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Multi-dimensional sensor placement optimization for Canton Tower focusing on application demands

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Abstract. Optimal sensor placement (OSP) technique plays a key role in the structural health monitoring (SHM) of large-scale structures. According to the mathematical background and implicit assumptions made in the triaxial effective independence (EfI) method, this paper presents a novel multi-dimensional OSP method for the Canton Tower focusing on application demands. In contrast to existing methods, the presented method renders the corresponding target mode shape partitions as linearly independent as possible and, at the same time, maintains the stability of the modal matrix in the iteration process. The modal assurance criterion (MAC), determinant of the Fisher Information Matrix (FIM) and condition number of the FIM have been taken as the optimal criteria, respectively, to demonstrate the feasibility and effectiveness of the proposed method. Numerical investigations suggest that the proposed method outperforms the original EfI method in all instances as expected, which is looked forward to be even more pronounced should it be used for other multi-dimensional optimization problems.

Keywords: optimal sensor placement; effective independence method; Canton Tower; sensitivity; robustness

1. Introduction

Large and complex civil infrastructures are being placed in new and extreme conditions for extended periods of time. The Canton Tower located in Guangzhou, China, assures a place among the supertall structures worldwide by virtue of its total height of 610 m (Ni *et al*. 2009). It consists of a 454 m high main tower and a 156 m high antenna mast. The main tower is a tube-in-tube structure consisting of a steel lattice outer structure and a reinforced concrete inner structure. The outer structure has a hyperboloid form, which is generated by the rotation of two ellipses, one at the ground level and the other at an imaginary horizontal plan 454 m above the ground. The tightening caused by the rotation between the two ellipses forms the characterizing “waist-line” of the tower. The antenna mast is made of a steel structure founded on the top of the main tower, the lower part of which is a steel lattice structure with an octagon cross-section and the upper part is a steel box structure. The hyperbolic shape makes the Canton Tower interesting and attractive from

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an aesthetics perspective, and it also makes it mechanically complex. Civil engineers require a reliable method to resolve their concerns about two fronts: first in the safe operation and maintenance of the project to ensure a long service life and second in insuring safety and efficiency of modern design practice (Housner *et al*. 1997). Both of these intentions can benefit from on-line monitoring of the structure, whether it is at the global level, using accelerometers, or at a local level using strain sensing elements.

An effective health monitoring system needs sensors to be located appropriately on the structure since the characterization of the dynamic behavior of a structure is possible only if a minimum amount of information is available. This, in turn, implies that designing efficient sensor networks is necessary for economical and efficient issues (Yi *et al*. 2011). Normally, the sensor placement may base on engineering judgment or trial-and-error approach. However, for a large-scale complicated structure like the Canton Tower, a systematic and efficient approach is needed to solve the problem. The problem of sensor placement can be investigated from several approaches, as can be seen from the abundance of literature. Among them, the Effective Independence (EfI) method is one of the most influential and commonly cited sensor placement approaches (Kammer *et al*. 1991, 1994). In this method, Kammer argued that the optimal arrangement for measuring and estimating structural vibration was that which minimized the norm (usually either trace or determinant) of the Fisher information matrix (FIM), which was constructed from the modal and measurement covariance matrices. There were several derivative methods based on the EfI method. The so-called energy optimization technique was derived from modal kinetic energy (MKE) and EfI by optimizing the kinetic energy matrix measured by candidate sensor locations (Heo *et al*. 1997). In one study (Coote *et al*. 2005), it was shown that the energy optimization technique appeared more favorable because the EfI resulted in clustering of sensors and did not reduce the off-diagonal Modal Assurance Criteria (MAC) terms particularly well. Other derivative methods were adding weights (residue weighted or mass weighted) for different mode shapes in order to compromise the EfI and other methods. For example, the EfI-DPR was a compromise between the EfI and an energetic approach, in which the modes in EfI were weighted by the corresponding driving-point residues (Meo and Zumpano 2005). Similar to the EfI, a SVD-based method directly decomposed the mass weighted information matrix (Park and Kim 1996). It complemented the EfI by providing a guide for an allowable number of degrees to be deleted at each iteration stage, which rendered the selection computation faster. Recently, Li *et al*. (2007) had discovered that the EfI was an iterated version of MKE with re-orthonormalized mode shapes though the QR decomposition and that the latter was an iterated version of the former for the case of a structure with equivalent identity mass matrix. With the aid of this connection, the EfI could be easily computed through the row norm of the orthonormal Q matrix (Li *et al*. 2009).

The state-of-the-practice is to select individual sensor locations/directions from a candidate set based upon one of several available criteria, which may results in the nonoptimal placement of triaxial sensors. Considering that, Kammer (Kammer and Tinker 2004, Kammer 2005) extended the EfI method and proposed a new technique, called the triaxial EfI which placed triaxial sensors as a single unit in an optimal fashion. Experimental results verified that triaxial sensor configurations produced by the triaxial EfI technique consistently produced larger FIM determinants than configurations generated by expanding uniaxial EfI results.

Generally, the underlying idea of the conventional optimization formulation is to identify a sensor layout that could maximize some performance measure (optimal criteria), such as target mode signal strength or linear independence. However, this kind of sensor placement strategy doesn’t consider the ill-posed of modal matrix which may cause the solving infeasible. In this paper, a new strategy is presented based upon the triaxial EfI for the design of a multi-dimensional sensor system that renders the corresponding target mode shape partitions as linearly independent as possible and, at the same time, maintains the stability of the modal matrix in the iteration process. The remainder of this paper is organized as follows. Section 2 deliveres the main feature and detailed implementation steps of the proposed method. Section 3 gives the optimization criteria used for sensors locations. Section 4 outlines the computing model of the Canton Tower.

