenderness ratio) in contrast to the thick CMEV-FG nanobeams. The negative/positive electric voltage increase /decrease the vibration frequencies of CMEV-FG nanobeams. Analogously a reverse trend in noticed for the magnetic potential. Further, the sign and magnitude of the electric voltage and magnetic potential plays an important role in deciding the influence of damping coefficient and internal damping constant on the frequency response of

CMEV-FG nanobeam. It is concluded that higher value of damping coefficient reduces the frequencies of CMEV-FG nanobeams drastically.

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