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A novel porosity-based homogenization scheme for propagation of waves in axially-excited FG nanobeams

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Abstract. Putting emphasis on the effect of existence of porosity in the functionally graded materials (FGMs) on the dynamic responses of waves scattered in FG nanobeams resulted in implementation of a novel porosity-based homogenization method for FGMs and show its applicability in a wave propagation problem in the presence of axial pre-load for the first time. In the employed porosity-dependent method, the coupling between density and Young's moduli is included to consider for the effective moduli of the FG nanobeam by the means of a more reliable homogenization technique. The beam-type element will be modeled via the classical theory of beams, namely Euler-Bernoulli beam theory. Also, the dynamic form of the principle of virtual work will be extended for such nanobeams to derive the motion equations. Applying the nonlocal constitutive equations of Eringen on the obtained motion equations will be resulted in derivation of the nanobeam's governing equations. Depicted results reveal that the dispersion responses of FG nanobeams will be decreased as the porosity volume fraction is increased which must be noticed by the designers of advanced nanosize devices who are interested in employment of wave dispersion approach in continuous systems for specific goals.

Keywords: wave propagation; porous materials; functionally graded materials (FGMs); nonlocal elasticity theory

1. Introduction

Because of the enhanced features of Functionally graded materials (FGMs) compared with the conventional laminates, FGMs are better alternatives for the purpose of designing mechanical elements (Ebrahimi *et al.* 2018). Thus, many researchers found it significant to probe the elastic characteristics of elements fabricated from FGMs. In one of the researches in this area, the natural frequency behaviors of circular FGM plates was studied by Ebrahimi and Rastgoo (2008) on the basis of the classical plate theory (CPT). Shen (2009) procured a comparative investigation dealing with the stability characteristics of plates consisted of FGM with intelligent actuators. Furthermore, an iteration-based approach is implemented by Huang and Li (2010) to compute the frequency of FG beams with respect to the variations in the beam's section area. On the other hand, a numerical analysis was conducted by Alshorbagy *et al.* (2011) to survey the time-dependent responses of FG beams. Şimşek *et al.* (2012) analyzed the dynamically affected elastic characteristics of FG beams subjected to a harmonic loading. Vibrational behaviors of FGM plates were investigated by Thai and Choi (2012) via a higher-order theory which is powerful enough to capture the influences of shear deflection. Ebrahimi (2013) studied frequency characteristics of plates consisted of FGMs with respect to the between electrical and mechanical properties of the structure. The nonlinear thermally influenced stability problem of either circular or annular plates made from FGM was studied by Ghiasian *et al.* (2014). Moreover, a general frequency analysis, including both free and forced oscillations, of FG beam-type elements was carried out by Şimşek (2015). On the other hand, the nonlinearity effects were included in an investigation proposed by Ghiasian *et al.* (2015) as well as those of thermal environment for the goal of analyzing the dynamic stability characteristics of FG beams. Jafarinezhad and Eslami (2017) considered for the effects of existence of a thermal shock on the mechanical behaviors of FGM plates. Gharibi *et al.* (2017) employed a mathematical series-based method to obtain the stress components of pressure vessels consisted of FGMs. Lately, a nonlinear mechanical analysis was procured by Tang and Yang (2018) dealing with the FG pipes.

Meanwhile a pin-moment model of flexoelectric actuators was presented by Wang *et al.* (2018) and an electro-hydrostatic actuator for hybrid active-passive vibration isolation by Henderson *et al.* (2018). Also Active vibration compensator on moving vessel by hydraulic parallel mechanism examined by Tanaka (2018).

Besides, the influence of porosity in materials like FGMs are of high importance. To be honest with you, one of the weak points of FGMs is their high probability of possessing defects and porosity because of their critically sensitive fabrication procedure. Henceforward, the porosity effects shall be regarded once a FGM is analyzed. Originated from this fact, some of the authors preferred to account for porosity's side effects in their analyses on the mechanical characteristics of FG structures. For instance, Wattanasakulpong and Ungbhakorn (2014) explored the

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nonlinear vibrational responses of FG porous beams with restrained ends. In a series of researches, the buckling analysis of FG plates is performed under various loadings while regarding for porosity effects (Jabbari *et al*. 2014, Mojahedin *et al*. 2014, 2016). Moreover, bending, buckling and vibration analyses of FG porous beams are fulfilled by Chen and colleagues (Chen *et al*. 2015, 2016a, b). In addition, Rezaei and Saidi (2016) used Carrera unified formulation (CUF) to study the vibrational behaviors of FG porous plates. Wang and Wu (2017) employed a higher-order shell theory to investigate the vibrational responses of FG porous shells. In another endeavor, Atmane *et al*. (2017) considered for thickness stretching and porosity effects in their vibrational analysis on FG beams. Zenkour (2018) utilized a refined quasi-3D plate theory for the goal of analyzing the bending responses of FG porous single-layered and sandwich plates. Most recently, Gupta and Talha (2018) introduced a sigmoid homogenization model on the basis of porosity effects to probe both bending and buckling behaviors of FG plates.