Section 5 shows the comprehensive evaluation of this novel method for the OSP in the Canton Tower. Finally, some conclusions are drawn in Section 6.

2. Multi-dimensional sensor placement method

Ambient excitations not only induce a vibration response of a structure in both the longitudinal and transversal directions, but also produce torsional effects which are more complex to be detected during full-scale experimental tests. These torsional effects are mainly due to the fact that the center of mass does not coincide with the stiffness center of the building. Thus, to detect the environmental-induced effects, the sensors placement on the structure must be designed in order to identify a preferable topology (i.e., combination and locations of the sensors) able to provide information on the horizontal and torsional response. In general, candidate nodes have six associated degrees of freedom (DOF), three translations and three rotations. However, the vertical displacement is much less than the two horizontal displacements for the high-rise structures, and thus the DOF of vertical translation is disregarded for each node in the computing model. This paper considers this case and the goal of the work presented here is to reformulate the EfI such that candidate sensor directions can be deleted by node and the stability of the modal matrix in the iteration process can be maintained.

The mathematical background and implicit assumptions made in the triaxial EfI method are adopted and extended here (Kammer and Tinker 2004, Kammer 2005). Instead of a candidate set of sensor locations, a candidate set of nodes with triaxes is chosen. Thus the FIM can be decomposed into the contributions from each candidate set of nodes with 5 DOFs in the form

$$Q = \Phi_5^T \Phi_5 = \sum_{i=1}^{n_n} \phi_{5i}^T \phi_{5i} = \sum_{i=1}^{n_n} Q_{5i} \tag{1}$$

where, Q is termed as the FIM; Φ_5 stands for the modal matrix, i.e., $\Phi_5 = [\Phi_x, \Phi_y, \Phi_{yz}, \Phi_{xz}, \Phi_{xy}]$, Φ_x and Φ_y are two translations, Φ_{yz}, Φ_{xz}, and Φ_{xy} represent three rotations in yz, xz and xy, respectively ; ϕ_{5i} means the target modal matrix partitioned to the five rows corresponding to the ith node; n_n denotes the number of candidate nodes.

The new FIM with the ith node deleted can be written as

$$Q^{5i} = Q - \phi_{5i}^T \phi_{5i} = Q[I_5 - Q^{-1} \phi_{5i}^T \phi_{5i}] \tag{2}$$

where, I_5 is a five dimensional identity matrix.

The determinant of the FIM with the ith node removed can be written as

$$\det\left(Q^{5i}\right) = \det\left(Q\right)\det\left(I_5 - \phi_{5i} Q^{-1} \phi_{5i}^T\right) = \det\left(Q\right)\det\left(I_5 - E_{5i}\right) \tag{3}$$

in which

$$E_{5i} = \phi_{5i} Q^{-1} \phi_{5i}^{T} = \begin{bmatrix} \phi_{5i1}Q^{-1}\phi_{5i1}^{T} & \phi_{5i1}Q^{-1}\phi_{5i2} & \phi_{5i1}Q^{-1}\phi_{5i3}^{T} & \phi_{5i1}Q^{-1}\phi_{5i4}^{T} & \phi_{5i1}Q^{-1}\phi_{5i5}^{T} \\ \phi_{5i2}Q^{-1}\phi_{5i1}^{T} & \phi_{5i2}Q^{-1}\phi_{5i2}^{T} & \phi_{5i2}Q^{-1}\phi_{5i3}^{T} & \phi_{5i2}Q^{-1}\phi_{5i4}^{T} & \phi_{5i2}Q^{-1}\phi_{5i5}^{T} \\ \phi_{5i3}Q^{-1}\phi_{5i1}^{T} & \phi_{5i3}Q^{-1}\phi_{5i2}^{T} & \phi_{5i3}Q^{-1}\phi_{5i3}^{T} & \phi_{5i3}Q^{-1}\phi_{5i4}^{T} & \phi_{5i3}Q^{-1}\phi_{5i5}^{T} \\ \phi_{5i4}Q^{-1}\phi_{5i1}^{T} & \phi_{5i4}Q^{-1}\phi_{5i2}^{T} & \phi_{5i4}Q^{-1}\phi_{5i3}^{T} & \phi_{5i4}Q^{-1}\phi_{5i4}^{T} & \phi_{5i4}Q^{-1}\phi_{5i5}^{T} \\ \phi_{5i5}Q^{-1}\phi_{5i1}^{T} & \phi_{5i5}Q^{-1}\phi_{5i2}^{T} & \phi_{5i5}Q^{-1}\phi_{5i3}^{T} & \phi_{5i5}Q^{-1}\phi_{5i4}^{T} & \phi_{5i5}Q^{-1}\phi_{5i5}^{T} \end{bmatrix} \tag{4}$$

where, E_{5i} is a 5×5 fully populated matrix containing the EfI values of the individual sensors corresponding to the ith node on the diagonal; ϕ_{5ir} means the rth row from the target mode partition corresponding to the ith node.

