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# Assessment of new 2D and quasi-3D nonlocal theories for free vibration analysis of size-dependent functionally graded (FG) nanoplates

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**Abstract.** In this present paper, a new two dimensional (2D) and quasi three dimensional (quasi-3D) nonlocal shear deformation theories are formulated for free vibration analysis of size-dependent functionally graded (FG) nanoplates. The developed theories is based on new description of displacement field which includes undetermined integral terms, the issues in using this new proposition are to reduce the number of unknowns and governing equations and exploring the effects of both thickness stretching and size-dependency on free vibration analysis of functionally graded (FG) nanoplates. The nonlocal elasticity theory of Eringen is adopted to study the size effects of FG nanoplates. Governing equations are derived from Hamilton's principle. By using Navier's method, analytical solutions for free vibration analysis are obtained through the results of eigenvalue problem. Several numerical examples are presented and compared with those predicted by other theories, to demonstrate the accuracy and efficiency of developed theories and to investigate the size effects on predicting fundamental frequencies of size-dependent functionally graded (FG) nanoplates.

**Keywords:** free vibration; functionally graded nanoplate; nonlocal elasticity; stretching effect

## 1. Introduction

Nowadays, a large number of microstructural systems used in modern technology components and devices are manufactured as nano-scale, so called "Nanotechnology" that is becoming increasingly important on account of their many advantages for example in nanomechanics, nanoelectronics, nanophotonics, medical instruments and solar cells. Recently, the nanoscale engineering materials become a great interest field of research inspirations in advanced Nanotechnology and the major restriction and implication of this branch is the ways in which mechanical behavior is described correctly of such materials (Akgoz and Civalek 2013, Kolahchi and Moniri Bidgoli 2016, Arani and Kolahchi 2016, Tahouneh 2016, Madani *et al*. 2016, Bilouei *et al*. 2016, Zamanian *et al*. 2017, Barati 2017, Kolahchi and Cheraghbak 2017, Kolahchi 2017, Hajmohammad *et al*. 2017, 2018a, b, c, Numanoğlu *et al*. 2018, Amnieh *et al*. 2018, Youcef *et al*. 2018, Golabchi *et al*. 2018, Karami *et al*. 2017, 2018a, b, c, d, 2019a, b, c, Hosseini and Kolahchi 2018, Fu *et al*. 2018, Gupta *et al*. 2018, Rahaeifard *et al*. 2009, Farajpour *et al*. 2019, Hussain and Naeem 2019, Emdadi *et al*. 2019, Boutaleb *et al*. 2019). As such, several size-dependent continuum models have been formulated and investigated to predict the mechanical behaviors of this kind of structures by including the small-scale effects. In fact, investigations from earlier studies have been mostly focused on three size-dependent continuum theories: nonlocal elasticity, modified couple stress and modified strain gradient theories. Based on modified strain gradient theory, the strain energy is usually included three additional gradients: the dilatation gradient, deviatoric stretch gradient and the symmetric curvature. The most well-known applications of this theory are reported by Nix and Gao (1998) and Lam *et al*. (2003). In broad terms, modified couple stress can be founded by introducing the symmetric curvature tensor representing as conjugate to micro-rotations in strain energy; Yang *et al*. (2002) have suggested and inspired this concept by modifying the classical couple stress given by Mindlin and Tiersten (1962), Toupin (1962) and Koiter (1969). In spite of all these theories mentioned above, the nonlocal elasticity theory is one of the most commonly used theories to investigate the small size effects on nanostructures mechanical behaviors. From conceptual theoretical frameworks of Eringen (1972), it should be noted that the classical elasticity of solid mechanics are applied to describe the stress tensor of a material point uniquely represented by the strain tensor at that same point; this

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assumption omits the small-scale effect. Thus, this effect plays an important role on mechanical behaviors of nano-size structures and should be taken into consideration. Alternatively, the nonlocal elasticity represents the stress tensor for a material point is given by the strain tensor of all material points in nanostructures. A good review is available for these theories and theirs related mechanical models in Thai *et al*. (2017). On the other hand, functionally graded materials are considered as new class of advanced composite materials with improved mechanical proprieties. These mechanical properties are varied continuously and smoothly through the thickness coordinate. By this way, the influence of stress concentration are eliminated and reduced in interfaces contrary to laminated composites. Moreover, a large-scale utilization of FGMs has accelerated their implementation in diverse engineering systems and applications at the macroscopic level (Avcar 2019, Meksi *et al*. 2019, She *et al*. 2018, Attia *et al*. 2018, Fakhar and Kolahchi 2018, Bourada *et al*. 2018, Avcar and Mohammed 2018, Zine *et al*. 2018, Bouhadra *et al*. 2018, Sekkal *et al*. 2017a, b, Abdelaziz *et al*. 2017, Mouffoki *et al.* 2017, Kolahchi *et al.* 2016a, b, 2017a, b, c, d, Bellifa *et al.* 2017a, b, Zidi *et al.* 2017, Benadouda *et al.* 2017, Barati and Shahverdi 2016, Beldjelili *et al*. 2016, Benferhat *et al.* 2016, Ahouel *et al.* 2016, Bounouara *et al.* 2016, Bousahla *et al.* 2016, Belkorissat *et al.* 2015, Mahi *et al*. 2015, Panda and Katariya 2015, Avcar 2015, Hamidi *et al*. 2015, Belabed *et al.* 2014, Ahmed 2014). Recently, FGMs have become a broad and most important class of advanced materials in a wide variety of nanotechnology applications, as novel high-performance materials. Including their high-performance utilization rates, FGMs have been provided quantitative evidence of various applications in the main workhorse of electronics and nano-electromechanical industries such as semiconductors, microchips and commercial MEMS and NEMS devices (Witvrouw and Mehta 2005, Hasanyan *et al*. 2008, Mohammadi-Alasti *et al*. 2011, Lee *et al*. 2011, Zhang and Fu 2012), thin films in the form of shape memory alloys (Fu *et al*. 2003, Lu *et al*. 2007), and atomic force microscopes (AFMs) to achieve high sensitivity and desired performance (Rahaeifard *et al*. 2009). In similarity to above applications, the effects of small scale also have studied experimentally in many researches (Fleck *et al*. 1994, Stolken and Evans 1998, Chong *et al*. 2001, Lam *et al*. 2003). Concludes that tremendous benefits can possess over from their micron- or nano-scale dimensions of thus structures, the significant of these factors on the mechanical properties such as Young's modules, Poisson's ratio and mass density, can also be size-dependency.