Once glazing at the enormously greater number of conducted researches in the field of nanotechnology in comparison with the conventional ones, it can be realized that the world is moving in the way which increases the rate of implementation of nanosize devices in different applications. Hence, the designers must be aware of the reactions of nanostructures to different mechanical excitations to be able to perform a reliable design. On the other hand, the effects of small scale cannot be included in a desired analysis by the means of utilizing classical theories of continuum mechanics. According to this reality, the first theory which was able to capture the effects of small scale on the responses of nanosize elements was developed by is called nonlocal elasticity. From then on, a large number of researchers devoted their scientific activities to study the mechanical features of nanosize continuous systems in the framework of this hypothesis. For instance, the issue of stability of mono-layered graphene was considered by Pradhan and Murmu (2010) in the framework of Eringen’s hypothesis. Via a similar procedure, Ansari *et al*. (2011) analyzed the dynamic feature of multi-layered graphene. Furthermore, Mahmoud *et al*. (2012) explored the natural frequency behaviors of a simple nanosize beam. Also, a finite element (FE) formulation was extended by Eltaher *et al*. (2013) to investigate the natural frequency of nanobeams. Later, the hygro-thermally influenced deflection characteristics of nanoplates were probed by Alzahrani *et al*. (2013). The thermo-elastic dynamic responses of single-walled carbon nanotubes (SWCNTs) were studied by Ebrahimi and Salari (2015b). Zenkour (2016) depicted a time-dependent deflection analysis dealing with the transient bending of nanosize plates made from pure graphene. In another endeavor, the Eringen’s theorem was extended by Ghorbanpour Arani *et al*. (2017) for the goal of surveying the viscoelastically-affected dispersion characteristics of waves scattered in the media. In another research, several static and dynamic analyses on the nanosize films fabricated from intelligent materials were performed by Farajpour *et al*. (2018). Lately, Ebrahimi *et al*. (2019) depicted nonlinear answers for the forced vibration problem in a graphene layer on the basis of the Eringen’s nonlocal theorem. Moreover, it must be declared that because of the great surface to volume ratio of nanostructures the effects of surface of the nanostructure are not negligible in tiny elements (Gurtin and Murdoch 1975). Also, it is proven that the only involved term in determination of the mechanical response of a continuous system in the nanoscale is not the nonlocal parameter and the effects of strain gradient must be included to enrich a more realistic data about the surveyed element (Fleck and Hutchinson 1993). One can gain more information about the size-dependent theories studying complementary references (Civalek 2013, Shafiei *et al*. 2016, Akgöz and Civalek 2017a, b, Sahmani *et al*. 2018, Ebrahimi and Dabbagh 2018a, b, Hosseini *et al*. 2018, Aydogdu *et al*. 2018, Bouadi *et al*. 2018, Kumar 2018, Bensaid *et al*. 2018, Salari *et al*. 2019, Bendaho *et al*. 2019).

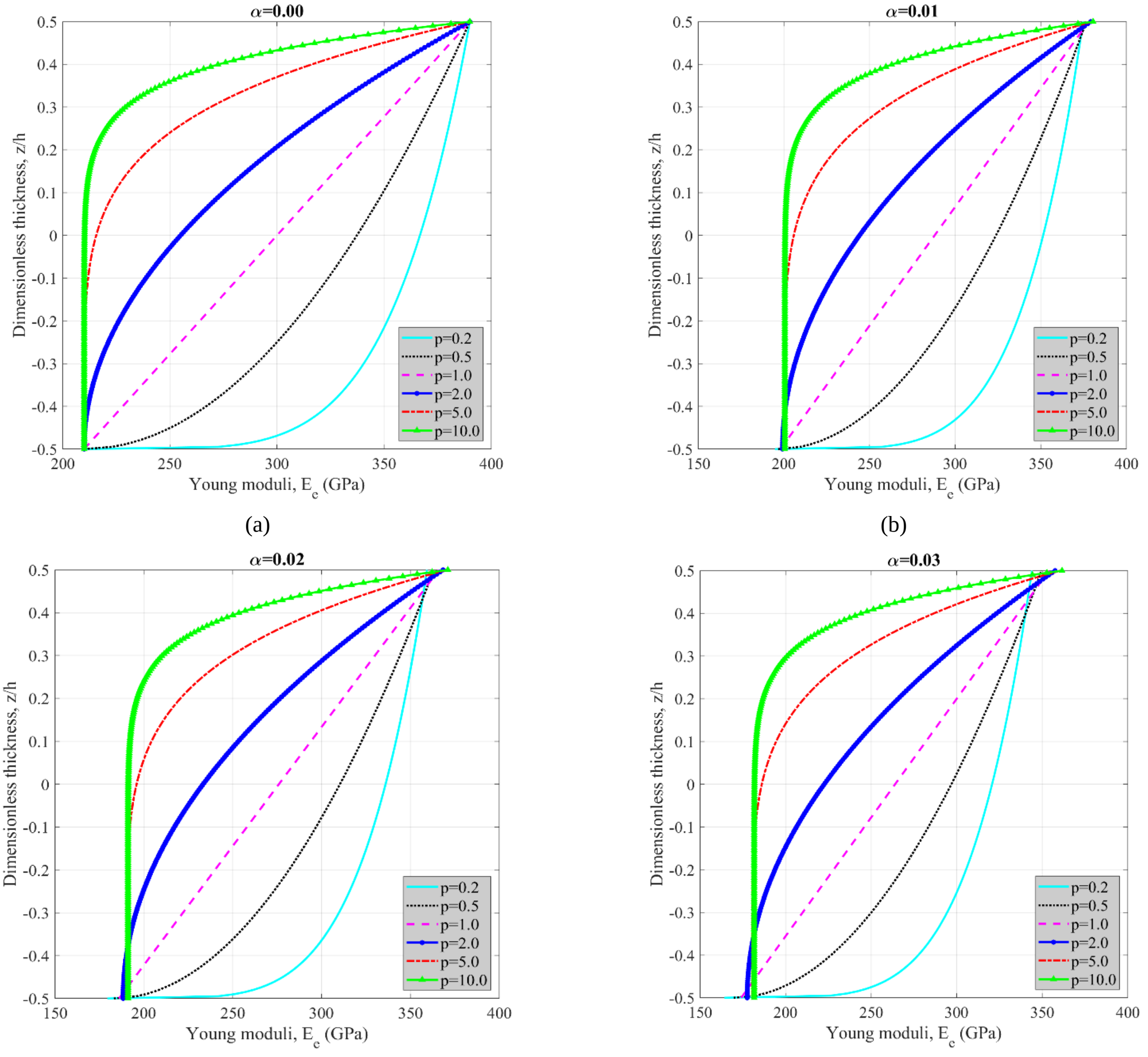
Furthermore, in the recent years, lots of researches can be found related to both time-dependent and -independent mechanical responses of nanosize structures made from FGMs. For example, Eltaher *et al*. (2012) studied the natural frequency properties of nanobeams consisted of an FGM. Moreover, the same problem was solved by Natarajan *et al*. (2012) for FGM plates. Timoshenko beam model was employed by Rahmani and Pedram (2014) to enrich the natural frequency of FG nanosize beams. In addition, influences of nonlinear infinitesimal strains were included by Nazemnezhad and Hosseini-Hashemi (2014) while probing the dynamic characteristics of FG nanobeams. Ebrahimi and Salari (2015a) analyzed both frequency and critical buckling temperatures of FG nanosize beams on the basis of the first-order hypothesis of beams. Also, the issue of buckling in FG nanosize beams was performed by Ebrahimi and Barati (2016) once influences of hygrothermal are included as well as those of magnetic one. In another article, the effect of material’s distribution in two directions is considered by Zamani Nejad *et al*. (2016) while analyzing the natural frequency behaviors of FG nanobeams. Meanwhile, the influence of different variants on the wave dispersion characteristics of FGM nanoscale plates is reported by Ebrahimi *et al*. (2016). Also, Ebrahimi *et al*. (2017) surveyed the wave propagation problem inside FG nanosize beams in the presence of the impact of beam’s rotation. Lately, Srividhya *et al*. (2018) modeled the bending problem of nanosize FGM plates with respect to the effect of nonlinear strain-displacement relation in the framework of a FE-based approximation.