Note that the determinant of Q^{5i} reduces to zero and the target modes are no longer independent when E_{5i} possesses an eigenvalue is equal to 1. This indicates that the ith node is vital to the independence of the target modes. Thereby, at least one of the five DOFs associated with the ith node must have an EfI value of 1.0. Without loss of generality, let the first DOF from the ith node be critical to the independence of the target modes. The (1,1) term in E_{5i} gives the corresponding EfI value as follows

$$\phi_{5i1} Q^{-1} \phi_{5i1}^{T} = 1 \tag{5}$$

Premultiplying both sides of Eq. (5) by ϕ_{5i1}^{T} produces

$$\phi_{5i1}^{T} \phi_{5i1} Q^{-1} \phi_{5i1}^{T} = P_{i1} \phi_{5i1}^{T} = \phi_{5i1}^{T} \tag{6}$$

In which

$$P_{i1} = \phi_{5i1}^{T} \phi_{5i1} Q^{-1} \tag{7}$$

$$Q = \Phi_5^{T} \Phi_5 = \sum_{i=1}^{n_n} \sum_{r=1}^{5} Q_{5ir} = \sum_{i=1}^{n_n} \sum_{r=1}^{5} \phi_{5ir}^{T} \phi_{5ir} \tag{8}$$

$$Q_{5ir} = \phi_{5ir}^{T} \phi_{5ir} \tag{9}$$

Premultiplying Q by P_{i1} produces

$$P_{i1} Q = \phi_{5i1}^{T} \phi_{5i1} Q^{-1} Q = \phi_{5i1}^{T} \phi_{5i1} = Q_{5i1} \tag{10}$$

In Eq. (10), the $P_{i1}Q$ can be expanded as follows

$$P_{i1} Q = P_{i1} \sum_{i=1}^{n_n} \sum_{r=1}^{5} Q_{5ir} = P_{i1} Q_{5i1} + P_{i1} Q_{5i2} + P_{i1} Q_{5i3} + P_{i1} Q_{5i4} + P_{i1} Q_{5i5} + \sum_{j=1 j \neq i}^{n_n} \sum_{r=1}^{5} P_{i1} Q_{5jr} \tag{11}$$

In which

$$P_{i1} Q_{5i1} = \phi_{5i1}^{T} \phi_{5i1} Q^{-1} \phi_{5i1}^{T} \phi_{5i1} \tag{12}$$

The Eq. (12) can be simplified by Eq. (5) as follows

$$P_{i1}Q_{5i1} = \phi_{5i1}^T\phi_{5i1} = Q_{5i1} \tag{13}$$

Thus, from Eqs. (13) and (11) can be further transformed into the form

$$P_{i1}Q = \sum_{i=1}^{n_n}\sum_{r=1}^{5} Q_{5ir} = Q_{5i1} + P_{i1}Q_{5i2} + P_{i1}Q_{5i3} + P_{i1}Q_{5i4} + P_{i1}Q_{5i5} + \sum_{j=1 j\neq i}^{n_n}\sum_{r=1}^{5} P_{i1}Q_{5jr} = Q_{5i1} \tag{14}$$

i.e.

$$P_{i1}Q_{5i2} + P_{i1}Q_{5i3} + P_{i1}Q_{5i4} + P_{i1}Q_{5i5} + \sum_{j=1 j\neq i}^{n_n}\sum_{r=1}^{5} P_{i1}Q_{5jr} = 0 \tag{15}$$

According to the matrix theory, matrices P_{i1} and Q_{5jr} are positively semidefinite, therefore, their product is also positively semidefinite. The individual terms in the sum of Eq. (13) must then vanish. Specifically

$$P_{i1}Q_{5ir} = 0 \qquad r = 2,\ 3,\ 4 \tag{16}$$

The Eq. (16) can be further transformed into the form

$$P_{i1}\phi_{5i}^T\phi_{5ir} = \phi_{5i}^T\phi_{5i1}Q^{-1}\phi_{5i1}^T\phi_{5ir} = \phi_{5i}^T\phi_{5i1}a_r = 0 \qquad r = 2,3,4,5 \tag{17}$$

In which

$$a_r = \phi_{si1}Q^{-1}\phi_{sir}^T \qquad r = 2,\ 3,\ 4 \tag{18}$$

From Eq. (18), it can be found that a_r is a constant. To ensure Eq. (17) established, the coefficients of the matrix should always be 0, thus $a_r = 0$.

The form of E_{5i} reduces to

$$E_{5i} = \phi_{5i}Q^{-1}\phi_{5i}^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \phi_{si2}Q^{-1}\phi_{si2}^T & \phi_{si2}Q^{-1}\phi_{si3}^T & \phi_{si2}Q^{-1}\phi_{si4}^T & \phi_{si2}Q^{-1}\phi_{si5}^T \\ 0 & \phi_{si3}Q^{-1}\phi_{si2}^T & \phi_{si3}Q^{-1}\phi_{si3}^T & \phi_{si3}Q^{-1}\phi_{si4}^T & \phi_{si3}Q^{-1}\phi_{si5}^T \\ 0 & \phi_{si4}Q^{-1}\phi_{si2}^T & \phi_{si4}Q^{-1}\phi_{si3}^T & \phi_{si4}Q^{-1}\phi_{si4}^T & \phi_{si4}Q^{-1}\phi_{si5}^T \\ 0 & \phi_{si5}Q^{-1}\phi_{si2}^T & \phi_{si5}Q^{-1}\phi_{si3}^T & \phi_{si5}Q^{-1}\phi_{si4}^T & \phi_{si5}Q^{-1}\phi_{si5}^T \end{bmatrix} \tag{19}$$

which obviously has an eigenvalue of 1.0.