Furthermore, a wide range of studies has been carried out to investigate the mechanical modeling of size-dependent nanoplate structures. Chakraverty and Behera (2015) investigated the effects of various parameters on free vibration analysis of nanoplates by using classical (Kirchhoff) plate theory and nonlocal elasticity theory. Bessaim *et al*. (2015) proposed a nonlocal quasi-3D trigonometric plate theory for free vibration behaviour of micro/nanoscale plates. This model considers both shear deformation and thickness stretching effects. Belkorissat *et al*. (2015) presented a new nonlocal hyperbolic refined plate model to study the vibration properties of functionally graded nanoplates. Daneshmehr *et al* (2015) used a higher order shear deformation plate theory and GDQ method to predict natural frequencies of functionally graded nanoplates by applying the Eringen's nonlocal theory. Zare *et al*. (2015) analyzed the natural frequencies of a functionally graded nanoplate with different boundary conditions. Bounouara *et al*. (2016) presented a zeroth-order shear deformation theory for free vibration analysis of functionally graded (FG) nanoscale plates, the equations of motion are obtained by using the nonlocal differential constitutive expressions of Eringen in conjunction with the zeroth-order shear deformation theory via Hamilton's principle. A four-variable plate theory based on nonlocal elasticity theory is developed and explored to study free vibrations of nanoplates in the thermal environments by Barati and Shahverdi (2016). Kiani *et al*. (2017) assessed the influence of both in-plane and out-of-plane elastic waves on the frequencies of nanoplates immersed in bidirectional magnetic fields, the equations of motion are formulated by nonlocal continuum theory of Eringen. Moreover, a detailed analysis is communicated to present the effects of small scale on exponentially graded nanoplates under hygro-thermo-mechanical loads (Sobhy 2017). In this study, the Eringen's differential form of nonlocal elasticity theory is adopted to derive governing equations for four-unknowns shear deformation plate model. Besseghier *et al*. (2017) discussed the influence of various parameters on free vibration analysis of nanoplates via a novel nonlocal refined trigonometric shear deformation theory. Ebrahimi and Heidari (2018) employed the Eringen's nonlocal elasticity theory in conjunction with surface elasticity theory to study linear and nonlinear free vibration behavior of FG nanoplates using Reddy's plate theory and generalized differential quadrature method. Hosseini-Hashemi and Khaniki (2018) presented a three dimensional dynamic simulation based on Eringen's nonlocal theory of functionally graded nanoplates under a moving load. In this work, Galerkin, state space and fourth-order Runge-Kutta methods are employed to solve the governing equations. Rong *et al*. (2018) proposed a new analytical approach to various mechanical problems in free vibration, buckling and forced vibration of Kirchhoff rectangular nanoplates based on Eringen's nonlocal elasticity theory. More recently and for better understanding the small scale effect on mechanical behaviors of FG nanoplates, Sobhy and Radwan (2017) developed a new quasi-3D nonlocal hyperbolic plate theory to analysis both vibrat on and buckling problems using Eringen's nonlocal theory, and proved that small scale effect is non-trivial and needs to be taken into consideration in order to study the mechanical behaviors of nanoplates for wide nanotechnology applications. Karami and Karami (2019) presented a recent buckling analysis of nanoplate-type temperature-dependent heterogeneous materials.

This study has two key aims. Firstly, the free vibration analysis is investigated for FG nanoplates by development of new 2D and quasi-3D nonlocal plate theories. Both theories describe a new description of displacement field, which includes undetermined integral terms. A hyperbolic variation is employed for all displacement parts across the thickness, which satisfies the stress-free boundary conditions on the upper and lower surfaces of the plate without requiring any shear correction factor. The equations of motion and boundary conditions are derived from Hamilton's principle. The Navier's method is adopted to derive the closed form solutions for simply supported FG nanoplates. Various numerical examples are presented and compared to those reported in the literature. The second part discusses the effect of small sca

le on free vibration of FG nanoplates based on parametrical studies such as nonlocal parameter, power index, nanoplate thickness and side to thickness ratio on size dependent frequency. Finally, the present results show the impact of small scale on free vibration response of nanoplaes and can be more useful for processing and design of nano-electro-mechanical devices, nanooscillators, and nanosensors.

## 2. Theoretical formulation

### 2.1 Constitutive equations

As mentioned above, the material properties of FG plates are assumed to vary continuously through the thickness and are dependent on the volume fraction of inclusions. The distribution of material properties is assumed to obey the power-law distribution as follows (Belabed *et al.* 2014, Bousahla *et al.* 2014, Meziane *et al.* 2014, Bourada *et al.* 2015, Yahia *et al.* 2015, Benahmed *et al.* 2017, Fahsi *et al.* 2017, Hachemi *et al.* 2017, Benchohra *et al.* 2018, Mehala *et al.* 2018, Fourn *et al.* 2018)

$$V_f(z) = V_m + (V_c - V_m)\left(\frac{2z+h}{2h}\right)^p \tag{1}$$

where $p$ is the power law index; and the subscripts $m$ and $c$ represent the metallic and ceramic constituents, respectively. The Young's modulus is given as

$$E(z) = E_c V_f(z) + E_m(1 - V_f(z)) \tag{2}$$

and the related Poisson's ratio is assumed to be constant for convenience. The density $\rho$ is given as

$$\rho(z) = \rho_c V_f(z) + \rho_m(1 - V_f(z)) \tag{3}$$

### 2.2 Kinematics

The displacement field of present theory is chosen to deduce a 2D and quasi 3D formulation for thick nanoplate problems and the following set of reduced number of unknowns are derived by these assumptions: (1) The transverse into extension, bending and undetermined integral terms; (2) the bending parts of the in-plane displacements are partitioned into bending and stretching parts; (3) the in-plane displacements are partitioned to those given by CPT ; and (4) the undetermined integral terms of the in-plane displacements give rise to hyperbolic variations of shear strains and hence to shear stresses through the thickness coordinate of the plate in such a way that the shear stresses vanished on the top and bottom surfaces of the plate. Based on these assumptions, the following displacement field relations can be obtained (Abualnour *et al.* 2018, Younsi *et al.* 2018, Boukhlif *et al.* 2019, Khiloun *et al.* 2019, Zaoui *et al.* 2019)

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y,t)dx \tag{4a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y,t)dy \tag{4b}$$

$$w(x,y,z,t) = w_0(x,y,t) + g(z)\varphi_z(x,y,t) \tag{4c}$$

where $u_0$ and $v_0$ denote the displacements along the $x$ and $y$ coordinate directions of a point on the mid-plane of the plate; $w_0$ is the bending part of the transverse displacement, respectively. The unknown $\int \theta(x,y,t)$ presents the undetermined integral term. The coefficients $k_1$ and $k_2$ depend on the geometry of plate. In this study, the shape function $f(z)$ is chosen based on the hyperbolic function proposed by Belabed *et al.* (2018) as

$$f(z) = \frac{\cosh\left(\frac{\pi}{2}\right)h^2\left(z\pi\cosh\left(\frac{\pi}{2}\right) - h\sinh\left(\frac{\pi z}{h}\right)\right)}{\frac{\pi}{2}\left(\cosh\left(\frac{\pi}{2}\right) - 1\right)} \tag{5}$$

In 2D case, the stretching effect presented by $\varphi_z$ is neglected. The non-zero strains associated with the displacement field in Eq. (4) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \tag{6a}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \tag{6b}$$

$$\varepsilon_z = g'(z)\varepsilon_z^0 \tag{6c}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial x} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{7a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\frac{\partial}{\partial y}\int \theta dx + k_2\frac{\partial}{\partial x}\int \theta dy \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2 \int \theta dy + \dfrac{\partial \varphi_z}{\partial y} \\ k_1 \int \theta dx + \dfrac{\partial \varphi_z}{\partial x} \end{Bmatrix} \tag{7b}$$

$$\varepsilon_z^0 = \varphi_z \tag{7c}$$

and

$$g'(z) = \frac{dg(z)}{dz} \tag{8}$$

The integral terms used in this formulation shall be resolved by Navier's method and can be expressed as

$$\frac{\partial}{\partial y}\int \theta dx = A' \frac{\partial^2 \theta}{\partial x \partial y}, \qquad \frac{\partial}{\partial x}\int \theta dy = B' \frac{\partial^2 \theta}{\partial x \partial y}, \qquad \int \theta dx = A' \frac{\partial \theta}{\partial x} \text{ and } \int \theta dy = B' \frac{\partial \theta}{\partial y} \tag{9}$$

The coefficients *A*' and *B*' are derived from Navier's method as

$$A' = -\frac{1}{\lambda^2}, \quad B' = -\frac{1}{\mu^2} \quad \text{and} \quad k_1 = \lambda^2, \quad k_2 = \mu^2 \tag{10}$$

where, $\lambda$ and $\mu$ are defined in Eq. (28).