Up to now, several analyses have been carried out by the authors for the goal of investigating the static and dynamic behaviors of FG nanosize elements. But, influence of porous being of the material on the mechanical responses of FG nanosize beams and plates are studied in just a couple of articles. In the only available work on the FG nanobeams, Ebrahimi and Dabbagh (2017) probed the wave dispersion problem of a porous nanoscale structure. Also, the nonlinear wave dispersion analysis of FG porous nanosize beams was procured by Barati (2017). In both of these research, Young’s modulus and density are supposed to be independent from each other, whereas, such a phenomenon does not happen in porous materials. Herein, a modified power-law model is introduced which connects elastic modulus and density via an integration method. The beam is modeled as a Euler beam and on the basis of the infinitesimal strains inside a beam the Navier equations are achieved. Size-dependency is prescribed according to nonlocal elasticity and the governing equations are solved via an analytical method to enrich the dynamic responses of dispersed waves in the nanostructure.

2. Theory and formulation

2.1 Modified power-law homogenization method

In this section, the effect of porosity on the effective mechanical properties of FG beams is covered in the framework of a modified porosity dependent homogenization scheme. As mentioned in the literature, Ebrahimi and Dabbagh (2017) considered the effect of using a porous FGM while analyzing the wave dispersion problem of FG nanobeams. They proposed a linear model to capture porosity. In other words, the influence of porosity on the effective material properties is presumed to be a linear function of the porosity volume fraction. Although this method can describe the decreasing impact of porosity, however, it cannot be presumed as a realistic homogenization method for the purpose of regarding for porosity effects (Zok and Levi 2001). Present method relates the effective Young's modulus of porous FGMs to the mass densities of both perfect and porous materials. Therefore, the homogenization is constructed on the basis of the primary definition of porosity. Henceforward, the effective material properties can be formulated as follows (Eltaher *et al*. 2018)



(a) (b)

(c) (d)

Fig. 1 Variation of the equivalent Young's moduli of FGM against dimensionless thickness for various gradient indices and porosity volume fractions

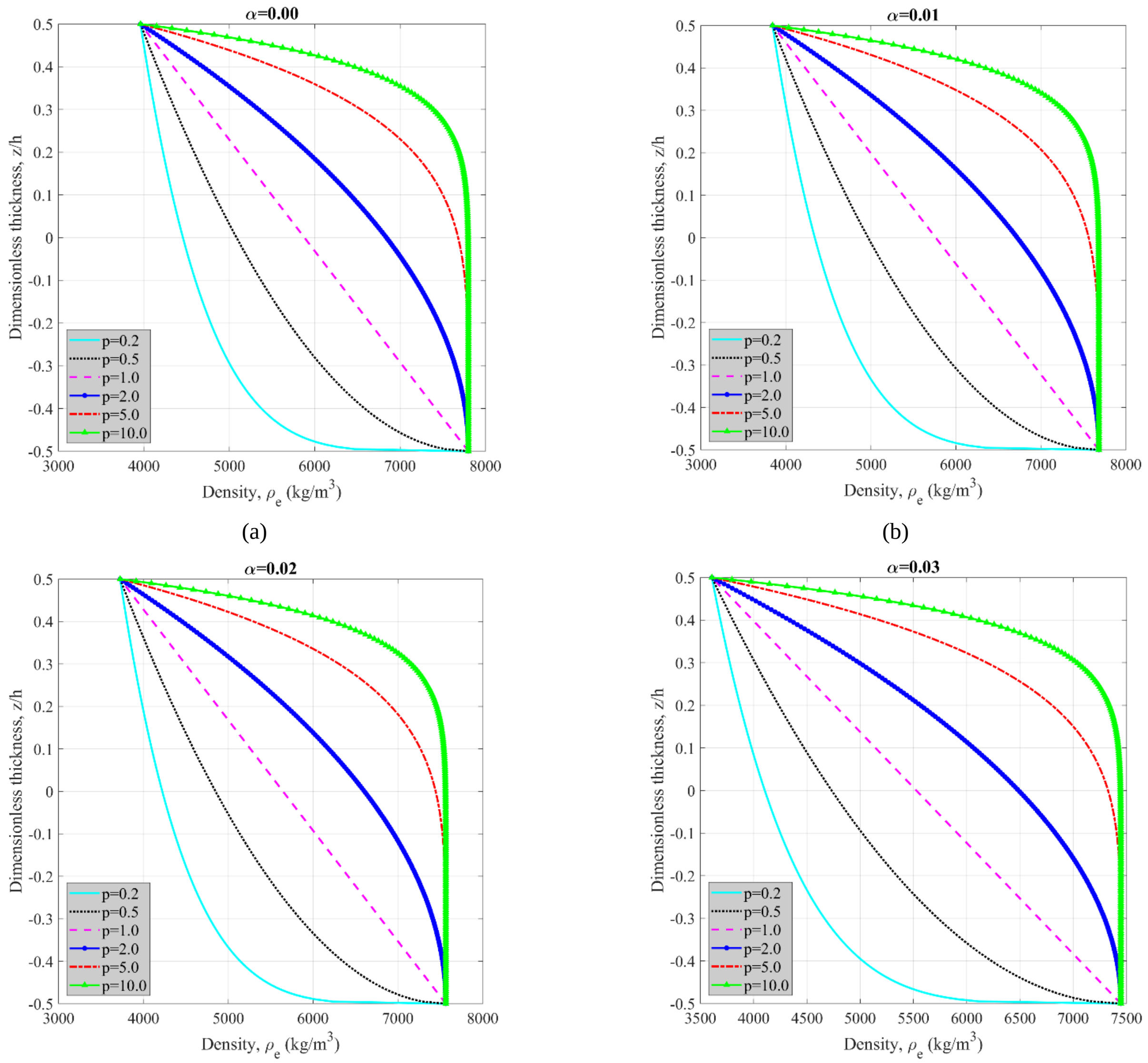
$$E(z) = (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + E_m - \frac{m_0 - m_1}{m_0}(E_c + E_m), \tag{1}$$