Therefore, if one of the sensor directions associated with the *i*th node is vital to the independence of the target modes, E_{5i} has an eigenvalue of 1.0 and the determinant of I_5-E_{5i} becomes zero. The FIM Q is positively definite, while Q^{5i} is at least positively semidefinite. The expression $I_k - Q^{-1}\phi_{5i}^T\phi_{5i}$ must then be at least positively semidefinite. The eigenvalues of $I_k - Q^{-1}\phi_{5i}^T\phi_{5i}$ can be expressed in the form

$$\lambda\left(I_k - Q^{-1}\phi_{5i}^T\phi_{5i}\right) = 1 - \lambda\left(Q^{-1}\phi_{5i}^T\phi_{5i}\right) \tag{20}$$

The sign definiteness of $I_k - Q^{-1}\phi_{5i}^T\phi_{5i}$ then requires that $\lambda\left(Q^{-1}\phi_{5i}^T\phi_{5i}\right) \le 1$. Based on its form, $Q^{-1}\phi_{5i}^T\phi_{5i}$ is also positively semidefinite. Therefore, the eigenvalues of $Q^{-1}\phi_{5i}^T\phi_{5i}$ must satisfy the inequality

$$0 \le \lambda\left(Q^{-1}\phi_{5i}^T\phi_{5i}\right) \le 1 \tag{21}$$

which then implies that

$$0 \le \lambda\left(I_k - Q^{-1}\phi_{5i}^T\phi_{5i}\right) \le 1 \tag{22}$$

Using this result, Eq. (3) indicates that the range on the determinant of $I_5 - E_{5i}$ is given by

$$0 \le \det\left(I_5 - E_{5i}\right) \le 1 \tag{23}$$

The five dimensional EfI measure (i.e., $\mathrm{EfI5}_i$) is then given by the expression

$$\mathrm{EfI5}_i = 1 - \det(I_5 - E_{5i}) \qquad 0 \le \mathrm{EfI5}_i \le 1 \tag{24}$$

As known, the sensitivity of the structural responses means the sensitivity extent of the structural responses by the performance measure which reflects certain performance of the structures. The robustness of the system is the characteristic of maintaining certain performance under certain perturbations. In the optimal sensor placement, this “robustness” shows the stability of the performance measure in the search process of most advantageous sensor measuring points. Generally, the underlying idea of the conventional OSP is to identify a sensor layout that could maximize some performance measure, such as the effective independence coefficients in the EfI. However, with the ability to detect and discriminate relevant data features increase, the system robustness may be decreased. That is to say, the objectives of structural responses’ sensitivity and system robustness have certain contradiction. Thus, to establish a judicious method for sensor placement, the structural response sensitivity and system robustness should be comprehensively considered. Unfortunately, the above derivation doesn’t incorporate any system robustness for ill-posed of modal matrix which may cause the solution infeasible.

Sensor placement belongs to a kind of inverse problems in engineering. In these one often has to solve operator equations of the first kind, which are usually ill-posed. It means that the hardest issue in the numerical computation of inverse problems is the instability of the solution with respect to the noise from the observation data; that is, small perturbations of the observation data may lead to large changes on the considered solution. Thus to ensure a feasible and stable numerical approximation solution, it is necessary adopting some appropriate strategies that can keep the system robust.

The operator equations of the first kind can be expressed as

$$Ax = y \qquad x \in F, \quad y \in D \tag{25}$$

where, A is the integral operators, or differential operator, or matrix; F denotes the parameter space and D means the observation space.

The vector of the measured structural responses denoted by y_s can be estimated as a combination of N mode shapes through the following expression

$$y_s = \Phi q + \omega = \sum_{i=1}^{N} q_i \phi_i + \omega \tag{26}$$

where, Φ is the matrix of FE model target mode shapes; q denotes the coefficient response vector; ω represents the sensor noise vector, which could be assumed stationary Gaussian white noise variance σ^2; N means the column number of Φ (n by N matrix, n being the number of the candidate sensor positions) and ϕ_i stands for the ith column of Φ that is the ith target mode shape selected.

By comparing the Eqs. (25) and (26), it can be easily found that A is equivalent to Φ. The condition number is a measure of stability or sensitivity of a matrix to numerical operations. Matrices with condition numbers near 1 are said to be well-conditioned. Matrices with condition numbers much greater than one are said to be ill-conditioned. Thus, the quality of system robustness can be measured by the condition number of the FIM corresponding to the target modal partitions. Known by the matrix theory, the condition number of the Φ is larger, the estimation error of the q is larger, accordingly the monitoring results of the located sensors are worse.

The 2-norm condition number of Φ can be expressed as

$$cond_2(\Phi) = \|\Phi\|_2 \|\Phi^{-1}\|_2 = \sqrt{\frac{\lambda_{\max}(\Phi^T \Phi)}{\lambda_{\min}(\Phi^T \Phi)}} \tag{27}$$

where, $\lambda_{\max}$ and $\lambda_{\min}$ are maximal and minimal eigenvalues $\Phi^T \Phi$, respectively.

By getting the eigenvalues of the conjugate matrix of the modal matrix, the corresponding singular values as well as the condition number and the norm of the matrix can be obtained. Thus, the relationship between the 2-norm condition number and 2-norm can be established by the singular value decomposition of the matrix.

The 2-norm of Φ can be expressed as

$$\|\Phi\|_2 = \lambda_{\max}(\Phi^T \Phi) = \sigma_1^2 \tag{28}$$

The singular value decomposition of matrix Φ can be expressed as

$$\Phi = U \Sigma V^T \tag{29}$$

where, $\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}$; $\Sigma_1 = diag(\sigma_1\ \sigma_{2.} ... \sigma_r)$ ($\sigma_1 > \sigma_{2.} > ... \sigma_r$) and $\Sigma_2 = 0$.

The singular value of matrix Φ is

$$\sigma_i = \sqrt{\lambda_i(\Phi^T \Phi)} \tag{30}$$

where, r is the number of nonzero eigenvalues, $i = 1, 2, ... r$.

