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Investigation of masonry elasticity and shear moduli using finite element micro-models

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Abstract. In this paper, a methodology for the estimation of masonry elasticity and shear moduli is presented, for linear elasticity considerations. The methodology is based on the assumption that for a "periodic" masonry wall, which is formed by the repetition of a basic unit containing blocks and mortar, the mechanical characteristics of the unit are representative of the characteristics of the entire wall. For their calculation, the finite element analysis method is used. A micro-model with finite elements simulating separately the blocks and the mortar is developed. An equivalent finite element model, using an homogenous material is also developed and assuming equivalence of strains for the two models, the homogenous material properties are estimated. The efficiency of the method and its applicability limits are investigated.

Keywords: modulus of elasticity; shear modulus; masonry; microanalysis; finite element method.

1. Introduction

For the evaluation of the response of masonry structures, masonry's special characteristics should be taken into consideration. Masonry peculiarities are mainly associated with its non-homogeneous and non-isotropic nature, which is related to its blocks and mortar layout. The lack of homogeneity is additionally caused by the deviation of mechanical properties, within the same structure. Especially in historical masonries, non-industrial construction techniques and poor standardization, result in lack of homogeneity, thus rendering unreliable the assumption of an homogenous material.

The finite element method is a widespread method for the numerical analysis of masonry structures, since it can offer a realistic simulation of their geometry and of their massive weight distribution. According to the desired accuracy of the analysis results, the finite element meshing differs. In most cases, for the evaluation of the response of masonry structures, an homogenous masonry material, that incorporates mechanical properties of both blocks and mortar, is assumed for the finite element discretization. This assumption is the basis of the finite element macroanalysis method. In the case that detailed analysis results are demanded, i.e., when the failure mode and the formation of cracks are investigated, then blocks and mortar can be simulated using different finite elements and each material can be described separately. The latter finite element meshing is used when microanalysis is performed. During the last decades a lot of research has been carried out in order to developed reliable micromodels

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(Antoine 1995, Bosiljkov, *et al.* 1997, Briccoli, *et al.* 1997, Cluni, *et al.* 2004, Pande, *et al.* 1989).

Comparing the two aforementioned methods, macroanalysis offers flexibility for the analysis of large-scale structures. It can be used to provide a valuable indication of the structural response, on which the structural design can be based. Microanalysis, on the other hand, is very time-consuming for the analysis of large-scale structures and it demands considerable computational capacity. However it provides the advantage of taking into account the different mechanical properties of blocks and mortars.

For the macroanalysis, the mechanical properties of the assumed homogenous material have to be defined. This is often made using theoretical or empirical formulae, which fit the available experimental data. Especially for the calculation of the Young's modulus of elasticity, E, and of the shear modulus, G, formulae commonly met in literature, correlate their values with masonry's strength in compression. Eurocode 6, suggests that, in lack of experimental data or in the absence of better information, the short term secant modulus of elasticity, E, under service conditions for all masonries may be taken as $1000 f_k$, where f_k is the characteristic compressive strength of masonry. Additionally, the shear modulus is calculated as $0{,}4E$.

In this paper, a methodology is presented for the calculation of elasticity and shear moduli of masonries, when an homogenous masonry material is assumed. For the calculation, blocks and mortars elasticityand shear moduli, as well as their geometrical properties are considered and the finite element method is used. In contrast with a lot formulae met in literature, for the application of the methodology no material strength values are necessary.

For the demonstration of the methodology, its application on a masonry wall, made of normal shape blocks, is made. Its efficiency is evaluated and its applicability limits are discussed.