$$\rho(z) = (\rho_c - \rho_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \rho_m - \frac{\alpha}{2}(E_c + E_m) \tag{2}$$

where *c* and *m* subscripts denote ceramic and metal phases, respectively. In addition, α stands for the porosity volume fraction. Also, m_0 and m_1 are the true and apparent mass densities, respectively and can be calculated as follows (Eltaher *et al*. 2018)

$$m_0 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(z)dz \,\text{at}\, \alpha = 0, m_1 = \int_{-h/2}^{h/2} \rho(z)dz \,\text{at}\, \alpha > 0 \tag{3}$$

In Fig. 1, the effect of existence of porosities in the media on the stiffness characteristics of FGM is illustrated. It can be realized that whenever the effect of porosities in the material is dismissed, stiffer material can be observed. However, as mentioned in the Introduction, the effects of porosity must be considered. It is clear that the elastic moduli of the used FGM will be decreased as the volume fraction of porosity is intensified. In other words, the stiffness of the continuous system consisted of FGMs will be lessened whenever a porous material is utilized. Moreover, Fig. 2 is allocated to show the effect of availability of porosity on the mass density of the implemented material. It is obvious that the density of the FGM will be lessened as same as its moduli while a larger volume fraction is selected for the porous FGM. So, it is of great importance to consider for the effects of porosity due to the remarkable reduction which can be seen in the mechanical properties of FGMs as the porosity volume fraction grows. It can be estimated that the dispersion



(a) (b)

(c) (d)

Fig. 2 Variation of the equivalent density of the FGM against dimensionless thickness for various gradient indices and porosity volume fractions curves of waves scattered in the FG nanobeam will experience a decrease as a nonzero value is assigned to the porosity volume fraction instead of certain zero.

2.2 Euler-Bernoulli beam theory

The equations of motion for the FG beam are modeled in the present research according to the classical beam theory. The displacement field of this theory can be written as (Eltaher *et al*. 2018)

$$u_x(x,z,t)=u(x,t)-z\frac{\partial w(x,t)}{\partial t} \tag{4}$$

$$u_z(x,z,t)=w(x,t) \tag{5}$$

in which, u and w correspond with the axial displacement and bending deflection of the beam. Therefore, the nonzero strains of the beam can be defined as (Eltaher *et al*. 2018)

$$\varepsilon_{xx}=\varepsilon_{xx}^0-z\kappa_x^0 \tag{6}$$

where

$$\varepsilon_{xx}^0=\frac{\partial u}{\partial x},\qquad \kappa_x^0=\frac{\partial^2 w}{\partial x^2} \tag{7}$$

2.3 Hamilton's principle

Now, Hamilton's principle is applied to obtain the Navier equations of FG beam as follows (Ebrahimi and Dabbagh 2017)

$$\int_0^t \delta(U+V-T)dt=0 \tag{8}$$

where U and T account for strain energy and kinetic energy, respectively; also, V denotes the work done by external loading applied on the structure. Now, the variation of strain energy can be formulated as (Ebrahimi and Dabbagh 2017)

$$\delta U=\int_V(\sigma_{xx}\delta\varepsilon_{xx})dV=\int_0^L\left(N(\delta\varepsilon_{xx}^0)-M(\delta\kappa_x^0)\right)dx \tag{9}$$

in above equation, the axial force (N) and bending moment (M) can be defined as

$$N=\int_A\sigma_{xx}dA\,,M=\int_A\sigma_{xx}zdA \tag{10}$$

Also, the variation of kinetic energy can be expressed as follows (Ebrahimi and Dabbagh 2017)

$$\begin{aligned}\delta T=\int_0^L&\left(I_0\left(\frac{\partial u}{\partial t}\frac{\partial \delta u}{\partial t}+\frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t}\right)\right.\\&-I_1\left(\frac{\partial u}{\partial t}\frac{\partial^2 \delta w}{\partial x\partial t}+\frac{\partial^2 w}{\partial x\partial t}\frac{\partial \delta u}{\partial t}\right)\\&\left.+I_2\left(\frac{\partial^2 w}{\partial x\partial t}\frac{\partial^2 \delta w}{\partial x\partial t}\right)\right)dx\end{aligned} \tag{11}$$

in which, the mass moments of inertia are defined as

$$(I_0,I_1,I_2)=\int_A(1,z,z^2)\rho(z)dA \tag{12}$$

Besides, the influence of the initial axial pre-load applied on the beam can be expressed in the following mathematical form

$$\delta V=\int_0^L\widetilde{N}\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x}dx \tag{13}$$

where $\widetilde{N}$ expresses the axial loading applied on the beam. Herein, once Eqs. (9), (11) and (13) are substituted in Eq. (8) and the coefficients of δu and δw are set to zero, the Navier equations of FG beam can be written as follows

$$\frac{\partial N}{\partial x}=I_0\frac{\partial^2 u}{\partial t^2}-I_1\frac{\partial^3 w}{\partial x\partial t^2}, \tag{14}$$

$$\frac{\partial^2 M}{\partial x^2}-\widetilde{N}\frac{\partial^2 w}{\partial x^2}=I_0\frac{\partial^2 w}{\partial t^2}+I_1\frac{\partial^3 u}{\partial x\partial t^2}-I_2\frac{\partial^4 w}{\partial t^2\partial x^2} \tag{15}$$

