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Damage detection of nonlinear structures with analytical mode decomposition and Hilbert transform

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Abstract. This paper proposes an analytical mode decomposition (AMD) and Hilbert transform method for structural nonlinearity quantification and damage detection under earthquake loads. The measured structural response is first decomposed into several intrinsic mode functions (IMF) using the proposed AMD method. Each IMF is an amplitude modulated-frequency modulated signal with narrow frequency bandwidth. Then, the instantaneous frequencies of the decomposed IMF can be defined with Hilbert transform. However, for a nonlinear structure, the defined instantaneous frequencies from the decomposed IMF are not equal to the instantaneous frequencies of the structure itself. The theoretical derivation in this paper indicates that the instantaneous frequency of the decomposed measured response includes a slowly-varying part which represents the instantaneous frequency of the structure and rapidly-varying part for a nonlinear structure subjected to earthquake excitations. To eliminate the rapidly-varying part effects, the instantaneous frequency is integrated over time duration. Then the degree of nonlinearity index, which represents the damage severity of structure, is defined based on the integrated instantaneous frequency in this paper. A one-story hysteretic nonlinear structure with various earthquake excitations are simulated as numerical examples and the degree of nonlinearity index is obtained. Finally, the degree of nonlinearity index is estimated from the experimental data of a seven-story building under four earthquake excitations. The index values for the building subjected to a low intensity earthquake excitation, two medium intensity earthquake excitations, and a large intensity earthquake excitation are calculated as 12.8%, 23.0%, 23.2%, and 39.5%, respectively.

Keywords: degree of nonlinearity; damage detection; analytical mode decomposition; Hilbert transform; earthquake excitations

1. Introduction

Vibration-based methods for system identification and damage detection have been widely studies as summarized in a comprehensive review by Doebling *et al*. (1996, 1998) and Sohn *et al*. (2004). The basic idea is that the modal parameters are functions of the physical properties of the structure, and changes in physical property will cause changes in the modal properties. Therefore,

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over the past decades, many structural modal parameter identification methods in both frequency and time domain were developed by different investigators such as: peak-picking method from power spectral densities (Bendat and Piersol 1993), natural excitation technique (NExT) method (James *et al*. 1995), and stochastic subspace identification method (Van *et al*. 1996). However, their research work was paid attention to positive and negative problem of linear system. For time-varying or nonlinear structures, the structural nonlinearities such as stiffness and damping force nonlinearities can introduce dynamic phenomena and behaviors that are dramatically different from those predicted by the linear theory. Brandon (1997, 1999) stated that the nonlinear response of a mechanical system was often overlooked and valuable information was lost when one discarded the time series data and focused on the spectral data. Therefore, the author advocated the use of time-domain system identification techniques such as ARMA model and autocorrelation function to retain the important nonlinear information. Although attempts were made to take advantage of nonlinear behaviours (Vakakis *et al*. 2004, Kerschen *et al*. 2006), it is still a challenge to identify a nonlinear system due to its highly individualistic nature.

Nonlinearities of structures under extreme loads, such as earthquakes, hurricanes, and tornados, are mainly due to the stiffness and boundary conditions vary rapidly or slowly over excitation duration, and the characteristic properties of structures often change over time. Therefore, structural identification for time-varying properties such as instantaneous natural frequency can be used for structural nonlinearity characterization. In recent years, various time-frequency analysis methods for time-varying and nonlinear structures identification were proposed, such as: Wigner-Ville distribution (WVD) (Qian and Chen 1994), Hilbert transform based methods (Huang *et al*. 1998, 1999), and wavelet transform theory (Mallat 1998, Lilly and Olhede 2010, Ghanem and Romeo 2000, Li *et al*. 2009, Yi *et al*. 2012, 2013). Based on the time-frequency analysis methods, the nonlinearities of structures can be further identified. Kerschen *et al*. (2006) and Pai and Hu (2006) used a popular empirical mode decomposition of vibration signal for nonlinear identification. Ta and Lardies (2006) proposed the continuous wavelet transform for identifying and quantifying nonlinearities of each vibration mode. More preferable nonlinear system identification methods without known the prior nonlinear models were also investigated by a few researchers. For example, Feldman (2007) proposed a Hilbert transform vibration decomposition together with the modal-spatial coordinate transform method for initial nonlinear characteristics identification; Chanpheng *et al*. (2012) proposed the degree of nonlinearity as a feature for damage detection on large civil structures with Hilbert transform method; Wang *et al*. (2003) and Feldman (2012) mapped a class of nonlinear system to a skeleton linear model and extracted the skeleton from the response data using the quadratic time-frequency distribution and the wavelet transform method.