### 2.3 Equations of motion

The equations of motion for nano-plates are derived using Hamilton's principle. This principle can be stated in an analytical form as follows (Al-Basyouni *et al*. 2015, Larbi Chaht *et al*. 2015, Attia *et al*. 2015, Bennoun *et al*. 2016, Boukhari *et al*. 2016, Draiche *et al*. 2016, Ait Atmane *et al*. 2017, Klouche *et al*. 2017, Bakhadda *et al*. 2018, Kaci *et al*. 2018, Adda Bedia *et al*. 2019, Bourada *et al*. 2019, Tlidji *et al*. 2019, Chaabane *et al*. 2019, Draoui *et al*. 2019)

$$\int_0^T (\delta U - \delta K) dt = 0 \tag{11}$$

where $\delta U$ is the variation of strain energy and $\delta K$ is the variation of kinetic energy. The variation of strain energy of the plate is given by

$$\begin{aligned} \delta U &= \int_{-h/2}^{h/2} \int_A \begin{bmatrix} \sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \sigma_z \delta \varepsilon_z \\ +\tau_{xy} \delta \gamma_{xy} + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz} \end{bmatrix} dA\, dz \\ &= \int_A \Big[ N_x \delta \varepsilon_x^0 + N_y \delta \varepsilon_y^0 + N_{xy} \delta \gamma_{xy}^0 + N_z \delta \varepsilon_z^0 \\ &\quad + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b \\ &\quad + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s \\ &\quad + S_{yz}^s \delta \gamma_{yz}^0 + S_{xz}^s \delta \gamma_{xz}^0 \Big] dA = 0 \end{aligned} \tag{12}$$

where *A* is the top surface and the stress resultants *N*, *M*, and *S* are defined by

$$\begin{Bmatrix} N_x, & N_y, & N_{xy} \\ M_x^b, & M_y^b, & M_{xy}^b \\ M_x^s, & M_y^s, & M_{xy}^s \end{Bmatrix} = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz, \tag{13a}$$

$$N_z = \int_{-h/2}^{h/2} \sigma_z g'(z) dz, \tag{13b}$$

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz}) g(z) dz. \tag{13c}$$

The variation of kinetic energy of the plate can be written in the form

$$\begin{aligned} \delta K &= \int_{-h/2}^{h/2} \int_A \left[ \dot{u} \delta \dot{u} + \dot{v} \delta \dot{v} + \dot{w} \delta \dot{w} \right] \rho(z)\, dA\, dz \\ &= \int_A \Big\{ I_0 \left( \dot{u}_0 \delta \dot{u}_0 + \dot{v}_0 \delta \dot{v}_0 + \dot{w}_0 \delta \dot{w}_0 \right) \\ &\quad - I_1 \left( \dot{u}_0 \frac{\partial \delta \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial x} \delta \dot{u}_0 + \dot{v}_0 \frac{\partial \delta \dot{w}_0}{\partial y} + \frac{\partial \dot{w}_0}{\partial y} \delta \dot{v}_0 \right) \\ &\quad + J_1 \left( (k_1 A') \left( \dot{u}_0 \frac{\partial \delta \dot{\theta}}{\partial x} + \frac{\partial \dot{\theta}}{\partial x} \delta \dot{u}_0 \right) + (k_2 B') \left( \dot{v}_0 \frac{\partial \delta \dot{\theta}}{\partial y} + \frac{\partial \dot{\theta}}{\partial y} \delta \dot{v}_0 \right) \right) \\ &\quad + I_2 \left( \frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial y} \frac{\partial \delta \dot{w}_0}{\partial y} \right) \\ &\quad + K_2 \left( (k_1 A')^2 \left( \frac{\partial \dot{\theta}}{\partial x} \frac{\partial \delta \dot{\theta}}{\partial x} \right) + (k_2 B')^2 \left( \frac{\partial \dot{\theta}}{\partial y} \frac{\partial \delta \dot{\theta}}{\partial y} \right) \right) \\ &\quad - J_2 \left( (k_1 A') \left( \frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta \dot{\theta}}{\partial x} + \frac{\partial \dot{\theta}}{\partial x} \frac{\partial \delta \dot{w}_0}{\partial x} \right) + (k_2 B') \left( \frac{\partial \dot{w}_0}{\partial y} \frac{\partial \delta \dot{\theta}}{\partial y} + \frac{\partial \dot{\theta}}{\partial y} \frac{\partial \delta \dot{w}_0}{\partial y} \right) \right) \\ &\quad + J_1^s \left( \dot{\varphi}_z\, \delta \dot{w}_0 \right) + K_2^s \dot{\varphi}_z\, \delta \dot{\varphi}_z \Big\} dA \end{aligned} \tag{14}$$

Here the mass inertias ($I_0$, $I_1$, $J_1$, $I_2$, $J_2$, $K_2$, $J_1^s$, $K_2^s$) are defined as follows

$$\left(I_0, I_1, J_1, I_2, J_2, K_2, J_1^s, K_2^s\right) = \int_{-h/2}^{h/2} \left(1, z, f(z), z^2, z\, f(z), f^2(z), g(z), g^2(z)\right) \rho(z) dz \tag{15}$$

Substituting the expressions for $\delta U$ and $\delta K$ from Eqs. (12) and (14) into Eq. (11) and integrating by parts, and collecting the coefficients of $\delta u_0$, $\delta v_0$, $\delta w_0$, $\delta\theta$ and $\delta\varphi_z$, the governing partial differential equations are obtained as follows

$$\begin{aligned} \delta u_0 &: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} + k_1 A' J_1 \frac{\partial \ddot{\theta}}{\partial x} \\ \delta v_0 &: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} + k_2 B' J_1 \frac{\partial \ddot{\theta}}{\partial y} \\ \delta w_0 &: \frac{\partial^2 M_x^b}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2}_e = I_0 \ddot{w}_0 \\ &\quad + I_1 \left( \frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - I_2 \nabla^2 \ddot{w}_0 + J_2 \left( k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) \\ &\quad + J_2^s \ddot{\varphi}_z \end{aligned} \tag{16}$$

$$\delta w_s: \; -k_1 A' M_x^s - k_2 B' M_y^s - (k_1 A' + k_2 B') \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1 A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} = -J_1 \left( k_1 A' \frac{\partial \ddot{u}_0}{\partial x} + k_2 B' \frac{\partial \ddot{v}_0}{\partial y} \right) + K_2 \left( (k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} + (k_2 B')^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) + J_2 \left( k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2} \right)$$
$$\delta \varphi_z: \; \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} - N_z = J_1^s \ddot{w}_0 + K_2^s \ddot{\varphi}_z \quad (16)$$