2.4 Nonlocal elasticity theory

Based upon the nonlocal constitutive equations, the stress state of a point inside a nanostructure is a function of strain of all adjacent points in addition to that point's strain. So, the stress-strain relationship can be described in the following form (Eringen 1972)

$$(1-\mu^2\nabla^2)\boldsymbol{\sigma}=\boldsymbol{C}:\boldsymbol{\varepsilon} \tag{16}$$

where **σ**, **ε** and **C** are stress, strain and elasticity tensors, respectively. Once extending Eq. (16) and integrating from it over the beam's cross-section, the following relations can be achieved for axial force and bending moment

$$(1-\mu^2\nabla^2)N=A\frac{\partial u}{\partial x}-B\frac{\partial^2 w}{\partial x^2}, \tag{17}$$

$$(1-\mu^2\nabla^2)M=B\frac{\partial u}{\partial x}-D\frac{\partial^2 w}{\partial x^2} \tag{18}$$

where

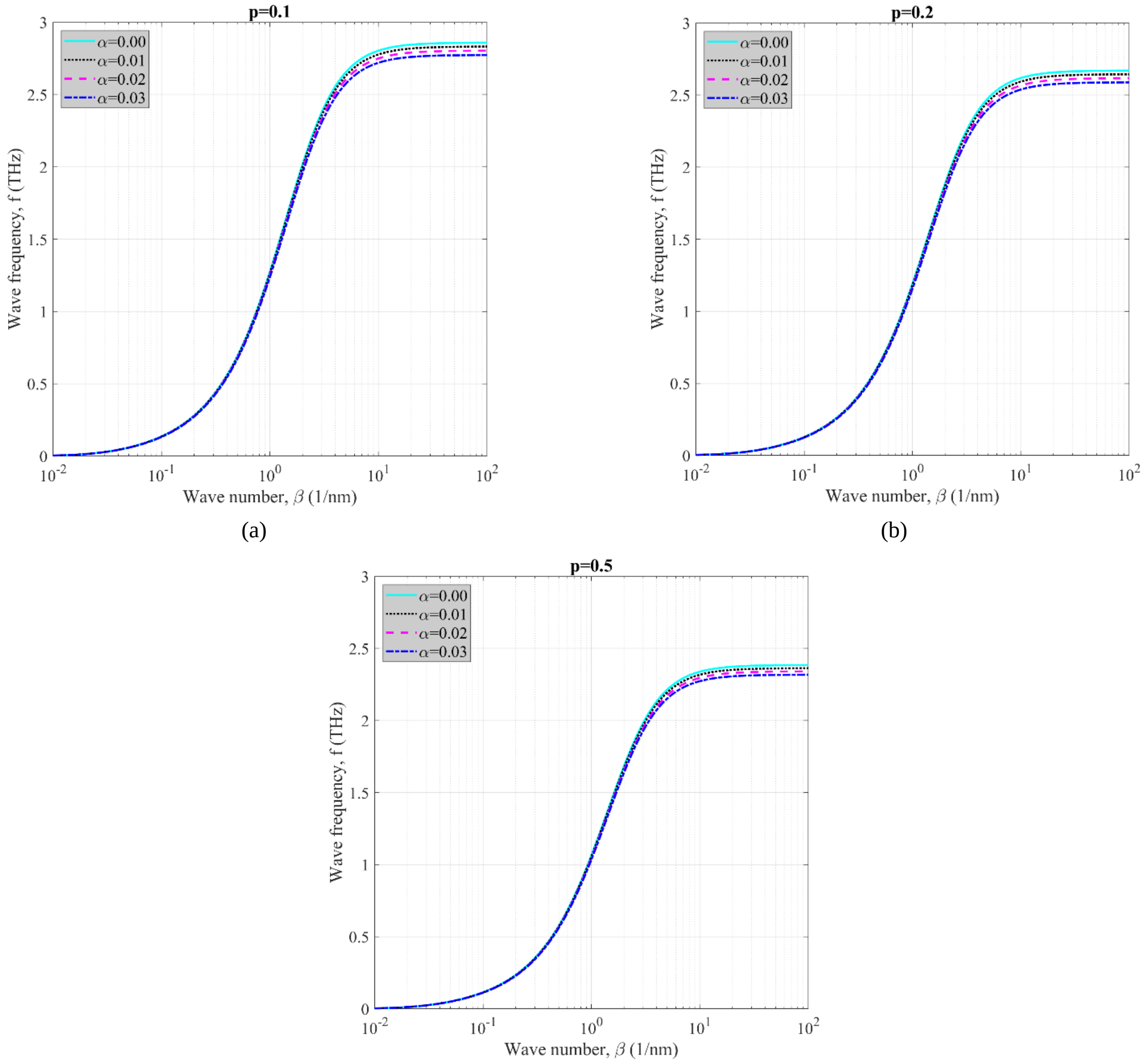
$$(A,B,D)=\int_A(1,z,z^2)dA \tag{19}$$

2.5 Governing equations

Now, the final governing equations of FG porous nanobeams can be achieved by substituting Eqs. (17) and (18) in Eqs. (14) and (15)

$$\begin{aligned}&A\frac{\partial^2 u}{\partial x^2}-B\frac{\partial^3 w}{\partial x^3}\\&+(1-\mu^2\nabla^2)\left(-I_0\frac{\partial^2 u}{\partial t^2}+I_1\frac{\partial^3 w}{\partial x\partial t^2}\right)=0,\end{aligned} \tag{20}$$

$$\begin{aligned}&B\frac{\partial^3 u}{\partial x^3}-D\frac{\partial^4 w}{\partial x^4}\\&+(1-\mu^2\nabla^2)\begin{pmatrix}-I_0\dfrac{\partial^2 w}{\partial t^2}-I_1\dfrac{\partial^3 u}{\partial x\partial t^2}\\+I_2\dfrac{\partial^4 w}{\partial x^2\partial t^2}-\widetilde{N}\dfrac{\partial^2 w}{\partial x^2}\end{pmatrix}=0\end{aligned} \tag{21}$$



(a) (b)

(c)

Fig. 3 Variation of wave frequency versus wave number for both perfect and porous materials by considering the influence of gradient index (μ = 0.5 nm)