Since the time-frequency analysis based method for structural nonlinearity identification need to characterize the responses' time-varying features, therefore, high quality signal processing techniques are quite necessary. However, only in recently years, a few new time-frequency analysis methods for non-stationary signal analysis such as ensemble empirical mode decomposition (Wu and Huang 2009), synchrosqueezing wavelet transform (Daubechies *et al*. 2011, Thakur and Wu 2011, Montejo and Vidot 2012, Wang *et al*. 2013b) were developed and proposed in literature. However, up to date, it is still a quite challenge to find a universal fully automatic decomposition in spite of signal adaptive properties (Braun and Feldman 2011).

In this paper, the recently developed AMD method (Chen and Wang 2012, Wang *et al*. 2013a) is applied to decompose the measured signal into several IMFs. Then, the instantaneous frequency of the decomposed IMF, which includes a slowly-varying part that represents the instantaneous frequency of the structure and a zero mean rapidly-varying part for a nonlinear structure, can be defined with Hilbert transform. To eliminate the rapidly-varying part effect, the instantaneous frequency is integrated over time duration, and the degree of nonlinearity index, which represents the damage severity of structure, is defined based on the integrated instantaneous frequency. The degree of nonlinearity indices are obtained based on the one-story hysteretic structure subjected to low, medium, and larger intensity earthquakes. Final, the proposed method is validated by the shake table test data of a seven-story building subjected to a low, two medium, and a large intensity earthquake excitations.

2. Degree of nonlinearity index with AMD and Hilbert transform

For a SDOF nonlinear system with mass m and excitation $f(t)$, the equation of motion can be described as

$$m\ddot{x}(t) + F(x,\dot{x}) = f(t) \tag{1}$$

For a nonlinear structure, the nonlinear restoring force $F(x,\dot{x})$ as function of time can be transformed into a multiplication form $m\omega_0^2(t)x(t)$ with a new fast time-varying natural frequency $\omega_0^2(t)$ and a system displacement solution $x(t)$ with an overlapping spectra (Feldman 1997). Similarly, the nonlinear damping force can also be transformed into a function of time as a multiplication: $2mh_0(t)\dot{x}(t)$ between the fast time-varying instantaneous damping coefficient $h_0(t)$ and the velocity. Thus the equation of motion of a nonlinear system can be written as

$$\ddot{x}(t) + 2h_0(t)\dot{x}(t) + \omega_0^2(t)x(t) = f(t)/m \tag{2}$$

When the instantaneous damping coefficient $h_0(t)$ and instantaneous natural frequency $\omega_0^2(t)$ vary slowly compared to the oscillatory of the displacement, which is satisfied in most engineering applications. Then, the instantaneous damping coefficient $h_0(t)$ and natural frequency $\omega_0^2(t)$ of the structural system can be solved as

$$\omega_0^2(t) = \omega^2(t) + \frac{f(t)x(t) + H[f(t)]H[x(t)]}{m[x^2(t) + (H[x(t)])^2]} + \frac{H[f(t)]x(t) - f(t)H[x(t)]}{x^2(t) + (H[x(t)])^2}\frac{\dot{A}(t)}{A(t)\omega(t)\mathrm{m}} - \frac{\ddot{A}(t)}{A(t)} + 2\frac{\dot{A}(t)}{A^2(t)} + \frac{\dot{A}(t)\dot{\omega}(t)}{A(t)\omega(t)} \tag{3}$$

$$h_0(t) = \frac{H[f(t)]x(t) - f(t)H[x(t)]}{x^2(t) + (H[x(t)])^2}\frac{1}{2\omega(t)m} - \frac{\dot{A}(t)}{A(t)} - \frac{\dot{\omega}(t)}{\omega(t)} \tag{4}$$

in which, $\omega(t)$ is the instantaneous frequency of the measured response $x(t)$, H[] represents the Hilbert transform of the function inside the square bracket, $A(t) = \sqrt{x^2(t) + (H[x(t)])^2}$ is the instantaneous amplitude of the measured response $x(t)$.