### 2.4 The nonlocal elasticity model for FG nanoplate

According to the nonlocal elasticity theory Eringen (1972), and as defined above, the nonlocal relationship between stress tensor components associated to the strain tensor components at each point $x$ in the Hookean solid can be expressed as

$$\sigma_{ij}(x) = \int_\Omega \alpha(|x' - x|, \tau) t_{ij}(x') d\Omega(x') \quad (17)$$

where $t_{ij}(x')$ are the components of the classical stress tensor at local point given as

$$t_{ij} = C_{ijkl} \varepsilon_{kl} \quad (18)$$

Herein, $\alpha$ presents the nonlocal modulus or kernel function, which considers the influence of the strain at the point $x'$ in the elastic solid, $|x' - x|$ is defined as the distance in Euclidean form, and $\tau$ is a material constant given as follows

$$\tau = \frac{e_0 a}{l} \quad (19)$$

where $l$ represents the relation of a characteristic internal length and a characteristic external length, $e_0$ is considered as a material constant which evaluated experimentally. Eringen (1983) showed that the nonlocal constitutive equation given in integral form can be expressed in an equivalent differential form as (Heireche *et al*. 2008, Berrabah *et al*. 2013, Benguediab *et al*. 2014, Adda Bedia *et al*. 2015, Aissani *et al*. 2015, Zemri *et al*. 2015, Besseghier *et al*. 2015, Eltaher *et al*. 2016, Akbas 2016, Ebrahimi and Shaghaghi 2016, Khetir *et al*. 2017, Bouafia *et al*. 2017, Kadari *et al*. 2018, Bouadi *et al*. 2018, Mokhtar *et al*. 2018, Yazid *et al*. 2018, Bensattalah *et al*. 2018)

$$(1 - (e_0 a) \nabla^2) \sigma_{kl}(x) = t_{ij} \quad (20)$$

Hence, $(e_0 a)$ is considered as nonlocal parameter, which presents the effects of small scale on mechanical responses of nanoplates. According to these definitions, the nonlocal linear constitutive relations of a FG nanoplate can be written as

$$(1 - e_0 a \nabla^2) \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \quad (21)$$

In which $\ell = (e_0 a)^2$ where ($\sigma_x$, $\sigma_y$, $\sigma_z$, $\tau_{yz}$, $\tau_{xz}$, $\tau_{xy}$) and ($\varepsilon_x$, $\varepsilon_y$, $\varepsilon_z$, $\gamma_{yz}$, $\gamma_{xz}$, $\gamma_{xy}$) are the stress and strain components, respectively. The computation of the elastic constants $C_{ij}$ are the plane stress reduced elastic constants, depend on which assumption of $\nu$, we consider. If $\varepsilon_z = 0$ then $C_{ij}$ are the plane stress reduced elastic constants, defined as

$$C_{11} = C_{22} = \frac{E(z)}{1 - \nu^2}, \quad C_{12} = \nu C_{11} \quad (22a)$$

$$C_{44} = C_{55} = C_{66} = G(z) = \frac{E(z)}{2(1 + \nu)} \quad (22b)$$

If $\varepsilon_z \neq 0$ (thickness stretching), then $C_{ij}$ are the three-dimensional elastic constants, given by

$$C_{11} = C_{22} = C_{33} = \frac{(1 - \nu)}{\nu} \lambda(z), \quad C_{12} = C_{13} = C_{23} = \lambda(z) \quad (23a)$$

$$C_{44} = C_{55} = C_{66} = G(z) = \mu(z) = \frac{E(z)}{2(1 + \nu)} \quad (23b)$$

where $\lambda(z) = \frac{\nu E(z)}{(1-2\nu)(1+\nu)}$ and $\mu(z) = G(z) = \frac{E(z)}{2(1+\nu)}$ are Lamé's coefficients. The moduli $E$, $G$ and the elastic coefficients $C_{ij}$ vary through the thickness according to Eq. (2). By substituting Eq. (6) into Eq. (21) and the subsequent results into Eq. (13), the stress resultants are readily obtained as

$$(1 - \ell \nabla^2) \begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} + \begin{bmatrix} L \\ L^a \\ R \end{bmatrix} \varepsilon_z^0 \quad (24a)$$

$$(1 - \ell \nabla^2) S = A^s \gamma \quad (24b)$$

$$N_z = R^a \varphi + L(\varepsilon_x^0 + \varepsilon_y^0) + L^a (k_x^b + k_y^b) + R(k_x^s + k_y^s), \quad (24c)$$

where

$$N = \{N_x, N_y, N_{xy}\}, \quad M^b = \{M_x^b, M_y^b, M_{xy}^b\}, \quad M^s = \{M_x^s, M_y^s, M_{xy}^s\}, \quad (25a)$$

$$\varepsilon = \{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\}, \quad k^b = \{k_x^b, k_y^b, k_{xy}^b\}, \quad k^s = \{k_x^s, k_y^s, k_{xy}^s\}, \quad (25b)$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \quad (25c)$$

$$B^s = \begin{bmatrix} B^s_{11} & B^s_{12} & 0 \\ B^s_{12} & B^s_{22} & 0 \\ 0 & 0 & B^s_{66} \end{bmatrix}, \quad D^s = \begin{bmatrix} D^s_{11} & D^s_{12} & 0 \\ D^s_{12} & D^s_{22} & 0 \\ 0 & 0 & D^s_{66} \end{bmatrix}, \quad H^s = \begin{bmatrix} H^s_{11} & H^s_{12} & 0 \\ H^s_{12} & H^s_{22} & 0 \\ 0 & 0 & H^s_{66} \end{bmatrix}, \tag{25d}$$

$$S = \left\{S^s_{xz}, S^s_{yz}\right\}, \quad \gamma = \left\{\gamma_{xz}, \gamma_{yz}\right\}, \quad A^s = \begin{bmatrix} A^s_{44} & 0 \\ 0 & A^s_{55} \end{bmatrix} \tag{25e}$$

$$\begin{Bmatrix} L \\ L^a \\ R \\ R^a \end{Bmatrix} = \int_{-h/2}^{h/2} \lambda(z) \begin{Bmatrix} 1 \\ z \\ f(z) \\ g'(z)\dfrac{1-\nu}{\nu} \end{Bmatrix} g'(z)dz \tag{25f}$$

### 2.5 Equations of motion in terms of displacements

Introducing Eq. (25) into Eq. (16), the equations of motion can be expressed in terms of displacements ($\delta u_0$, $\delta v_0$, $\delta w_0$, $\delta\theta$, $\delta\varphi_z$) and the appropriate equations take the form

$$\begin{aligned} &A_{11}d_{11}u_0 + A_{66}d_{22}u_0 + \left(A_{12} + A_{66}\right)d_{12}v_0 - B_{11}d_{111}w_0 \\ &-\left(B_{12} + 2B_{66}\right)d_{122}w_0 + B^s_{66}\left(k_1A' + k_2B'\right)d_{122}\theta \\ &+\left(B^s_{11}k_1 + B^s_{12}k_2\right)d_1\theta + Ld_1\varphi \\ &= \left(1 - \ell\nabla^2\right)\left(I_0\ddot{u}_0 - I_1 d_1\ddot{w}_0 + J_1k_1A' d_1\theta\right), \end{aligned} \tag{26a}$$

$$\begin{aligned} &A_{22}d_{22}v_0 + A_{66}d_{11}v_0 + \left(A_{12} + A_{66}\right)d_{12}u_0 - B_{22}d_{222}w_0 \\ &-\left(B_{12} + 2B_{66}\right)d_{112}w_0 + B^s_{66}\left(k_1A' + k_2B'\right)d_{112}\theta \\ &+\left(B^s_{11}k_1 + B^s_{12}k_2\right)d_2\theta + Ld_2\varphi \\ &= \left(1 - \ell\nabla^2\right)\left(I_0\ddot{v}_0 - I_1 d_2\ddot{w}_0 + J_1k_2B' d_2\theta\right), \end{aligned} \tag{26b}$$