3. Solution procedure

Here, an analytical solution method is applied to solve the governing equations. The displacement field's components are supposed to be (Ebrahimi and Dabbagh 2017)

$$\begin{Bmatrix} u \\ w \end{Bmatrix} = \begin{Bmatrix} U\,exp[i(\beta x - \omega t)] \\ W\,exp[i(\beta x - \omega t)] \end{Bmatrix} \tag{22}$$

where, U and W are wave amplitudes, β is wave number and ω is the circular frequency of dispersed waves. Substituting for u and w from Eq. (22) in the Eqs. (20) and (21), the following eigenvalue equation is obtained

$$([K] - \omega^2[M])\begin{bmatrix} U \\ W \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{23}$$

Once the above eigenvalue equation is solved for ω, the wave frequency of propagated waves can be reached. The aforementioned equation can be solved by setting the determinant of $[K] - \omega^2[M]$ part to be zero. Indeed, via this procedure, the non-trivial response will be obtained for the presented problem. Moreover, the phase velocity can be computed by dividing wave frequency to wave number as (Ebrahimi and Dabbagh 2017)

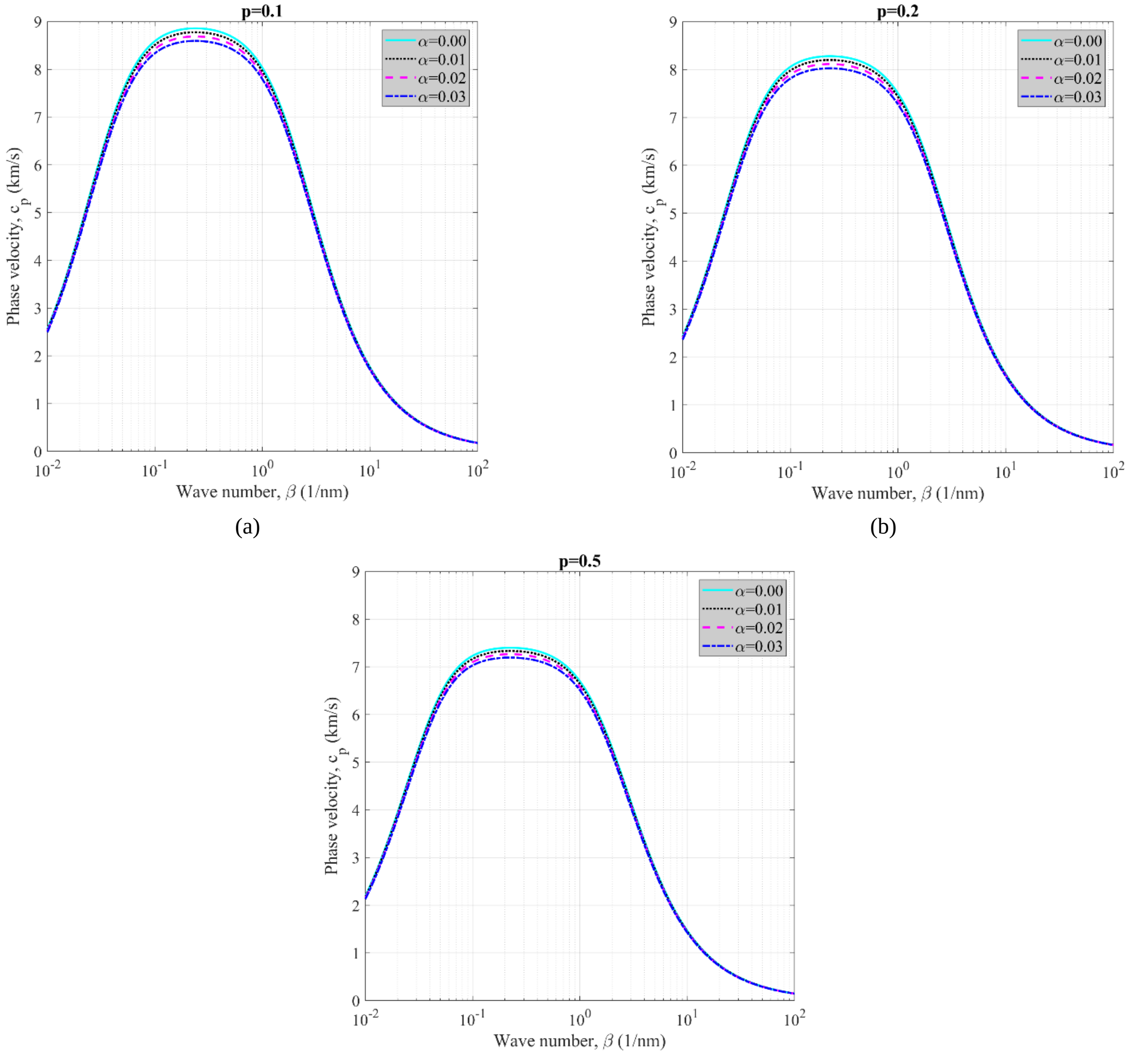
$$c_p = \frac{\omega}{\beta} \tag{24}$$

4. Numerical results

Present part is devoted to study the influences of various parameters on the wave propagation behaviors of FG nanobeams. In this study, the material properties of ceramic are E_c = 390 GPa, ρ_c = 3960 kg/m^3; also, the metal's mechanical properties are E_m = 210 GPa, ρ_m = 7800 kg/m^3. The presented formulation is validated by comparing the natural frequency responses of ours with those of Eltaher *et al*. (2012) for nonlocal parameters of 0, 1, 2, 3, 4 and 5 while the gradient index is presumed to be 0.1, 0.5, 1 and 5. It can be realized that the errors between the results reported by Eltaher *et al*. (2012) and those achieved by us are negligible. It is worth mentioning that this difference is appeared due to different types of solutions utilized in our modelling and theirs. Eltaher *et al*. (2012) used a FE approximation in their article, whereas, we implemented an analytical method.

In the first case study, the influence of porosity volume fraction on the wave frequency of the scattered waves is depicted in Fig. 3 by plotting the variations of wave frequency versus wave number. In addition, the impact of changing the gradient index of the employed FGM is included, too. Clearly, the wave frequency is affected by changing the volume fraction of porosities in the media in great wave numbers. In fact, the wave frequency of the dispersed waves will be lessened as the volume fraction of porosities is raised. The reason of such a reducing impact is that in the case of using a porous material, the stiffness of the material decreases and due to the direct relation between the stiffness and frequency, the dynamic response of the continua lessens. On the other hand, it can be figured out that the greater is the implemented gradient index, the lesser will be the obtained frequency. This phenomenon happens because of the fact that as the gradient increases the material's composition moves toward ceramic-rich phase which possesses lower stiffness in comparison with the metallic phase. Henceforward, the stiffness reduction will be resulted in a decrease in the frequency of the propagated waves.