According to Eqs. (30) and (27) can be written as

$$cond_2(\Phi) = \frac{\sigma_1}{\sigma_r} \tag{31}$$

As well known in matrix analysis theory, different matrix norms are equivalent in the sense that one norm can be always bounded in a range by another norm with appropriate constant scaling factors. The trace, the determinant and the maximum singular value are just different norms of a matrix (Golub and Van 1996). Therefore, E_{5i} matrix and 2-norm of Φ are the same in essence.

It can be remarked from Eqs. (28) and (31) that the 2-norm condition number of Φ may not be able to obtain the maximum value when the 2-norm of Φ gets the maximum value. However, the procedure for selecting the best sensor placements usually required by the condition number is minimized while the FIM is maximized.

Owing to the FIM and condition number equally important for the sensor placement, a new method considering the coordination of sensitivity and robustness is given here.

$$\mathrm{EfI5}_i^{new} = \mathrm{EfI5}_i + \frac{1}{cond_2\left(\Phi_i\right)} \tag{32}$$

where, Φ_i is the modal matrix after ith node deleted in Φ.

Since the order of magnitude for $cond_2\left(\Phi_i\right)$ is about 10^9, $1/cond_2\left(\Phi_i\right)$ is nearly 0 and the range of values for the $\mathrm{EfI5}_i$ of $\mathrm{EfI5}_i^{new}$ is 0 to 1. To better express the coordination between them, the condition number is transferred as follows

$$\gamma_i = \frac{z_i - z^-}{z^+ - z^-} \tag{33}$$

where, z_i is the value of $cond_2\left(\Phi_i\right)$ after the ith deleted; z^+ and z^- are the maximum and minimum, respectively.

From Eq. (33), the value of γ changes from 0 to 1 and has the same order of magnitude with $\mathrm{EfI5}_i$, which means the sensitivity and robustness can be better coordinated. Thus, the Eq. (32) can be written as

$$\mathrm{EfI5}_i^{new} = \mathrm{EfI5}_i + \gamma_i \tag{34}$$

Outline of the proposed method is as follows:

Step (1): Carry out the modal analysis and determine the number of mode shapes needed to be selected;

Step (2): Calculate the FIM Q and $\mathrm{EfI5}_i$ by Eqs. (1) and (24), respectively;

Step (3): Carry out loop calculations in candidates nodes to obtain the $cond_2\left(\Phi_i\right)$ and then calculate the γ_i by Eq. (33);

Step (4): Calculate the $\mathrm{EfI5}_i^{new}$ by Eq. (34) and sort them from largest to smallest, and delete the smallest one;

Step (5): Update the modal matrix, repeat Step (2) to Step (4) until the number of sensors needing to be placed on the building has been reached.

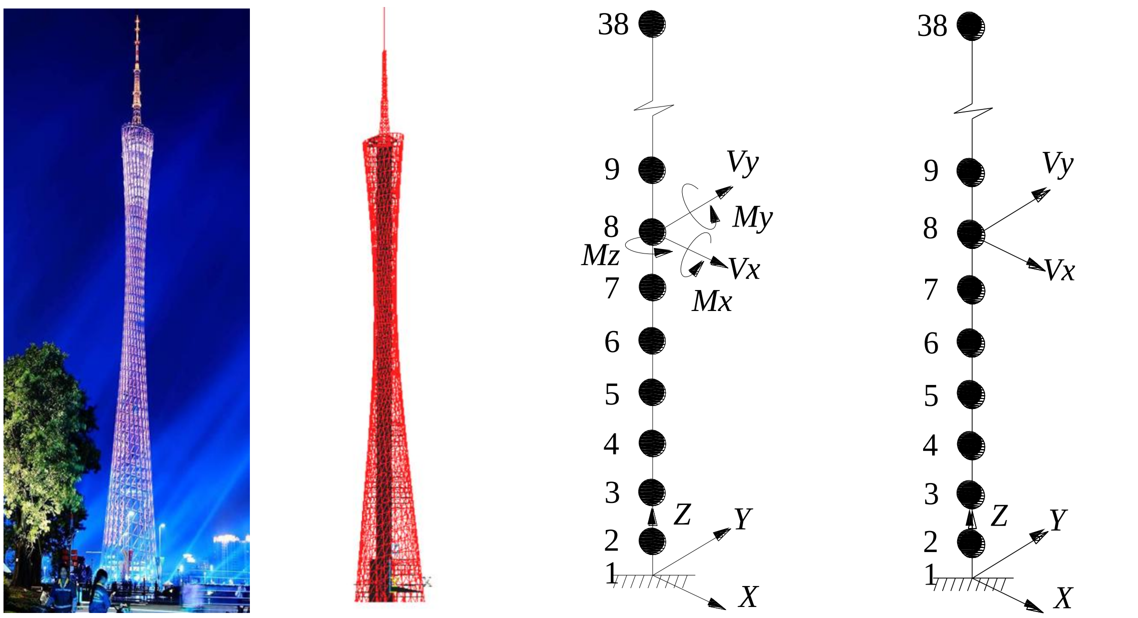
What need to be mentioned is that the proposed method can be easily generalized to the other case, such as two dimensions which mean only the translational DOFs are considered for possible sensor installation.

3. Formulation of computing model

The proposed method is applied and compared with standard approaches on the Canton Tower (Fig. 1(a)) to demonstrate its effectiveness.