2. Characteristics of periodic masonries

The proposed methodology applies to periodic masonries. These masonries are characterized by a basic unit, consisting of blocks and mortar. When the basic unit is repeated, the final shape of the masonry wall is formed. Two indicative types of such basic units, forming two different masonry types,

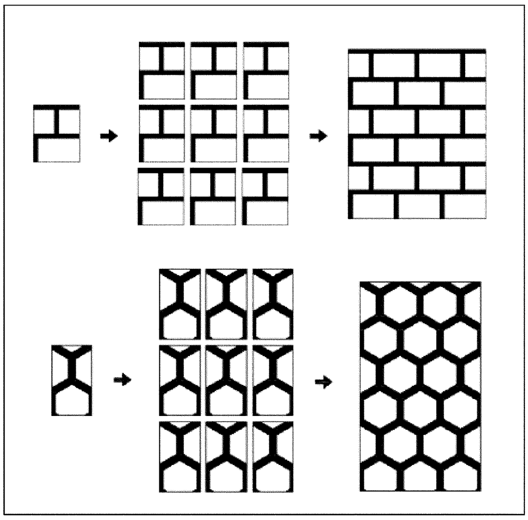


Fig. 1 Different types of masonry basic units

are shown in Fig. 1. Dark shaded areas represent mortar joints and white areas represent blocks.

For the masonry walls that were investigated in this paper, masonry basic units were considered to be identical, presenting no geometrical or mechanical properties deviations, within the same wall. In real structures, this assumption can be made for brickwork or for ashlar masonries of very good construction quality. Under these conditions, the basic unit's properties are representative of the entire wall's properties. Additionally, even in the case of slight variations of this type, across a wall, the basic unit's properties can provide an estimation for the entire wall's mechanical properties. As a result, in this paper, the investigation of masonry elasticity and shear moduli is focused on the basic unit.

3. Theoretical calculation of masonry's modulus of elasticity

For the evaluation of masonry elastic properties, masonry can be considered as a three-dimensional multi-layered material, made of blocks and mortar joints, which are combined so as to form various geometrical shapes. Subsequently, its elastic properties strongly depend on blocks and mortar properties.

In this paper, as an introductory remark, an analytical approach for the evaluation of masonry elasticity and shear moduli is presented. The response of a masonry system, made by two blocks and a mortar joint, under axial stress, is investigated, in order to calculate the elastic properties of an equivalent homogenous system. The elasticity moduli E_1, E_2 and E_3 are calculated for loadings applied in the directions 1, 2 and 3, as shown in Figs. 2, 3 and 4, respectively. For the calculation of the shear moduli G_{12}, G_{23} and G_{13}, the loadings that are shown in Figs. 5, 6 and 7 are considered.