Similar plots are depicted in Fig. 4 for the variations of the phase velocity of the dispersed waves. In this diagram, the phase velocity of the FG nanobeams is drawn against the wave number for both porous and non-porous FGMs. Again, the dispersion curves will be lessened as the porosity volume fraction is increased due to the physical reason



(a)

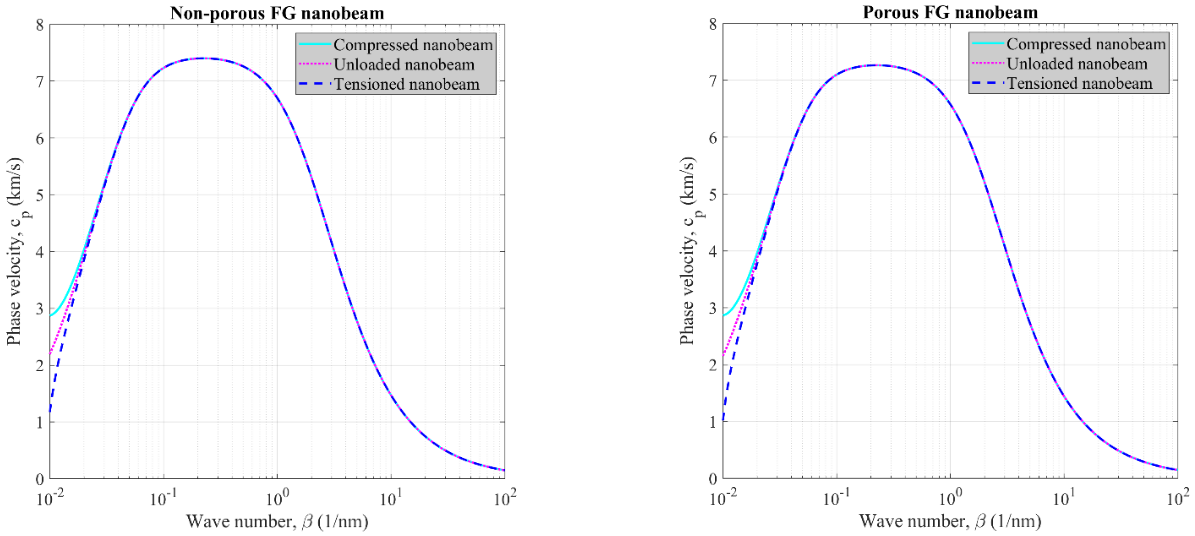
(b)

(c)

Fig. 4 Variation of phase velocity versus wave number for both perfect and porous materials by considering the influence of gradient index (μ = 0.5 nm) mentioned in the interpretation of Figs. 1 and 3. However, there is a difference between the phase speed curves with those of wave frequency. Indeed, the diagrams of phase speed will be affected in the mid-range wave numbers, whereas, those of wave frequency will be influenced only in large wave numbers. So, the velocity of waves in porous FG nanobeams will be affected in the wave number region from 0.05 (1/nm) to 2 (1/nm), while, this range for the wave frequency is from 2 (1/nm) to infinity.

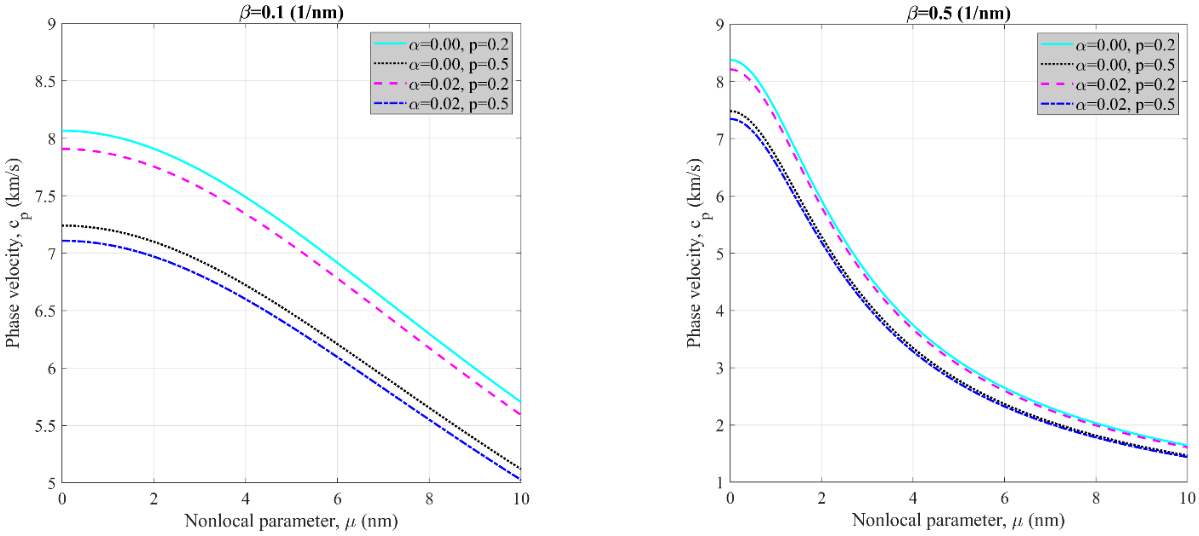
Furthermore, the effect of axial pre-load on the dispersion curves of nanosize beams consisted of both non-porous and porous media is depicted in Fig. 5. As same as former illustrations, it can be found that the phase velocity of the dispersed waves will be decreased as the volume fraction of porosities in the material is assumed to be a nonzero value. Besides, it can be understood that the nanostructure reacts to the applied pre-load in small wave numbers. Actually, the phase speed of the propagated waves becomes smaller whenever the nanobeam is subjected to an axial pre-tension. Reversely, the phase velocity will be aggrandized while the nanostructure is subjected to an axial pre-compression. Thus, it must be noticed that the behaviors of flexural waves can be varied once a pre-load is applied on the nanobeam in a limited range of small wave numbers.