$$\begin{aligned} &B_{11}d_{111}u_0 + \left(B_{12} + 2B_{66}\right)d_{122}u_0 + \left(B_{12} + 2B_{66}\right)d_{112}v_0 \\ &+B_{22}d_{222}v_0 - D_{11}d_{1111}w_0 - 2\left(D_{12} + 2D_{66}\right)d_{1122}w_0 \\ &-D_{22}d_{2222}w_0 + \left(D^s_{11}k_1 + D^s_{12}k_2\right)d_{11}\theta + 2D^s_{66}\left(k_1A' + k_2B'\right)d_{1122}\theta \\ &-\left(D^s_{12}k_1 + D^s_{22}k_2\right)d_{22}\theta + L^a\left(d_{11}\varphi + d_{22}\varphi\right) \\ &= \left(1 - \ell\nabla^2\right)\left(I_0\ddot{w}_0 + I_1\left(d_1\ddot{u}_0 + d_2\ddot{v}_0\right)\right. \\ &\quad \left. -I_2\left(d_{11}\ddot{w}_0 + d_{22}\ddot{w}_0\right) + J_2\left(k_1A' d_{11}\ddot{\theta} + k_2B' d_{22}\ddot{\theta}\right) + J^s_1\ddot{\varphi}\right) \end{aligned} \tag{26c}$$

$$\begin{aligned} &-\left(B^s_{11}k_1 + B^s_{12}k_2\right)d_1u_0 - B^s_{66}\left(k_1A' + k_2B'\right)d_{122}u_0 \\ &-\left(B^s_{12}k_1 + B^s_{22}k_2\right)d_2v_0 - B^s_{66}\left(k_1A' + k_2B'\right)d_{112}v_0 \\ &+\left(D^s_{11}k_1 + D^s_{12}k_2\right)d_{11}w_0 + 2D^s_{66}\left(k_1A' + k_2B'\right)d_{1122}w_0 \\ &-\left(D^s_{12}k_1 + D^s_{22}k_2\right)d_{22}w_0 - H^s_{11}k_1^2\theta - 2H^s_{12}k_1k_2\theta \\ &-H^s_{66}\left(k_1A' + k_2B'\right)^2 d_{1122}\theta - H^s_{22}k_2^2\theta + A^s_{44}\left(k_1A'\right)^2 d_{11}\theta \\ &+A^s_{55}\left(k_2B'\right)^2 d_{22}\theta + A^s_{44}\left(k_1A'\right)d_{11}\phi + A^s_{55}\left(k_2B'\right)d_{22}\varphi \\ &= \left(1 - \ell\nabla^2\right)\left(-J_1\left(k_1A' d_1\ddot{u}_0 + k_2B' d_2\ddot{v}_0\right) + J_2\left(k_1A' d_{11}\ddot{w}_0 + k_2B' d_{22}\ddot{w}_0\right)\right. \\ &\left. -K_2\left(\left(k_1A'\right)^2 d_{11}\ddot{\theta} + \left(k_2B'\right)^2 d_{22}\ddot{\theta}\right)\right) \end{aligned} \tag{26d}$$

$$\begin{aligned} &L\left(d_1u_0 + d_2v_0\right) - L^a\left(d_{11}w_0 + d_{22}w_0\right) + \left(R - A^s_{44}\right)d_{11}w_s \\ &+R^a\varphi_z - A^s_{44}d_{11}\varphi_z - A^s_{55}d_{22}\varphi_z = \left(1 - \ell\nabla^2\right)\left(J^s_1\left(\ddot{w}_b + \ddot{w}_s\right) + K^s_2\ddot{\varphi}_z\right) \end{aligned} \tag{26e}$$

where $d_{ij}$, $d_{ijl}$ and $d_{ijlm}$ are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, \quad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \qquad (i, j, l, m = 1,2). \tag{27}$$

### 2.6 Analytical solutions

Consider a simply supported rectangular nanoplate with length $a$ and width $b$. Based on the Navier's solution method, the following expansions of displacements ($u_0$, $v_0$, $w_0$, $\theta$, $\varphi_z$) are assumed as (Hebali *et al*. 2014, Houari *et al*. 2016)

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \\ \varphi_z \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}e^{i\omega t}\cos(\lambda x)\sin(\mu y) \\ V_{mn}e^{i\omega t}\sin(\lambda x)\cos(\mu y) \\ W_{mn}e^{i\omega t}\sin(\lambda x)\sin(\mu y) \\ \Xi_{mn}e^{i\omega t}\sin(\lambda x)\sin(\mu y) \\ \Phi_{mn}e^{i\omega t}\sin(\lambda x)\sin(\mu y) \end{Bmatrix} \tag{28}$$

where $U_{mn}$, $V_{mn}$, $W_{mn}$, $\Xi_{mn}$ and $\Phi_{mn}$ unknown parameters must be determined, $\omega$ is the eigen-frequency associated with $(m, n)^{th}$ eigen-mode, $\lambda = m\pi/a$ and $\mu = m\pi/b$. Substituting Eq. (28) into Eq. (26), the analytical solutions can be obtained from the matrix-vector system

$$\left( \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{12} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{13} & a_{23} & a_{33} & a_{34} & a_{35} \\ a_{14} & a_{24} & a_{34} & a_{44} & a_{45} \\ a_{15} & a_{25} & a_{35} & a_{45} & a_{55} \end{bmatrix} - \alpha\omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} & 0 \\ 0 & m_{22} & m_{23} & m_{24} & 0 \\ m_{13} & m_{23} & m_{33} & m_{34} & m_{35} \\ m_{14} & m_{24} & m_{34} & m_{44} & 0 \\ 0 & 0 & m_{35} & 0 & m_{55} \end{bmatrix} \right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \Xi_{mn} \\ \Phi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{29}$$

in which

$$\begin{aligned} &a_{11} = -\left(A_{11}\lambda^2 + A_{66}\mu^2\right) \quad a_{12} = -\lambda\,\mu\left(A_{12} + A_{66}\right) \\ &a_{13} = \lambda\left[B_{11}\lambda^2 + \left(B_{12} + 2B_{66}\right)\mu^2\right] \\ &a_{14} = \lambda\left[k_1B^s_{11} + k_2B^s_{12} - \left(k_1A' + k_2B'\right)B^s_{66}\,\mu^2\right] \\ &a_{15} = L\lambda, \quad a_{22} = -\left(A_{66}\lambda^2 + A_{22}\mu^2\right), \\ &a_{23} = \mu\left[\left(B_{12} + 2B_{66}\right)\lambda^2 + B_{22}\mu^2\right], \\ &a_{24} = \mu\left[k_2B^s_{22} + k_1B^s_{12} - \left(k_1A' + k_2B'\right)B^s_{66}\lambda^2\right], \\ &a_{25} = L\mu, \quad a_{33} = -\left(D_{11}\lambda^4 + 2\left(D_{12} + 2D_{66}\right)\lambda^2\mu^2 + D_{22}\mu^4\right) \\ &a_{34} = -k_1\left(D^s_{11}\lambda^2 + D^s_{12}\mu^2\right) + 2\left(k_1A' + k_2B'\right)D^s_{66}\lambda^2\,\mu^2 - k_2\left(D^s_{12}\lambda^2 + D^s_{22}\mu^2\right), \\ &a_{35} = -L^a\left(\lambda^2 + \mu^2\right) \\ &a_{44} = -k_1\left(H^s_{11}k_1 + H^s_{12}k_2\right) - \left(k_1A' + k_2B'\right)^2 H^s_{66}\lambda^2\mu^2 \\ &-k_2\left(H^s_{12}k_1 + H^s_{22}k_2\right) - \left(k_1A'\right)^2 A^s_{55}\lambda^2 - \left(k_2B'\right)^2 A^s_{44}\mu^2, \\ &a_{45} = -\left(k_1A' A^s_{55}\lambda^2 + k_2B' A^s_{44}\mu^2\right) + R\left(k_1 + k_2\right) \\ &a_{55} = -\left(A^s_{55}\lambda^2 + A^s_{44}\mu^2 + R^a\right), m_{11} = m_{22} = -I_0, m_{13} = \lambda I_1, \end{aligned} \tag{30}$$