3.1. Formulation of five-dimensional FE model

In order to provide input data for the proposed method, a FE model of the tower should be built first. Here, a fine three-dimensional FE model of the Canton Tower constructed of by Lin *et al.* (2010) using the ANSYS software (ANSYS, Inc., Canonsburg, PA, USA) is adopted, as shown in Fig. 1(b). The 3D full-order model contains 122,476 elements, 84,370 nodes, and 505,164 DOFs in total. In the model, the PIPE16 and BEAM44 are employed to model the outer structure, antenna mast, and connection girders between inner and outer structures. Four-node and three-node shell elements with six DOFs at each node are used to model the shear walls of the inner structure and the floor decks. It's clear that the computational run-time will be long to carry out modal analysis for an FE model involving 505,164 DOFs. To facilitate the model analysis and other model-related studies, an equivalent reduced-order FE model is also formulated by Lin *et al.* as depicted in and Fig. 1(c). In the reduced-order model, the whole structure is characterized by 37 beam elements, with 27 elements for the main tower and 10 elements for the antenna mast. Since the vertical displacement is disregarded, as a result, each node has two horizontally translational DOFs and three rotational DOFs, and the reduced-order model has a total of 185 unconstrained DOFs.



(a) Overview (b) Full-scale FE model (c) Five-dimensional FE model (d) Two-dimensional FE model

Fig.1 The Canton Tower and its computing model

3.2. Formulation of two-dimensional FE model

In some cases, only translational DOFs could be considered for possible sensor installation as rotational DOFs may be difficult to measure. Thus, the five-dimensional FE model should be reduced again. A thought of taking the horizontal DOF as the master DOF and rotational DOF as the slave DOF, and reducing the slave DOF by the model reduction is implemented by Yi *et al*. (2011). According to the analytical results in reference [5], the Iterated Improved Reduced System (IIRS) (Friswell *et al*. 1995) method is adopted here due to its faster convergence and higher accuracy. Fig. 1(d) demonstrates the two-dimensional FE model of the Canton Tower.

4. Objective function

It should be noted that an objective function stresses one perspective whereas another pays more attention to another aspect. That means the effectiveness of a certain sensor placement method depends on the evaluation criteria to some extent. Thus, compromises of several objective functions need to be made to verify the effectiveness of the proposed method. In this section, three influential criteria are selected both from historical points of view and from their impacts on practices and the development of sensor placement theory.

Objective function (1): MAC

The first is the biggest value in all the off-diagonal elements in the MAC matrix. The reason for the selection of these fitness functions is that the MAC matrix will be diagonal for an optimal sensor placement strategy so that the size of the off-diagonal elements can be taken as an indication of the fitness.

$$f_1 = \max_{i \neq j} \left\{ \text{MAC}_{ij} \right\} \tag{35}$$

Objective function (2): Determinant value of the FIM

Maximizing FIM would lead to the minimization of the covariance matrix and, thus, the best state estimate the vector of target modal coordinates from the perspective of statistics. According to the Fedorov's study (1972), the determinant value of the FIM for the best linear estimate is largest for all linear unbiased estimators. However, the determinant value is sometimes very small. In order to highlight the effectiveness of the proposed method, the ratio of the determinant value of the FIM is adopted here.

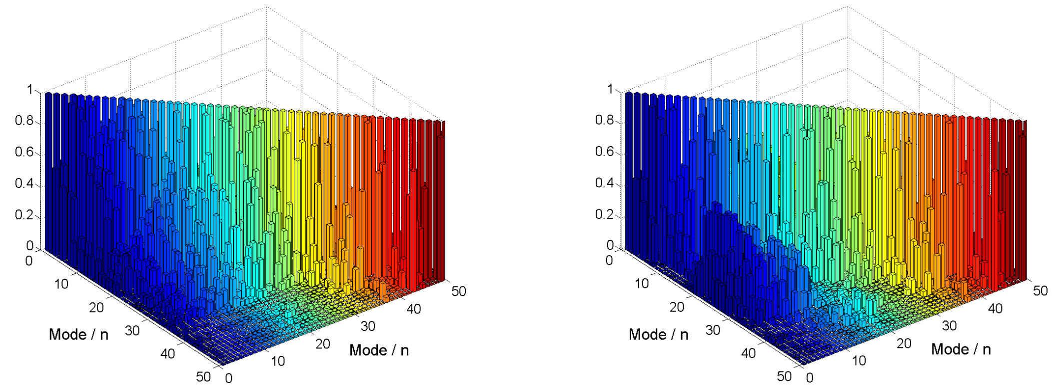
$$f_2 = \frac{Det(\text{FIM})_i}{Det(\text{FIM})_1} \tag{36}$$

where, $Det(\text{FIM})_i$ means the determinant of the FIM after the *i*th node deleted and $Det(\text{FIM})_1$ denotes the determinant of the original FIM.

Objective function (3): Condition number value of the FIM

The quality of system robustness can be measured by the condition number value of the FIM corresponding to the target modal partitions, which directly respects the extent of linear dependence between mode shape vectors. The smaller the condition number is, the better the linear dependence will be.

$$f_3 = \min\{cond(\text{FIM})\} \tag{37}$$



(a) EfI2 (b) EfI2new

Fig. 2 MAC values obtained by EfI2 and EfI2new

5. Comparative studies with different methods

To show the performance improvement achieved by the proposed method, two cases are carried out and their features are compared. Wang *et al*. (2007) analyzed the dynamic characteristics of the Canton Tower by the ANSYS software; he found that at least 10 modes should be considered according to the modal mass participation ratio when the vibration response of the structure was investigated. In order to improve the results of optimal sensor layout, the first 50 modes of the Canton Tower are selected to calculate. Here, it's assumed that the number of sensors needing to be placed on the building is 25 and thus optimal location is the target of this paper. For simplicity, the original method in 2 and 5 dimensional proposed by Kimmer are termed the EfI2 and EfI5, respectively; and the paper proposed method in 2 and 5 dimensional are termed as the EfI2new and EfI5new, respectively.