In reality, the damping coefficients and derivatives of the envelope of the analytical signal are much less than the natural frequency, so their influence can be ignored $(\frac{\dot{A}}{A} = \frac{\ddot{A}}{A} = 0)$. As a result, the instantaneous frequency of the signal leads to

$$\omega^2(t) = \omega_0^2(t) - \frac{f(t)x(t) + H[f(t)]H[x(t)]}{m[x^2(t) + (H[x(t)])^2]} \tag{5}$$

If the earthquake excitation f has zero mean value, the second term of Eq. (5) can be also approximately considered as the zero mean fast time-varying function. As one can see from Eq. (5) the instantaneous frequency $\omega(t)$ of the decomposed measured response includes a slowly-varying part $\omega_0(t)$ which represents the instantaneous frequency of the structure and a zero mean rapidly-varying part for a nonlinear structure under an earthquake excitation $f(t)$.

Similarly, an n DOF nonlinear system can be transferred as a time-varying linear system, the equation of motion can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}(t)\dot{\mathbf{x}}(t) + \mathbf{K}(t)\mathbf{x}(t) = \mathbf{f}(t) \tag{6}$$

where $\mathbf{M}$, $\mathbf{C}(t)$, and $\mathbf{K}(t)$ are constant mass, time-varying damping and stiffness matrices, respectively; $\boldsymbol{f}(t)$ is the external load vector. The Eq. (6) can be transformed into modal spatial coordinate

$$\ddot{q}_i(t) + 2h_{0i}(t)\dot{q}_i(t) + \omega_{0i}^2(t)q_i(t) = \frac{\phi_i^T f(t)}{M_i}(i = 1,2,\ldots,n) \tag{7}$$

in which, $M_i = \phi_i^T M \phi_i$ is the i^{th} modal mass, ϕ_i is the ith mode shape vector and $\omega_{0i}(t)$ is the natural frequency of the i^{th} modal response. Since the external load is assumed zero mean value, the instantaneous frequency of the modal response $\omega_i^2(t)$ can be expressed as similar as Eq. (3), which yields

$$\omega_i^2(t) = \omega_{0i}^2(t) - \frac{\frac{\phi_i^T f(t)}{M_i}q + H\left[\frac{\phi_i^T f(t)}{M_i}\right]H[q(t)]}{[q^2(t) + (H[q(t)])^2]} \tag{8}$$

Again, for vibration with zero mean earthquake excitation, the second term of Eq. (8) is the zero mean fast time-varying function.

Since the instantaneous frequency of the modal responses is time-varying function, therefore, an adaptive or time-varying bandpass filter is needed to extract the modal responses from the measured response. In this paper, the recently developed AMD method (Chen and Wang 2012) is extended to extract the modal responses from the measured signals of nonlinear structures with time-varying frequencies. The goal is to extract each individual component (IMF) with time-varying frequency from a general measured response by properly selecting two time-varying bisecting frequencies that cover the component frequency at any time instant. As mentioned in the reference (Wang 2011), the time-varying bisecting frequency can be preliminary selected from the wavelet scalogram.

For a measured response $x(t)$ with n individual modal responses with frequencies: $\omega_1(t), \omega_2(t), \ldots, \omega_n(t)$, By selecting n-1 time-varying bisecting frequencies:

$\omega_{bi}(t)\in(\omega_i(t),\omega_{i+1}(t))\,(i=1,2,\ldots,n-1)$ each individual signal can be determined by

$$x_1^{(d)}=s_1(t),\ldots,x_i^{(d)}(t)=s_i(t)-s_{i-1}(t),\ldots,x_n^{(d)}(t)=x(t)-s_{n-1}(t) \tag{9}$$

$$s_i(t)=\sin[\theta_{bi}(t)]H\{x(t)\cos[\theta_{bi}(t)]\}-\cos[\theta_{bi}(t)]H\{x(t)\sin[\theta_{bi}(t)](i=1,2,\ldots,n-1)\} \tag{10}$$

in which, $\theta_{bi}(t)=\int_{-\infty}^{t}\omega_{bi}(\boldsymbol{\tau})\,\mathrm{d}\tau$ is the phase angle of the ith bisecting frequency.