On the other hand, the coupled influences of porosity and material's composition on the variation of phase velocity against nonlocal parameter is presented in Fig. 6 with respect to the influence of the employed wave number. In this diagram, the stiffness-softening phenomenon reported by Eringen (1972) can be seen easily. In fact, the phase velocity becomes smaller in the cases of choosing a higher nonlocal parameter. In such a condition, the stiffness



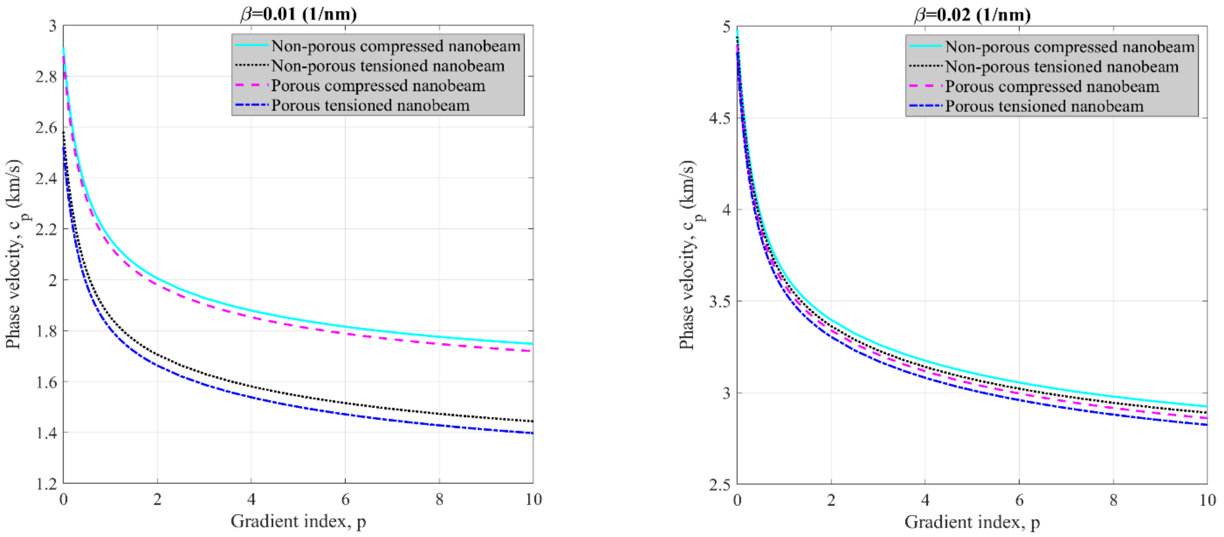
(a) (b)

Fig. 5 Variation of phase velocity versus wave number with for nanobeams with different types of axial loading (μ = 0.5 nm)



(a) (b)

Fig. 6 Variation of phase velocity versus nonlocal parameter for various gradient indices and porosity effects with respect to the influence of wave number



(a) (b)

Fig. 7 Variation of phase velocity versus gradient index for both non-porous and porous materials with respect to the influence of wave number and applied axial loading (μ = 0.5 nm)

Table 1 Comparison of the natural frequencies of FG nanobeams (L/h = 20)

	p = 0.1		p = 0.5		p = 1		p = 5	
μ	Eltaher *et al.* (2012)	Present	Eltaher *et al.* (2012)	Present	Eltaher *et al.* (2012)	Present	Eltaher *et al.* (2012)	Present
0	9.2129	9.1887	7.8061	7.7377	7.0904	6.9885	6.0025	5.9371
1	8.7889	8.7663	7.4458	7.3820	6.7631	6.6672	5.7256	5.6641
2	8.4166	8.3972	7.1312	7.0712	6.4774	6.3865	5.4837	5.4257
3	8.0887	8.0712	6.8533	6.7966	6.2251	6.1386	5.2702	5.2150
4	7.7964	7.7804	6.6057	6.5518	6.0001	5.9174	5.0797	5.0271
5	7.5336	7.5189	6.3830	6.3316	5.7979	5.7185	4.9086	4.8582

of the nanostructure will be lessened which results in a reduction in the dynamic response of the system. Moreover, again it can be observed that either employment of a greater gradient index or selecting a porous material will lessen the stiffness of the nanostructure and due to this reason the phase velocity will be decreased accordingly. Also, it can be found that the influence of the nonlocality can be better observed as the wave number is assumed to be 0.5 (1/nm) instead of 0.1 (1/nm).

As the last illustration, the variation of phase velocity versus gradient index is drawn in Fig. 7 for both porous and non-porous FGM nanobeams which are either tensioned or compressed. According to the diagram, an increment in the gradient index results in a reduction in the phase velocity of the nanobeam because of the decrease which happens in the stiffness of the nanobeam. It can be seen that non-porous nanostructures possess higher wave speeds in comparison with the porous ones. Also, the effect of wave number on the phase speed change of the dispersed waves is shown in this diagram. In other words, only with replacing wave number of 0.01 (1/nm) with 0.02 (1/nm) it can be seen that the effect of other involved parameters on the velocity variations will be decreased. So, the effects of wave number must be considered by designers working in this area.

5. Conclusions

In the framework of a newly developed porosity-dependent homogenization scheme, this article was tended to analyze the dispersion behaviors of FG nanosize beam-type elements while the nanostructure is subjected to an axial pre-load. After deriving the motion equations on the basis of the Hamilton's principle and obtaining the governing equations according to the nonlocal theory of elasticity, an analytical method was utilized to reach the dynamic response of the nanobeam. Now, the most crucial highlights of the paper will be reviewed in the following sentences:

- Existence of porosity in the FGM of the nanobeam result in a reduction in the stiffness of the nanobeam which leads to a decrease in the wave frequency and phase speed of the dispersed waves.
- The velocity of scattered waves in the nanostructure can be amplified by pre-compressing the nanobeam. Obviously, pre-tension results in decrease of the dynamic response of the nanobeam.
- The increment of gradient index behaves in a same way which existence of porosity in the media does.

- The dispersion response of the nanobeam can be lessened by adding the value of the nonlocal parameter.

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