$$m_{14} = -k_1 A' \lambda J_1, m_{23} = \mu I_1, m_{24} = -k_2 B' \mu J_1,$$
$$m_{33} = -\left(I_0 + I_2\left(\lambda^2 + \mu^2\right)\right), m_{34} = J_2\left(k_1 A' \lambda^2 + k_2 B' \mu^2\right),$$
$$m_{35} = -J_1^s, m_{44} = -K_2\left(\left(k_1 A'\right)^2 \lambda^2 + \left(k_2 B'\right)^2 \mu^2\right) m_{55} = -K_2^s,$$
$$\alpha = \left(1 + \ell\left(\lambda^2 + \mu^2\right)\right). \tag{30}$$

## 3. Results and discussion

In order to assess the accuracy of proposed theories, the numerical results are presented and compared to verify and discuss the effects of small scale in predicting the natural frequencies of simply supported functionally graded nanoplates. The material properties of FG nanoplates used in current study are listed in Table 1. Dimensionless frequency is utilized as follow

$$\varpi = \omega \frac{a^2}{h} \sqrt{\rho_m / E_m} \tag{31}$$

The first case is performed for thick isotropic square nanoplates. This example aims to verify the effect of small scale on fundamental frequency for various modes; the obtained results are compared with those computed by other nanoplate theories such as the two-variable refined plate theory of Malekzadeh and Shojaee (2013), higher and quasi-3D hyperbolic plate theory solutions of Sobhy and Radwan (2017). Table 2 presents the computed non-dimensional fundamental frequency. Obviously, Table 2 shows that the computations obtained by the new proposed theories are in excellent agreement with those predicted by higher and quasi-3D plate theories for all modes of vibration. It should be noted that above-mentioned higher and quasi-3D theories use the same number of unknowns and the accuracy of proposed theories is marked by using a new description of displacement field. It is also marked that the non-local parameter is non-trivial and the non-dimensional frequency becomes even worse when the nonlocal elasticity is omitted. Due to small-scale effect and as expected, as the non-local parameter increases, the non-dimensional frequency continues to decrease for all modes of vibration. Thus, the nonlocal elasticity should be taken into account for information related to the small-scale length of nanoplates.

Next FG nanoplates with various values of thickness ratio ($a/h$), nonlocal parameter $\mu$ and material index parameter are analysed, another situation will improve the efficiently of current theories. After inspection of Table 3, it is pertinent to note that the compared theory given by Sobhy and Radwan (2017) and the proposed theory is based on the same assumptions (in particular $\varepsilon_z = 0$) and number of unknowns. It is stated that the present theory solutions are correlated closely the obtained solutions of Sobhy and

Table 1 Material properties used in the FG nanoplate

| Properties | Metal (aluminum) | Ceramic (Alumina $Al_2O_3$) |
|---|---|---|
| $E$ (GPa) | 70 | 380 |
| $v$ | 0.30 | 0.30 |
| $\rho$ (kg/m$^3$) | 2702 | 3800 |

Table 2 Comparison of non-dimensional fundamental frequency $\varpi$ of simply supported homogeneous square nanoplate

| $a/h$ | Mode | $\mu^2$ | Theory | | | | |
|---|---|---|---|---|---|---|---|
| | | | Malekzadeh[(a)] | Sobhy[(b)] | Sobhy[(b)] | Present | Present |
| | | | $\varepsilon_z = 0$ | $\varepsilon_z = 0$ | $\varepsilon_z \neq 0$ | $\varepsilon_z = 0$ | $\varepsilon_z \neq 0$ |
| 10 | (1,1) | 0 | 0.093029 | 0.093031 | 0.093228 | 0,093031 | 0.093228 |
| | | 1 | 0.085016 | 0.085017 | 0.085197 | 0.085017 | 0.085197 |
| | | 2 | 0.078771 | 0.078772 | 0.078939 | 0.078772 | 0.078939 |
| | | 3 | 0.073726 | 0.073728 | 0.073884 | 0.073728 | 0.073884 |
| | (2,2) | 0 | 0.340640 | 0.340649 | 0.342212 | 0.340649 | 0.342212 |
| | | 1 | 0.254640 | 0.254643 | 0.255812 | 0.254643 | 0.255812 |
| | | 2 | 0.212120 | 0.212114 | 0.213087 | 0.212114 | 0.213088 |
| | | 3 | 0.167040 | 0.185599 | 0.186450 | 0.185599 | 0.186451 |
| | (3,3) | 0 | 0.644000 | 0.688960 | 0.688998 | 0.688960 | 0.688998 |
| | | 1 | 0.410490 | 0.410468 | 0.413491 | 0.410468 | 0.413492 |
| | | 2 | 0.320550 | 0.320538 | 0.322898 | 0.320538 | 0.322899 |
| | | 3 | .271840 | 0.271858 | 0.273861 | 0.271858 | 0.273861 |
| 20 | (1,1) | 0 | 0.023864 | 0.023864 | 0.023895 | 0.023864 | 0.023895 |
| | | 1 | 0.021808 | 0.021808 | 0.021837 | 0.021808 | 0.021837 |
| | | 2 | 0.020206 | 0.020206 | 0.020233 | 0.020206 | 0.020233 |
| | | 3 | 0.018912 | 0.018912 | 0.018937 | 0.018912 | 0.018937 |

[(a)] Given by Malekzadeh and Shojaee (2013); [(b)] Given by Sobhy and Radwan (2017)

Table 3 Comparison of non-dimensional fundamental frequency $\bar{\omega}$ of simply supported FG square nanoplate ($\varepsilon_z$ = 0)