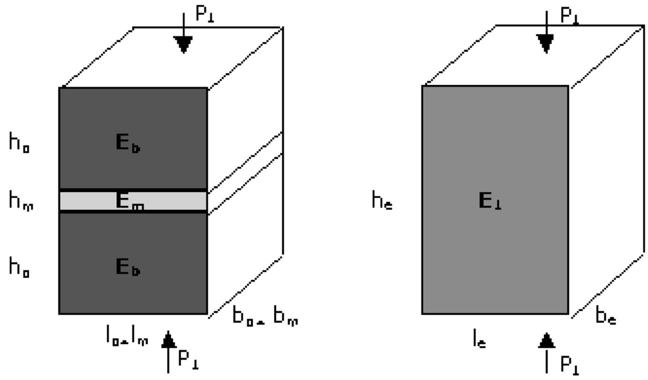


Fig. 2 Applied loading for the calculation of E_1

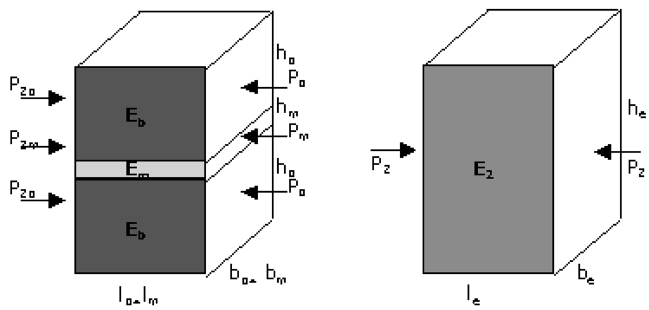


Fig. 3 Applied loading for the calculation of E_2

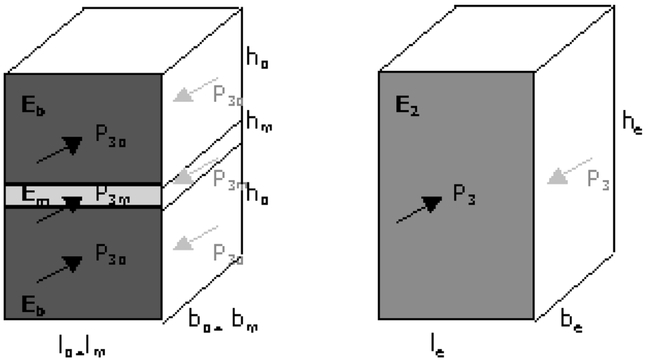


Fig. 4 Applied loading for the calculation of E_3

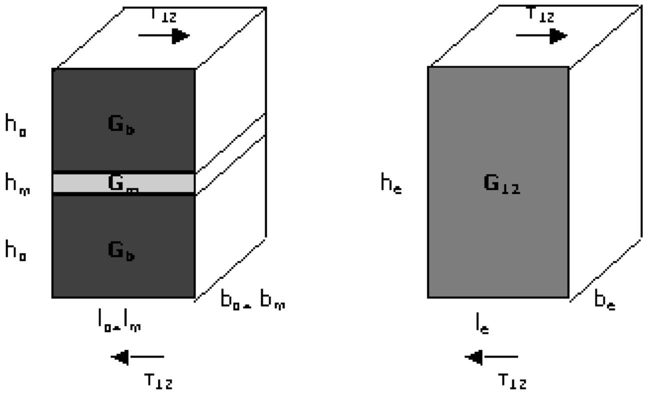


Fig. 5 Applied loading for the calculation of G_{12}

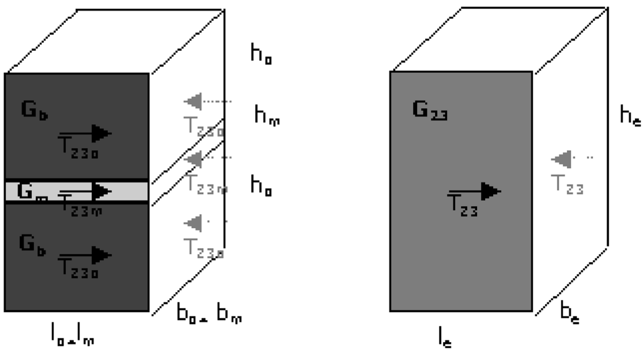


Fig. 6 Applied loading for the calculation of G_{23}

For loading vertical to the bed joint (Fig. 2), the Eqs. (1)-(3) and (4) apply:

$$\Delta h_b = \frac{1}{E_b} \cdot \frac{P}{b_b \cdot l_b} \cdot h_b \tag{1}$$

$$\Delta h_m = \frac{1}{E_m} \cdot \frac{P}{b_m \cdot l_m} \cdot h_m \tag{2}$$

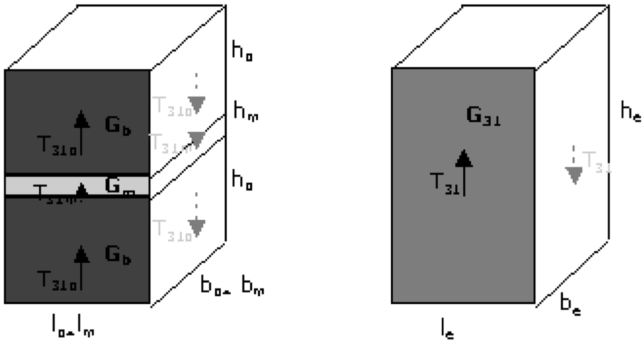


Fig. 7 Applied loading for the calculation of G_{13}

$$\Delta h_e = \frac{1}{E_1} \cdot \frac{P}{b_e \cdot l_e} \cdot h_e \tag{3}$$

Equivalence of strains for the two systems of Fig. 2, results in Eq. (4).

$$\Delta h_e = 2\Delta h_b + \Delta h_m \tag{4}$$

And, consequently, the modulus of elasticity, E_1, of the equivalent homogenous system, for loading vertical to the bed joint is given by Eq. (5).

$$\frac{E_1}{E_b} = \frac{\frac{E_m}{E_b} \cdot \left(2 + \frac{h_m}{h_b}\right)}{\frac{h_m}{h_b} + 2 \cdot \frac{E_m}{E_b}} \tag{5}$$

where E_b: block's elasticity modulus; E_m: mortar's elasticity modulus; h_b: block's height and h_m: mortar's height.