Case (1): EfI2 and EfI2new are compared

Figs. 2(a) and 2(b) demonstrate the MAC values obtained by the EfI2 and EfI2new, respectively, using the first objective function. A close look at the results presented in Fig. 2 indicates that two methods are little similar from the intuitive. Therefore, in order to highlight the effectiveness of the proposed method, another figure is plotted in each of the modes (Fig. 3). It is evident from Fig. 3 that all of the maximum MAC off-diagonal values obtained by the EfI2new in each of the modes are smaller than EfI2 method, which means the proposed method is superior to the original EfI method in keep the linear independence of the modal vectors.

Fig. 4 compares the ratio of determinant values for each of the truncation analyses over the 12 iterations. In this case the method of EfI2new clearly maintains larger values than the EfI2 method although two curves are close to each other, which implies using the proposed method could results in a sensor configuration possessing a smaller estimate error covariance matrix yielding better state estimates. However, what need points out is that with the iteration increased (especially after 5 iterations), the values become more and more small which reflect the FIM gradually become ill-conditioned. The reason for such phenomenon is that the removal of more and more additional sensors would render the target modal partitions dependent.

Results for the condition number of FIM are presented in Fig. 5. It's clearly that the method of EfI2new could yields a configuration with a much smaller condition number than the corresponding EfI2 configuration, which results in less sensitivity in the estimates to analytical modeling error. Also, the condition number resulting form the EfI2new method seems to be much more stable and predictable than the condition number resulting from sensor truncation based on the EfI2 method that can be easily verified by Fig. 5. As illustrated in Fig. 5, after 10 iterations the condition number obtained by the EfI2 method become dramatically instable while the proposed method can always remain stable. The sensor locations retained in the final set of 25 obtained by the EfI2new method are listed in Table 1.

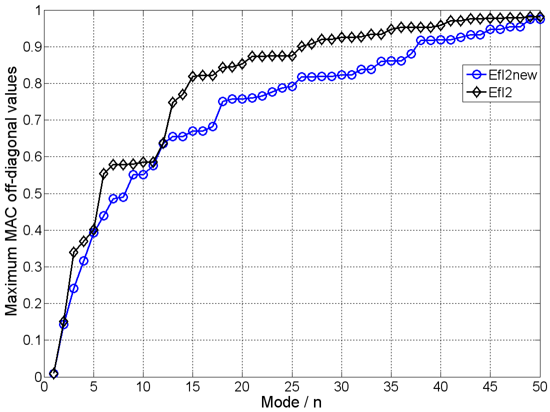


Fig. 3 Maximum MAC off-diagonal value in each of the modes

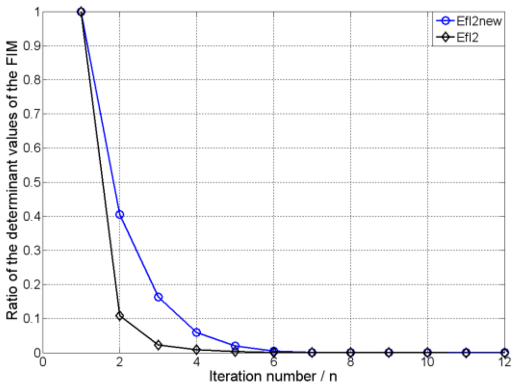


Fig. 4 Variation curves of the ratio of determinant values of the FIM with the iteration increased

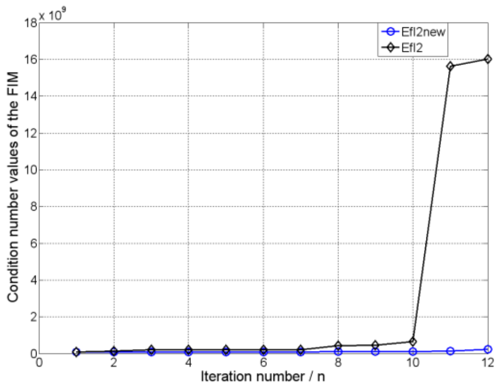
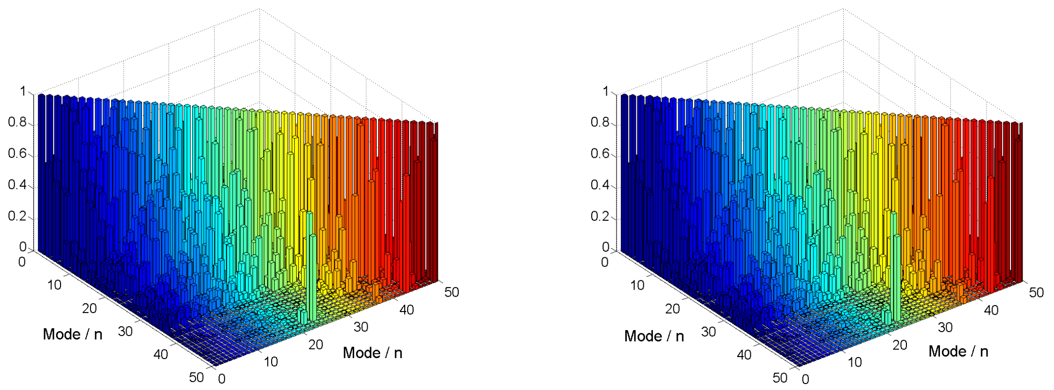


Fig. 5 Variation curves of the condition number values of the FIM with the iteration increased