| *a*/*h* | *p* | Nonlocal parameter *μ* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | | 0.5 | | 1 | | 1.5 | | 2 | |
| | | Sobhy[(a)] | Present | Sobhy[(a)] | Present | Sobhy[(a)] | Present | Sobhy[(a)] | Present | Sobhy[(a)] | Present |
| 5 | Ceramic | 5,10702 | 5,10702 | 4,98549 | 4,98549 | 4,66713 | 4,66713 | 4,24976 | 4,24975 | 3,81763 | 3,81761 |
| | 1 | 3,01860 | 3,01765 | 2,94677 | 2,94580 | 2,75859 | 2,75768 | 2,51190 | 2,51110 | 2,25648 | 2,25575 |
| | 5 | 2,42443 | 2,42445 | 2,36674 | 2,36676 | 2,21560 | 2,21562 | 2,01747 | 2,01747 | 1,81232 | 1,81233 |
| | Metal | 2,11261 | 2,11262 | 2,06234 | 0,06234 | 1,93064 | 1,93065 | 1,75799 | 1,75799 | 1,57923 | 1,57924 |
| 10 | Ceramic | 1,38829 | 1,38830 | 1,35525 | 1,35526 | 1,26871 | 1,26871 | 1,15525 | 1,15525 | 1,03778 | 1,30778 |
| | 1 | 0,82250 | 0,82293 | 0,80292 | 0,80334 | 0,75165 | 0,75205 | 0,68443 | 0,68479 | 0,61484 | 0,61516 |
| | 5 | 0,66485 | 0,66515 | 0,64903 | 0,64932 | 0,60758 | 0,60785 | 0,55325 | 0,55350 | 0,49699 | 0,49722 |
| | Metal | 0,57695 | 0,57596 | 0,56322 | 0,56323 | 0,52725 | 0,52726 | 0,48010 | 0,48011 | 0,43128 | 0,43129 |
| 20 | Ceramic | 0,35558 | 0,35558 | 0,34712 | 0,34712 | 0,32495 | 0,32496 | 0,29589 | 0,29589 | 0,26581 | 0,26581 |
| | 1 | 0,21083 | 0,21098 | 0,20581 | 0,20595 | 0,19267 | 0,19280 | 0,17544 | 0,17556 | 0,15760 | 0,15771 |
| | 5 | 0,17077 | 0,17086 | 0,16671 | 0,16680 | 0,15606 | 0,15615 | 0,14210 | 0,14218 | 0,12765 | 0,12772 |
| | Metal | 0,14799 | 0,14800 | 0,14447 | 0,14448 | 0,13524 | 0,13525 | 0,12315 | 0,12315 | 0,11063 | 0,11063 |
| 50 | Ceramic | 0,05730 | 0,05730 | 0,05593 | 0,05593 | 0,05236 | 0,05236 | 0,04768 | 0,04768 | 0,04283 | 0,04283 |
| | 1 | 0,03398 | 0,03400 | 0,03317 | 0,03320 | 0,03105 | 0,33108 | 0,02827 | 0,02830 | 0,02540 | 0,02542 |
| | 5 | 0,02754 | 0,02756 | 0,02688 | 0,02690 | 0,02517 | 0,02518 | 0,02291 | 0,02293 | 0,02058 | 0,02060 |
| | Metal | 0,02385 | 0,02386 | 0,02329 | 0,02329 | 0,02180 | 0,02180 | 0,01985 | 0,01985 | 0,01783 | 0,01774 |

[(a)] Given by Sobhy and Radwan (2017)

Table 4 Comparison of non-dimensional fundamental frequency $\bar{\bar{\omega}}$ of simply supported FG square nanoplate ($\varepsilon_z$ = 0)

| *a*/*h* | *p* | Nonlocal parameter *μ* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | | 0.5 | | 1 | | 1.5 | | 2 | |
| | | Sobhy[(a)] | Present | Sobhy[(a)] | Present | Sobhy[(a)] | Present | Sobhy[(a)] | Present | Sobhy[(a)] | Present |
| 5 | Ceramic | 5,12377 | 5,12377 | 5,00184 | 5,00184 | 4,68243 | 4,68243 | 4,26370 | 4,26370 | 3,83015 | 3,83015 |
| | 1 | 3,04445 | 3,04260 | 2,97200 | 2,97020 | 2,78221 | 2,78053 | 2,53340 | 2,53187 | 2,27580 | 2,27442 |
| | 5 | 2,44172 | 2,44052 | 2,38362 | 2,38244 | 2,23140 | 2,23030 | 2,03185 | 2,03085 | 1,82525 | 1,82435 |
| | Metal | 2,12231 | 2,12231 | 2,07180 | 2,07181 | 1,93950 | 1,93951 | 1,76606 | 1,76606 | 1,58648 | 1,58648 |
| 10 | Ceramic | 1,39015 | 1,390158 | 1,35707 | 1,357077 | 1,27041 | 1,270416 | 1,15680 | 1,156806 | 1,03917 | 1,039178 |
| | 1 | 0,82782 | 0,82785 | 0,80812 | 0,80815 | 0,75652 | 0,75654 | 0,68886 | 0,68889 | 0,61882 | 0,61884 |
| | 5 | 0,66890 | 0,66875 | 0,65299 | 0,65284 | 0,61129 | 0,61115 | 0,55662 | 0,55650 | 0,50002 | 0,49991 |
| | Metal | 0,57817 | 0,57818 | 0,56441 | 0,56442 | 0,52837 | 0,52837 | 0,48112 | 0,48112 | 0,43220 | 0,43220 |
| 20 | Ceramic | 0,35584 | 0,35585 | 0,34737 | 0,34738 | 0,32519 | 0,32519 | 0,29611 | 0,29611 | 0,26600 | 0,26600 |
| | 1 | 0,21205 | 0,21209 | 0,20701 | 0,20704 | 0,19379 | 0,19382 | 0,17646 | 0,17649 | 0,15851 | 0,15854 |
| | 5 | 0,17175 | 0,171726 | 0,16767 | 0,167640 | 0,15696 | 0,156934 | 0,14292 | 0,14290 | 0,12839 | 0,12837 |
| | Metal | 0,14819 | 0,14819 | 0,14466 | 0,14467 | 0,13543 | 0,13543 | 0,12331 | 0,12332 | 0,11078 | 0,11078 |
| 50 | Ceramic | 0,05733 | 0,05733 | 0,05596 | 0,05597 | 0,05239 | 0,05239 | 0,04770 | 0,04771 | 0,04285 | 0,04286 |
| | 1 | 0,03417 | 0,03418 | 0,03335 | 0,03336 | 0,03122 | 0,03123 | 0,02843 | 0,02844 | 0,02554 | 0,02555 |
| | 5 | 0,02769 | 0,02769 | 0,02703 | 0,02703 | 0,02531 | 0,02531 | 0,02304 | 0,02304 | 0,02070 | 0,02070 |
| | Metal | 0,02388 | 0,02389 | 0,02331 | 0,02332 | 0,02182 | 0,02183 | 0,01987 | 0,01988 | 0,01785 | 0,01786 |

[(a)] Given by Sobhy and Radwan (2017)

Radwan (2017). As can be seen, the obtained non-dimensional fundamental frequency $\bar{\omega}$ decreases when material index parameter increases for all values of thickness ratio ($a/h$). The significant decrease is due to the material transforming from the fully ceramic material to the fully metal material and the ceramic materials are considered as relatively stiff and strong-stiffnesses and strengths compared to metallic materials. Furthermore, increasing nonlocal parameter results a decrease in the non-dimensional fundamental frequency. These findings might be explained by the fact that the nonlocal elasticity includes the small-scale effects presented as nonlocal parameter in which corrects prediction of nanoplates fundamental frequencies.

Finally, the last example is performed to present the stretching effect on free vibration analysis of thick FG square nanoplates, for this aim, the obtained results are compared with the hyperbolic quasi 3D plate theory ($v$) solutions of Sobhy and Radwan (2017). The values of non-dimensional fundamental frequency are given in Table 4 for different values of thickness ratio a/h, material index parameter and nonlocal parameter $\mu$. Since the higher order nanoplate theory omits stretching effect, it over-estimates the frequency of thick nanoplates, noticeably. As it can be seen, the obtained values of non-dimensional fundamental frequencies which consider the stretching effect are higher than those obtained by higher order nanoplate theory ($\varepsilon_z$ = 0) and decrease when the thickness ratio $a/h$ increases for all values of nonlocal parameter $\mu$.

Due to the fact that the two-dimensional nanoplate theories assume the transverse displacement through the thickness as constant and for thick nanoplate theories must be considered the transverse shear and normal deformations at the same time. In fact, this affect can play a crucial role on the thick FG nanoplates and should be taken into consideration.