Respectively, for loading in parallel with the bed joint (Fig. 3), Eqs. (6)-(8) and (9) apply.

$$\Delta l_b = \frac{1}{E_b} \cdot \frac{P_b}{b_b \cdot h_b} \cdot l_b \tag{6}$$

$$\Delta l_m = \frac{1}{E_m} \cdot \frac{P_m}{b_m \cdot h_m} \cdot l_m \tag{7}$$

$$\Delta l_e = \frac{1}{E_e} \cdot \frac{P}{b_e \cdot h_e} \cdot l_e \tag{8}$$

$$\Delta l_e = \Delta l_b = \Delta l_m \tag{9}$$

Using Eqs. (6)-(8) and (9) and as Eq. (10) applies, the modulus of elasticity of the equivalent homogenous system, for loading in parallel with the bed joint, E_2, is given by Eq. (11).

$$P = 2 \cdot P_b + P_m \tag{10}$$

$$\frac{E_2}{E_b} = \frac{2 + \frac{E_m}{E_b} \cdot \frac{h_m}{h_b}}{2 + \frac{h_m}{h_b}} \tag{11}$$

In the same way, the modulus of elasticity E_3 is calculated for the loading of Fig. 4 by the Eq. (12).

$$\frac{E_2}{E_b} = \frac{2 + \frac{E_m}{E_b} \cdot \frac{h_m}{h_b}}{2 + \frac{h_m}{h_b}} \tag{12}$$

The calculation of G_{12} is based on Eqs. (13), (14) and (15)

$$\gamma_b = \frac{\tau_{12}}{G_b} \cdot \frac{\delta_b}{h_b} = \frac{\tau_{12}}{G_b} \cdot \delta_b = \frac{\tau_{12} \times h_b}{G_b} \tag{13}$$

$$\gamma_m = \frac{\tau_{12}}{G_m} \cdot \frac{\delta_m}{h_m} = \frac{\tau_{12}}{G_m} \cdot \delta_m = \frac{\tau_{12} \times h_m}{G_m} \tag{14}$$

$$\gamma_e = \frac{\tau_{12}}{G_{12}} \cdot \frac{\delta_e}{2 \times h_b + h_m} = \frac{\tau_{12}}{G_{12}} \cdot \delta_e = \frac{\tau_{12} \times (2 \times h_b + h_m)}{G_{12}} \tag{15}$$

where γ_b: block shear strain; γ_m: mortar shear strain; γ_e: homogenous system shear strain; δ_b: block horizontal displacement; δ_m: mortar horizontal displacement; δ_e: homogenous system horizontal displacement; G_b: block's shear modulus and G_m: mortar's shear modulus.

Equivalence of the two systems presumes equivalence of strains as expressed by Eq. (16).

$$\delta_e = 2 \times \delta_b + \delta_m \tag{16}$$

and the shear modulus G_{12} is equal to:

$$\frac{G_{12}}{G_b} = \frac{\frac{G_m}{G_b} \cdot \left(2 + \frac{h_m}{h_b}\right)}{\frac{h_m}{h_b} + 2 \cdot \frac{G_m}{G_b}} \tag{17}$$

In the case of Fig. 6, the calculation of G_{23} is based on Eq. (18).

$$\tau_{23} \times h_e = 2 \times \tau_{23b} \times h_b + \tau_{23m} \times h_m \tag{18}$$

which leads to Eq. (19).

$$\frac{G_{23}}{G_b} = \frac{2 + \frac{G_m}{G_b} \cdot \frac{h_m}{h_b}}{2 + \frac{h_m}{h_b}} \tag{19}$$