As observed from Eqs. (9) and (10), AMD functions like a suite of bandpass filters. Since the bisecting frequency varies with time, reflecting the time-frequency analysis of a data series, the related filter with a time-varying bisecting frequency $\omega_{bi}(t)$ is referred to as an adaptive lowpass filter, which is schematically illustrated in Fig. 1.

For the measured response of the l^{th} degree $x_l(t)$ can be expressed as

$$x_l(t)=\sum_{i=1}^{n}\phi_{li}q_i \tag{11}$$

in which, ϕ_{li} is the lth element of the i^{th} mode shape vector. Therefore, the decomposed ith modal response $x_l^{(i)}(t)$ using Eq. (10) from l^{th} degree measured response $x_l(t)$ can be written as

$$x_l^{(i)}(t)=\phi_{li}q_i(t) \tag{12}$$

For an n DOF nonlinear system, ϕ_{li} is slow time-varying function, therefore, the analytical signal $Z_l^{(i)}$ of the ith decomposed response $x_l^{(i)}(t)$ can be further expressed as

$$Z_l^{(i)}=\phi_{li}q_i(t)+H[\phi_{li}q_i(t)]=\phi_{li}(q_i(t)+H[q_i(t)])=\phi_{li}A_i(t)e^{j\int\omega_i(t)dt} \tag{13}$$

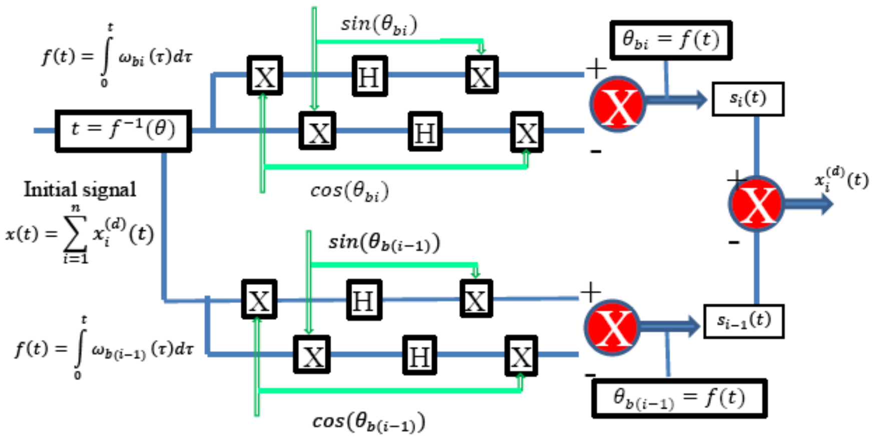

Fig. 1 Block diagram of an adaptive bandpass filter with two bisecting frequencies: $\omega_{b(i-1)}(t)$ and $\omega_{bi}(t)$

Eq. (13) clearly demonstrates that the instantaneous frequency of the decomposed signal is equal to the instantaneous frequency of the response $q_i(t)$.

Again, the instantaneous frequency of the decomposed response includes a slowly-varying part and a rapidly-varying part. The rapidly-varying part can be filtered out by AMD with a suitable bisecting frequency. Another way to eliminate the rapidly-varying part is to integrate the instantaneous frequency over time duration. Therefore, the phase of the decomposed response can describe the structural nonlinearity during the vibration. The nonlinearity index E can be defined as

$$E = \sqrt{\frac{\int \left[\int_0^t \omega_n(\boldsymbol{\tau})\,\mathrm{d}\tau - \omega_l t \right]^2 \mathrm{d}t}{\int \left[\int_0^t \omega_n(\boldsymbol{\tau})\,\mathrm{d}\tau \right]^2 \mathrm{d}t}} \tag{14}$$

In which, $\omega_n(\tau)$ is the time-varying frequency of the modal response of structure performed nonlinear phenomena, while ω_l is the constant natural frequency of the structure performed linear behavior. The nonlinearity index E also represents the damage severity of the structure during vibration, therefore, E is also called as a damage index.