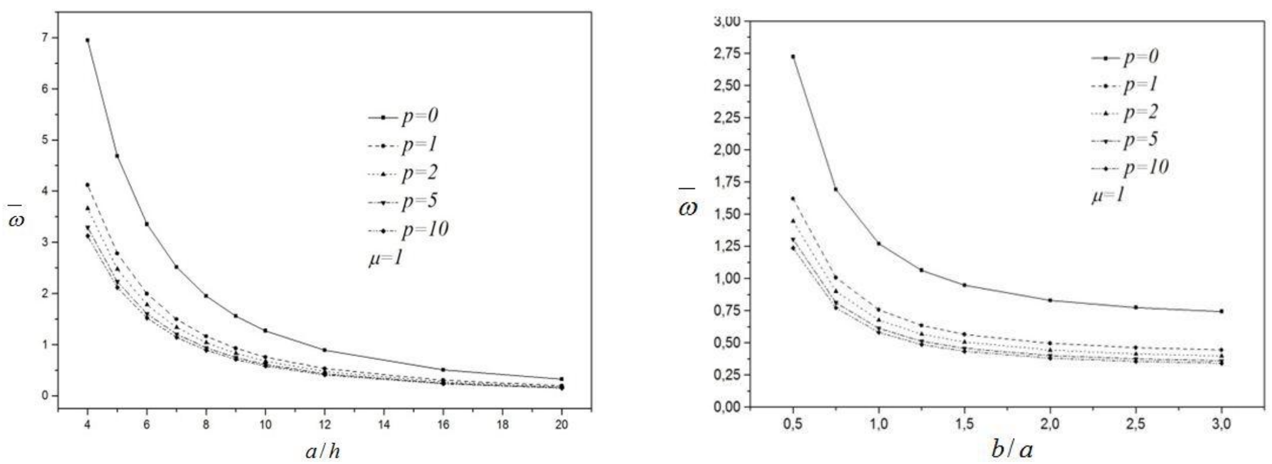
The numerical results for free vibration nanoplates with different geometrical and material parameters are given in graphical form in Figs. 1 and 2. Fig. 1(a) illustrates the effect of thickness ratio ($a/h$) on non-dimensional fundamental frequency of nanoplate obtained for different values of power law index $p$ with nonlocal parameter $\mu$ = 1 nm.

It is observed that the obtained fundamental frequency decreases when the thickness ratio ($a/h$) is increases. As mentioned above, the ceramic material is considered as a stiff material compared to metallic material. Indeed, the highest values are presented in ceramic material and the lowest values are observed in the metallic one, according to the material proprieties of each material can be used. The thickness ratio contributes to a reduction in the difference fundamental frequency from thick to thin nanoplates. The variation of non-dimensional fundamental frequency versus the aspect ratio ($b/a$) and nonlocal parameter μ is presented in Fig. 1(b). According to the figure, the increase in aspect ratio ($b/a$) leads to a decrease in non-dimensional fundamental frequency for all power law index $p$ values, on the other hand, the fully ceramic nanoplate provides results higher than those with fully metallic nanoplate. The different results of used power law index $p$ are slightly influenced by the increase in aspect ratio ($b/a$).

Fig. 2 shows the variation of non-dimensional fundamental frequency versus thickness ratio ($a/h$) and the aspect ratio ($b/a$) respectively for various values of the nonlocal parameter $\mu$. It can be seen explicitly the effect of small scale on non-dimensional fundamental frequency, since the continuum mechanics omits the dependency size dealing with nanostructures, hence the effect of nonlocal parameter plays an important role and enhances fundamental frequency in thus nanostructures. Another observation is seen that increase of aspect ratio ($b/a$) and thickness ratio ($a/h$) decreases non-dimensional fundamental frequency.

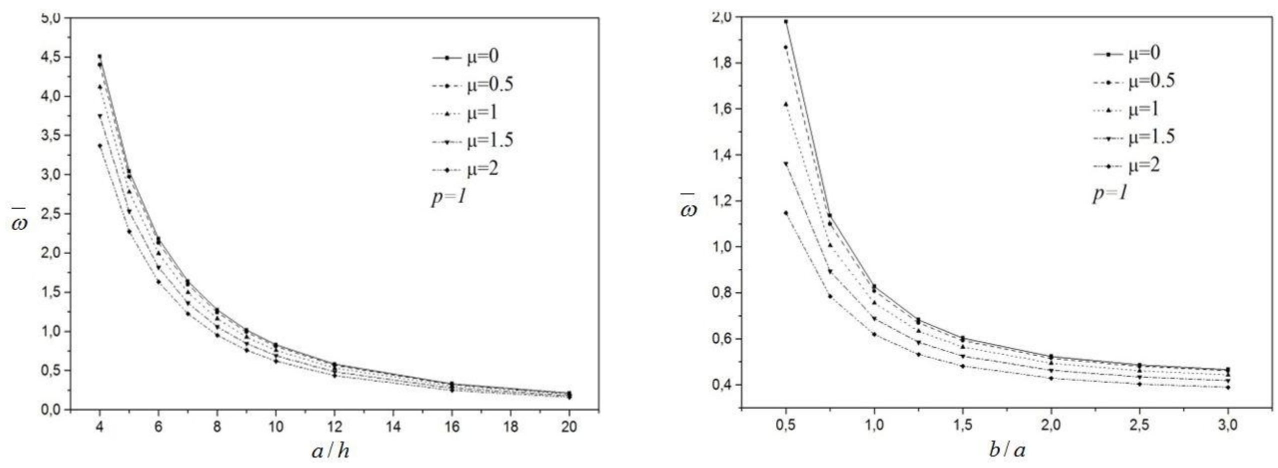
## 4. Conclusions

Based on new displacement fields and nonlocal elasticity, a theoretical study on the effects of small scale on free vibration analysis of functionally graded material




(a) (b)

Fig. 1 Variation of non-dimensional fundamental frequency versus power law index $p$ of $Al/Al_2O_3$ square nanoplates ($\mu$ = 1 nm): and (a) the thickness ratio $a/h$; (b) the aspect ratio ($b/a$)




(a) (b)

Fig. 2 Variation of non-dimensional fundamental frequency versus nonlocal parameter $\mu$ of $Al/Al_2O_3$ square nanoplates ($p = 1$): and (a) the thickness ratio $a/h$; (b) the aspect ratio ($b/a$)

(FGM) nanoplates is formulated. Unlike other higher order and quasi-3D shear nanoplate theories, the proposed new 2D and quasi-3D theories contain a new description of displacement by using undetermined integral terms. Mechanical proprieties of FG nanoplate are assumed to be graded smoothly through thickness coordinate according power law material distribution. The equations of motion are derived using Hamilton's principle and nonlocal constitutive relations of Eringen. Afterwards, the exact closed-form solutions are derived for simply supported FG thick rectangular nanoplates by employing Navier's analytical method. In order to assess the accuracy of the present theories, a comparative study is founded on various factors such as thickness ratio, aspect ratio and nonlocal parameter and where a closely agreement was observed in all cases. Additionally, the both stretching and small-scale effects have been also reported. Finally, these findings may provide strong potential usefulness in the future works and will make many contributions to offer valuable insights of mechanical responses for design and development in nanotechnology applications. Furthermore, future investigated would usefully supplement and extend to other mechanical behaviors as thermo-mechanical FG plate/shell at nanoscale. An improvement of present formulation will be considered in the future work to consider the thermal effect (Bouderba *et al*. 2013 and 2016, Tounsi *et al*. 2013, Zidi *et al*. 2014, Chikh *et al*. 2017, El-Haina *et al*. 2017, Menasria *et al*. 2017, Cherif *et al*. 2018, Semmah *et al*. 2019).

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