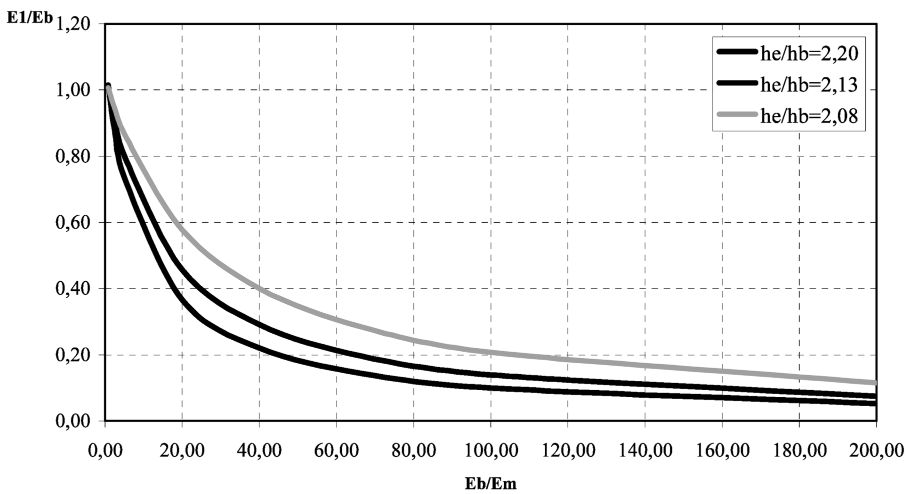


Fig. 8 Influence of blocks and mortar properties to E_1

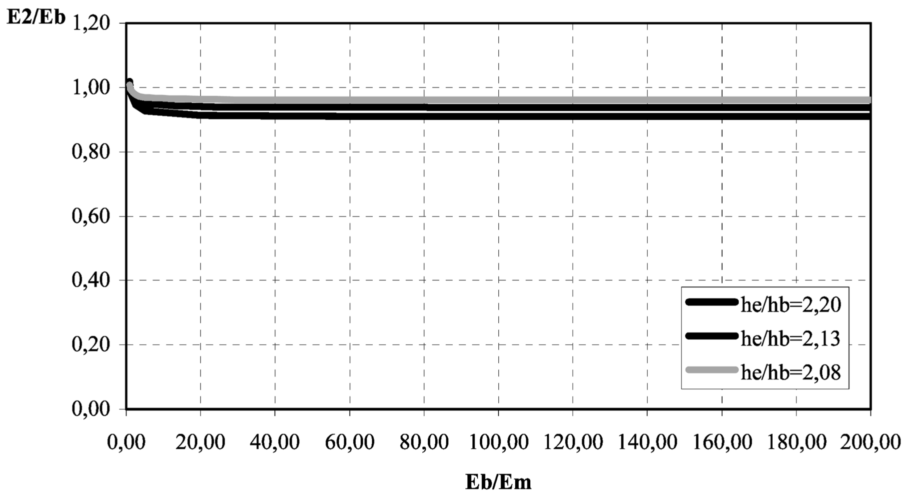


Fig. 9 Influence of blocks and mortar properties to E_2

And respectively G_{13} (Fig. 7) is calculated by Eq. (20).

$$G_{13} = \frac{G_b \times 2 \times h_b + G_m \times h_m}{2 \times h_b + h_m} \tag{20}$$

Indicatively, the influence of blocks and mortar properties to the elasticitymodulus of the equivalent system, is shown in Figs. 8 and 9.

4. The finite element method procedure

In this paper, for the calculation of the equivalent elasticity and shear moduli of an homogenous masonry material, the finite element method is used (Lourenço 1996). A micromodel of the basic unit and its equivalent macromodel are developed. The same normal and shear stresses are applied on both models and equivalence of strains is assumed.