Table 1 Sensor configuration of the Canton Tower obtained by EfI2new method

Sensor number	1	2	3	4	5	6	7	8	9	10	11	12	13
Node number	1	2	5	7	8	9	10	12	13	14	15	16	17
Sensor number	14	15	16	17	18	19	20	21	22	23	24	25	
Node number	18	19	20	24	26	27	29	30	31	33	35	36	



(a) EfI5 (b) EfI5new

Fig. 6 MAC values obtained by EfI5 and EfI5new

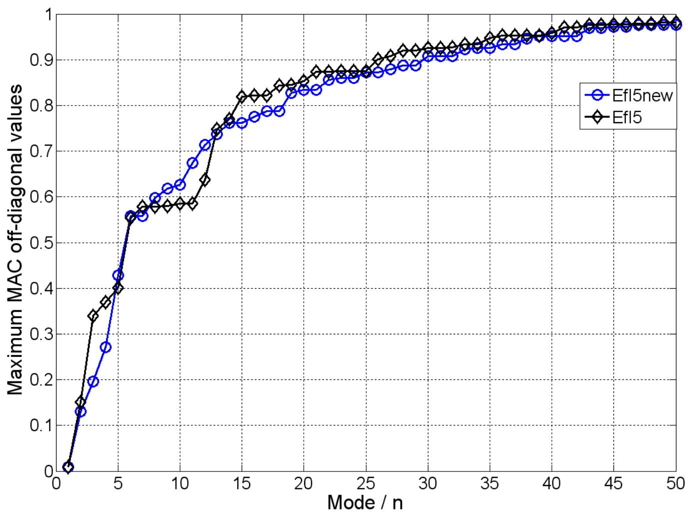


Fig.7 Maximum MAC off-diagonal value in each of the modes

Case (2): EfI5 and EfI5new are compared

Further to demonstrate the effectiveness of the proposed method, the maximum MAC off-diagonal values obtained by the EfI5 and EfI5new methods are compared each other in Fig. 6 and Fig. 7, respectively. Note that the use of EfI5new method yields slightly better results although the values closely approximate the results derived from truncating the sensor locations based on the EfI5 method. The ratio of determinant values is also monitored during the iteration sequence to track the goodness of the selected sensor sets in Fig. 8. In this case, the results derived from EfI5new method come close to duplicating but outperforms these results for the EfI5new method. While compared condition numbers with each other in Fig. 9, the performance of the EfI5new method is found to be of the better performance as expected. Accordingly, the sensor locations retained in the final set of 25 obtained by EfI52new method are listed in Table 2.

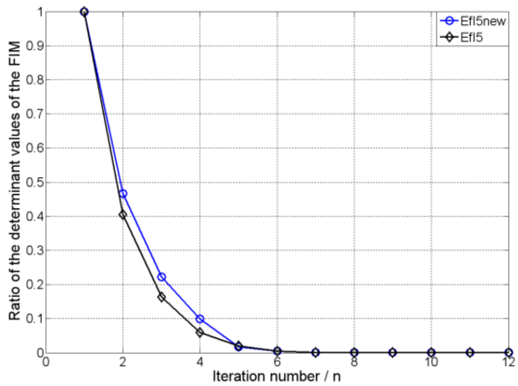


Fig. 8 Variation curves of the ratio of determinant value of the FIM with the iteration increased

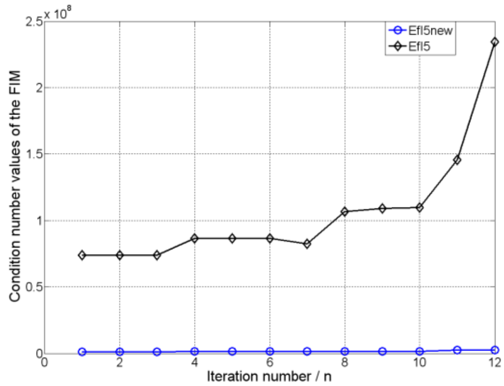


Fig. 9 Variation curves of the condition number values of the FIM with the iteration increased

Table 2 Sensor configuration of the Canton Tower obtained by EfI5new method

Sensor number	1	2	3	4	5	6	7	8	9	10	11	12	13
Node number	1	3	5	7	8	9	10	12	13	14	15	16	17
Sensor number	14	15	16	17	18	19	20	21	22	23	24	25	
Node number	19	20	24	25	27	28	29	31	33	35	36	37	

6. Conclusions

Considering the characteristics of the OSP techniques in the high-rise structures, this paper presents an effective method for the optimal design of SHM system sensor arrays. The state-of-the-practice is to select individual sensor locations that could maximize some performance measure, such as target mode signal strength or linear independence. However, this kind of sensor placement strategy doesn't consider the ill-posed of modal matrix which may cause the solving infeasible. The proposed method based upon the triaxial EfI for the design of a multi-dimensional sensor system that could render the corresponding target mode shape partitions as linearly independent as possible and, at the same time, maintains the stability of the modal matrix in the iteration process. The method for ranking sensor locations presented in this paper was applied to locate sensors for structural health monitoring of the Canton Tower. For demonstration purposes, three influential criteria are selected and two cases are carried out to show the performance improvement achieved by the proposed method. Numerical investigations suggest that the proposed method outperforms the original EfI method in all instances as expected, which is expected to be even more pronounced should it be used for other multi-dimensional optimization problems. In addition, it is important to note that the proposed method in its present form does not determine how many sensors are required to identify the target modes in the presence of sensor noise, high modal density, and sensor failure. The number of sensors required to guarantee identification of the target modes is a subject of further research.

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