3. Numerical simulations

A one-story shear building with hysteretic behavior is considered as a numerical example. It has mass of *m*=100 kg, elastic stiffness of *k*=12.5 kN/m, and damping coefficient *c*=299.3 N-sec/m. The hysteretic behavior of the building is represented by a Bouc-Wen model (Wen 1976). The equation of motion of the shear building can be described as

$$m\ddot{x} + c\dot{x} + \alpha kx + (1-\alpha)kz = f \tag{15}$$

$$\dot{z} = \frac{A\dot{x} - v(\beta\dot{x}|z|^n - \gamma|\dot{x}||z|^{n-1}z)}{\eta} \tag{16}$$

in which α is the rigidity ratio, A, β, γ, n are the hysteresis shape parameters (if $n = \infty$, the elastoplastic hysteresis case is obtained). In this paper, a concrete structural system is considered and the corresponding hysteresis shape parameters are selected based on the research done by Kunnath *et al*. (1997). The hysteresis shape parameters are set as: $\alpha = 0.1$, $A = 1$, $\beta = 0.05$, $\gamma = 0.95$, $n = 2$. v and η are degrading parameters, which can be described as [35]

$$v = 1.0 + s_v \varepsilon \tag{17}$$

$$\eta = 1.0 + s_\eta \varepsilon \tag{18}$$

$$\varepsilon = (1-\alpha)kz\dot{x} \tag{19}$$

in which, S_v controls the amount of strength deterioration, and S_η controls the rate of stiffness decay. In this paper, S_v and S_η are set to 0.025 and 0.25, respectively.

To verify the effective of the proposed method, four earthquake excitations including a low intensity earthquake EQ1, two medium intensity earthquakes EQ2 and EQ3, and a large intensity earthquake EQ4, are used to generate the responses of the nonlinear system by using the fourth order Runge-Kutta method. The low intensity earthquake record EQ1 was the VNUY longitudinal component from the 1971 San Fernando earthquake. The two medium intensity records EQ2 and EQ3 were the VNUY transverse component record from 1971 San Fernando earthquake and the WHOX longitudinal component from the Northridge 1994 earthquake. The large intensity record EQ4 is the Sylmar Olive View Med 360o component record from the 1994 Northridge earthquake. The ground time histories of the four earthquakes with sampling frequency of 240 Hz are presented in Fig. 2. The acceleration response spectra of the input ground motions as well as the design spectrum for 5% damping are presented in Fig. 3. The Bouc-Wen nonlinear force-displacement hysteretic loops with various excitations are presented in Fig. 4.

To estimate the nonlinearity indices, the generated accelerations with 5% noise-signal-ratio white noise are assumed as measured responses. The modal response of a generated acceleration can be extracted by using the proposed AMD method with bisecting frequency of 1.8 Hz. The instantaneous frequency of the modal response can then be estimated from the defined analytical signal. The instantaneous frequencies of the nonlinear system subjected the above four earthquakes are presented in Fig. 5. The corresponding nonlinearity indices are presented in Table 1. As one can see from Fig. 5, the instantaneous frequency of the measured acceleration indeed includes a slowly-varying part and a rapidly-varying part for the nonlinear structure subjected to an earthquake excitation. The nonlinearity indices for the building subjected to a low intensity earthquake excitation, two medium intensity earthquake excitations, and a large intensity earthquake excitation are equal to 10.2%, 21.3%, 22.9%, and 42%, respectively. The structure subjected to low, medium, and large intensity earthquake excitations represents it is minor, medium, and severe damaged, therefore, the nonlinearity index also represents the damage severity of the structure during vibration.