Initially, for the macromodel, some staring values of the elastic properties E_1, E_2, E_3, G_{12}, G_{23} and G_{13} are assumed. The final elastic properties are calculated through an iterative procedure using Eqs. (21)-(25) and (26). The need for this iterative procedure comes as a result of the differential strains between the blocks and the mortar, which are developed only in the case of the micromodel.

$$E_1^{(r+1)} = \frac{u_{1MA}^{(r)} \cdot E_1^{(r)}}{u_{1MI}} \tag{21}$$

$$E_2^{(r+1)} = \frac{u_{2MA}^{(r)} \cdot E_2^{(r)}}{u_{2MI}} \tag{22}$$

$$E_3^{(r+1)} = \frac{u_{3MA}^{(r)} \cdot E_3^{(r)}}{u_{3MI}} \tag{23}$$

$$G_{12}^{(r+1)} = \frac{\gamma_{12MA}^{(r)} \cdot G_{12}^{(r)}}{\gamma_{12MI}} \tag{24}$$

$$G_{23}^{(r+1)} = \frac{\gamma_{23MA}^{(r)} \cdot G_{23}^{(r)}}{\gamma_{23MA}} \tag{25}$$

$$G_{13}^{(r+1)} = \frac{\gamma_{13MA}^{(r)} \cdot G_{13}^{(r)}}{\gamma_{13MI}} \tag{26}$$

where r: the repetition number of the iterative procedure and $[U]^T=[u_1\ u_2\ u_3\ \gamma_{12}\ \gamma_{23}\ \gamma_{13}]$ the displacements matrix. The iterative procedure is terminated when the Eq. (27) is satisfied:

$$\lambda_1 < \lambda = \frac{U_{MI}}{U_{MA}^r} < \lambda_2 \tag{27}$$

where λ_1 and λ_2 the acceptable errors.

5. Application of the procedure on a single-leaf wall

5.1. The finite element models

The described procedure is applied on the single-leaf masonry wall, which is illustrated in Fig. 10. In the same figure, the finite element discretization for the basic unit (micromodel) is also shown. The wall dimensions are 4,90×2,88 m^2, blocks dimensions are 0,19×0,11×0,09 m^2 and the thickness of horizontal and vertical joints is 1 cm. Blocks are considered solid. In contrast with horizontal mortar joints, vertical mortar joints are interrupted.

For the calculation of the wall elastic properties, the masonry material was considered orthotropic, while blocks and mortar were considered isotropic. All materials were assumed linear-elastic, which applies for small strains, where masonry cracking and mortar's nonlinear response can be neglected.

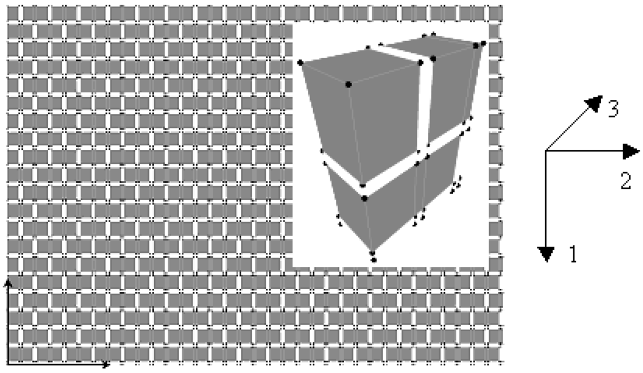


Fig. 10 Finite element micromodel

Table 1 Material properties for the micromodel

Material	*E* (KPa)	*G* (KPa)	ν
Block	10,00×10^6	4,35×10^6	0,15
Mortar	1,00×10^6	4,35×10^5	0,15