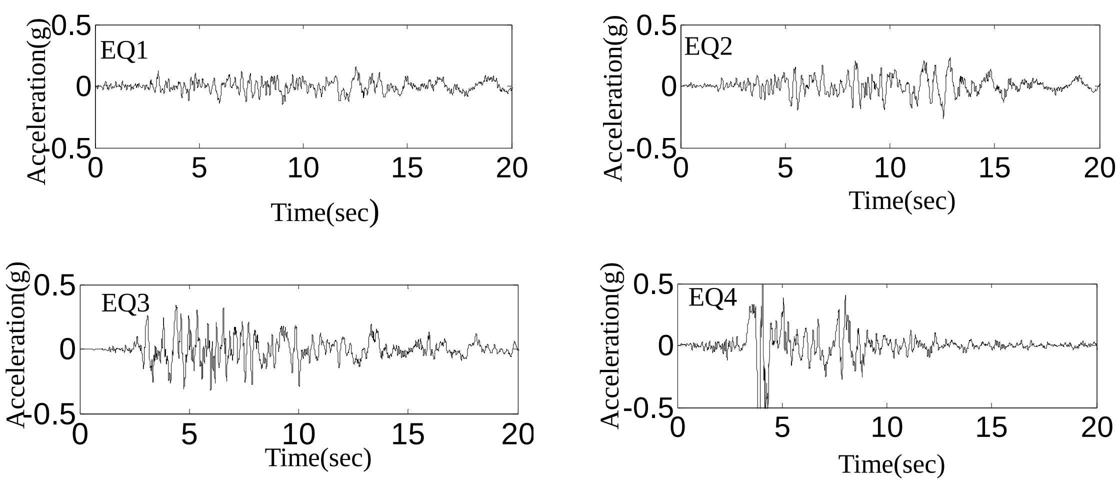

Fig. 2 ories of the ground motions

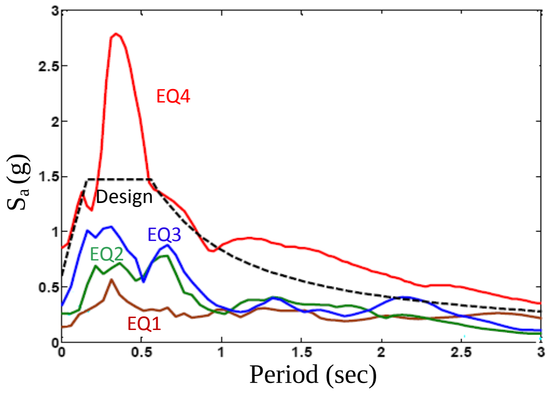

Fig. 3 Acceleration response spectra of the ground motions

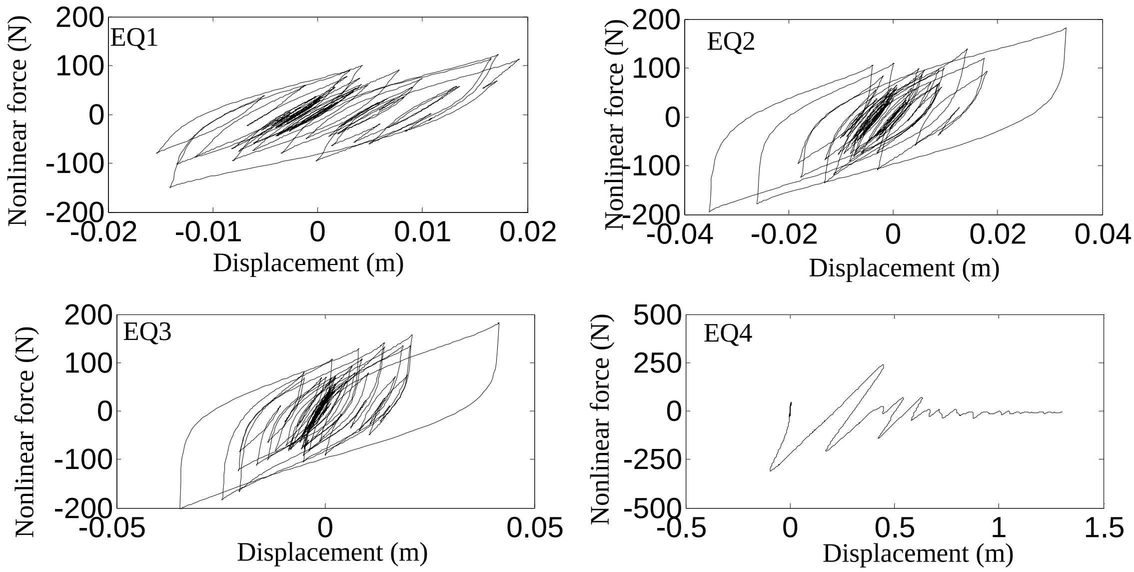

Fig. 4 Bouc-Wen nonlinear force-displacement hysteretic loops with various earthquake excitations

Table 1 Nonlinearity index of the system subjected to various earthquake excitations

Excitation	EQ1	EQ2	EQ3	EQ4
Nonlinearity index *E* (%)	10.2	21.3	22.9	42.0

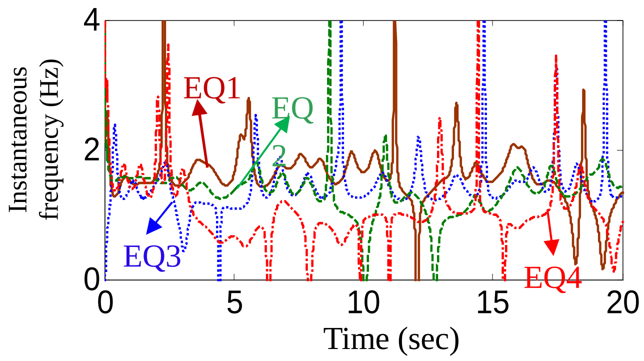

Fig. 5 Instantaneous frequencies of the measured accelerations generated with various earthquake excitations

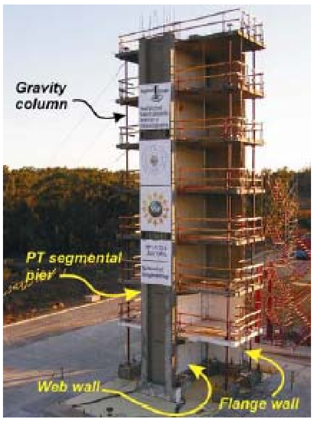

Fig. 6 Test specimen

4. Shake table test validation

To validate the proposed method for structural nonlinearity quantification, a full-scale 7-story reinforced concrete residential building is considered. The shake table test was conducted by Panagiotou *et al*. (2011) and the specimen is illustrated in Fig. 6. The same earthquake excitations including a low intensity earthquake EQ1, two medium intensity earthquakes EQ2 and EQ3, and a large intensity earthquake EQ4 were considered. More details of the shake table test can be found in Panagiotou *et al*. (2011). In this paper, only the acceleration of the top floor is used to estimate the instantaneous frequency and the nonlinearity index. The acceleration responses and their Fourier spectra of the top floor with various earthquake excitations are presented in Figs. 7 and 8, respectively. The instantaneous frequencies of the measured responses are further obtained based on the proposed method and presented in Fig. 9. Again, the instantaneous frequency of the measured acceleration indeed includes a slowly-varying part and a rapidly-varying part for the nonlinear structure subjected to an earthquake excitation. The corresponding nonlinearity indices are presented in Table 2. From the observation, the building subjected to low, medium, and large intensity earthquake excitations represents it is minor, medium, and severe damaged. It can be seen from Table 2, the nonlinearity indices for the building subjected to a low intensity earthquake excitation, two medium intensity earthquake excitations, and a large intensity earthquake excitation are equal to 12.8%, 23.0%, 23.2%, and 39.5%, respectively.