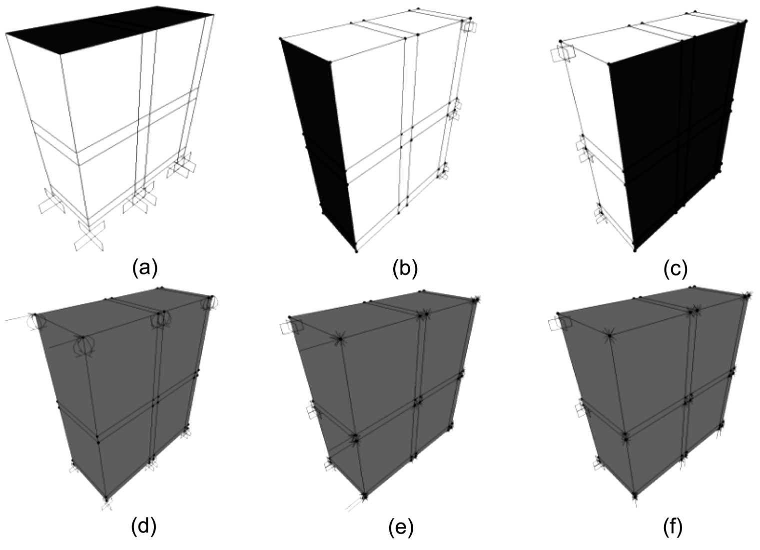


Fig. 11 Applied loadings for the calculation of (a) E_1; (b) E_2; (c) E_3; (d) G_{12}; (e) G_{23} (f) G_{13}

Material properties are presented in Table 1.

Two different models were developed. The first one was a micro-model where blocks and mortar were simulated separately and the second one was a macro-model where an homogenous masonry material was considered. Both models were developed using the finite element method and the software SAP 2000 v.10. Three-dimensional "solid" finite elements, activating three translational degrees of freedom on each node, were used. Blocks and mortar were interconnected though finite element joints. The starting values of the macromodel properties, were: $E_1^{(r=0)}$ = 5,00×10^6 KPa, $E_2^{(r=0)}$ = 5,00×10^6 KPa, $E_3^{(r=0)}$ = 5,00×10^6 Kpa, $G_{12}^{(r=0)}$ = 2,17×10^6 KPa, $G_{23}^{(r=0)}$= 2,17×10^6, $G_{13}^{(r=0)}$ = 2,17×10^6 KPa.

5.2. Calculation of elastic properties

For the calibration of the macro-model, homogenous material, six loadings were applied to both models, considering different restraints and joint constraints, as illustrated in Fig. 11. In Fig. 11(a), 11(b) and 11(c), a stress equal to 1000 KN/m^2 is applied on the dark-shaded area. In Fig. 11(d), 11(e) and 11(f), each concentrated load is equal to 5 KN.

Indicatively, the iterative procedures for the calculation of E_3 and G_{12} are shown in Tables 2 and 3. It is noticeable that when for the micromodel, blocks and mortar present considerable differential strains, as in the case of Fig. 12, more iterations are needed in order to achieve equivalence of strains for the micromodel and the macromodel.

The calculated elastic properties of the equivalent homogenous masonry material are: E_1 = 5,616×10^6 KPa, E_2 = 6,717×10^6 KPa, E_3 = 8,280×10^6 KPa, G_{12} = 2,4477×10^6 KPa, G_{23} = 3,629×10^6, G_{13} = 23,629×10^6 KPa.

Table 2 Calculation of E_3

u_{3MI} (m)	r	$E_3^{(r)}$ (MPa)	$u_{3MA}^{(r)}$ (m)	$\lambda = u_{3MI}/u_{3MA}^{(r)}$
9,959E-06	0	5000	1,650E-06	0,6036
	1	8284	9,954E-06	1,0005
	2	8280	9,959E-06	1,0000

Table 3 Calculation of G_{12}

γ_{12MI} (m)	r	$G_{12}^{(r)}$ (MPa)	$\gamma_{12MA}^{(r)}$ (m)	$\lambda=\gamma_{12MI}/\gamma_{12MA}^{(r)}$
3,868E-04	0	2170	4,192E-04	0,9229
	1	2351	3,9714E-04	0,9741
	2	2414	3,903E-04	0,9911
	3	2436	3,880E-04	0,9971
	4	2443	3,873E-04	0,9989
	5	2447	3,868E-04	1,0000