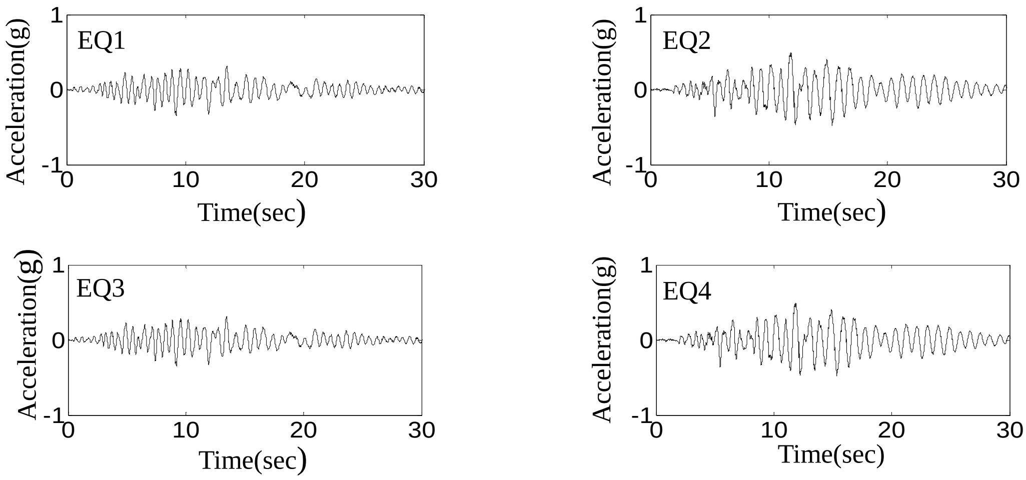

Fig. 7 Acceleration responses on the top floor with various earthquake excitations

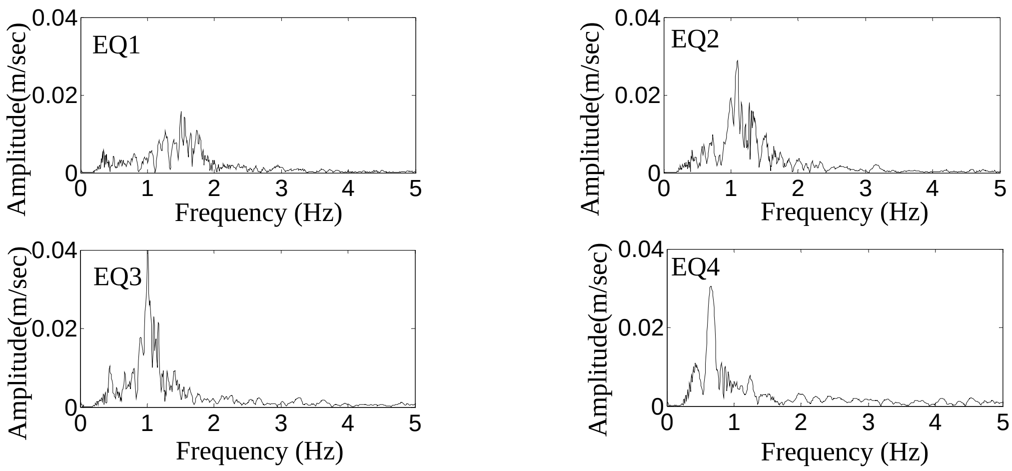

Fig. 8 Fourier spectra of acceleration responses on the top floor with various earthquake excitations

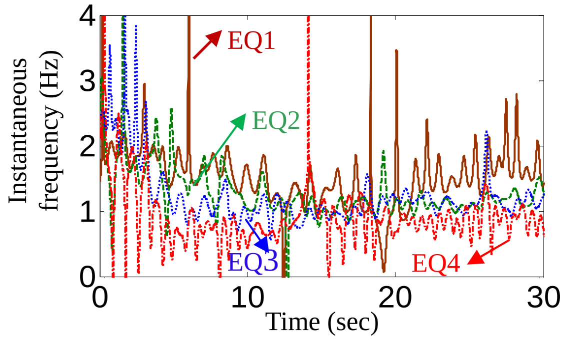

Fig. 9 Instantaneous frequencies of the measured accelerations with various earthquake excitations

Table 2 Nonlinearity index of the building subjected to various earthquake excitations

Excitation	EQ1	EQ2	EQ3	EQ4
Nonlinearity index E (%)	12.8	23.0	23.2	39.5

5. Conclusions

This paper proposes an analytical mode decomposition (AMD) and Hilbert transform method for structural nonlinearity quantification and damage detection under earthquake loads. The instantaneous frequency of the measured response is extracted by using the proposed method. Then the degree of nonlinearity index, which represents the damage severity of structure, is defined based on the integrated instantaneous frequency. A one-story shear building with hysteretic behaviour subjected to low, medium, and larger intensity earthquake excitations is simulated as a numerical example. Final, the proposed method is validated by the shake table test data of a seven-story building subjected to a low, two medium, and a large intensity earthquake excitations. From both numerical simulations and shake table test validations, the nonlinearity indices are approximately equal to 12%, 23%, and 40% when a concrete building is minor, medium, and severe damaged.

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