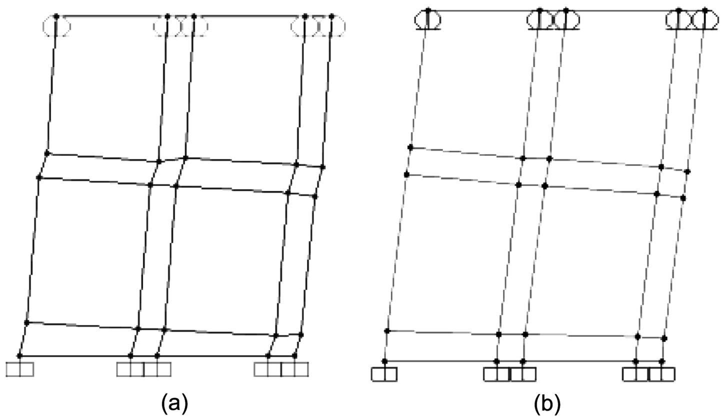
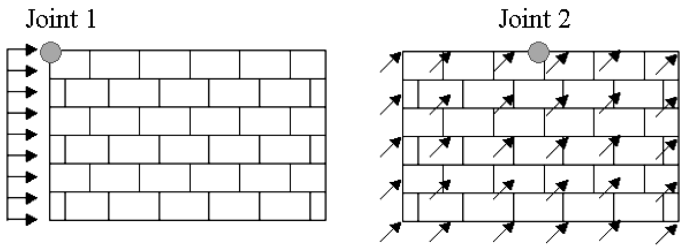


Fig. 12 Deformed shape of the micromodel (a) and the macromodel (b) for the loading shown in Fig. 11(d)



(a) (b)

Fig. 13 (a) horizontal in-plane stress and (b) horizontal out of plane stress

Table 4 Micromodel and macromodel displacements

Loading	Displacements	Micro-model	Macro-model	u_{MI}–u_{MA}/u_{MI}
in-plane	u_2 (m) (joint 1)	1,818E-04	1,721 E-04	0,05
out of plane	u_3 (m) (joint 2)	2,589E-02	2,477E-02	0,04

5.3. Comparison of microanalysis and macroanalysis model

For the evaluation of the efficiency of the proposed methodology, the response of the micro-model and the macro-model of the entire wall were compared. The applied loadings, which are illustrated in Fig. 13, are an horizontal in-plane stress equal to 200 KN/m^2 and an horizontal out of plane stress equal to 1 KN/m^2.

The comparison of the response of the two models is, indicatively, presented in Table 4, in terms of displacements. The elastic analysis results reveal that for the proposed homogenous material properties, the response of the macro-model expresses with accuracy the response of the detailed microanalysis model. In this way, it is proved that the presented methodology can provide a very useful tool for the evaluation of masonry elastic properties, which can be used for the analysis of large-scale structures with finite element macromodels.

6. Conclusions

In this paper, a methodology has been proposed for the analytical evaluation of masonry elastic properties and, in particular, of its elasticity and shear moduli, using the finite element method. The methodology is based on the consideration that the masonry is a multi-layered material, consisting of blocks and mortar and, as a result, the calculation of its mechanical properties can be made directly, by taking into account its composite materials properties.

On this basis, an homogenisation technique was developed. The technique applies to masonries presenting periodicity in their form, thus consisting of identical units that when repeated, form the final masonry layout. Under these conditions, the basic unit properties are representative of the entire wall properties. As a result, the investigation of masonry elasticity and shear moduli was focused on the investigation of the unit properties.

The proposed procedure was demonstrated through its application on a single-leaf masonry wall. Two different finite element models were developed for the unit. The first one was a micromodel where blocks and mortar were simulated separately and the second one was a macromodel where an homogenous masonry material was considered. The homogenous masonry material was considered orthotropic. E_1, E_2, E_3, G_{12}, G_{23}, G_{13} for the macromodel were calculated presupposing equivalence of strains for the micro-model and the macro-model. Their final values were obtained through an iterative procedure, based on Hook's law elasticity considerations.

The efficiency of the proposed method was evaluated through the comparison of displacements between the micromodel and the macro-model, for an entire wall, and it was proved that the suggested methodology led to reliable results.

The proposed methodology can provide a useful tool for the evaluation of masonry elastic properties. However, for the wide application of the methodology, it is important that further investigation is made, concerning, among others, the deviation of material properties within the same structure, the non-linearity of the mortar and the lack of full mortar joints in between blocks. Especially in the case of historical structures, where structural monitoring often achieves values smaller than those reported here, the influence of past damage and cracks on the final elastic properties should be further investigated.

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