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Physics-guided neural networks for bridge health monitoring: A state-of-the-art review

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Abstract. Appropriately utilizing the data recorded by the bridge health monitoring (BHM) system to identify structural damage, evaluate structural safety, and predict structural serviceability is the core work in the community of BHM. Neural network models are more flexible to describe multi-source, multi-dimension and non-linear relationships, comparing with traditional statistical and regression analysis, and have been widely used for data-driven evaluation of bridge performance. But it is easily influenced by noise and errors that are difficult to eliminate in the monitoring data. Physics-guided neural networks (PGNNs), which combine physical information with neural networks, have stronger accuracy, robustness, and reliability, and are becoming promising tools for bridge performance evaluation. In the past few years, numerous researchers all over the world paid intensive attention on this topic. This paper summarizes the latest developments of PGNN methods for BHM. The commonly used PGNNs are classified into three categories, including the physics-guided loss function, the physical data enhancement and the digital twin. Following that, the applications of the three types of PGNNs are presented through a summary of relevant literature. Finally, the challenges and prospects of PGNN methods in the field of BHM are discussed.

Keywords: artificial neural network; bridge health monitoring; digital twin; physics-guided neural network; structural safety evaluation

1. Introduction

Bridges play an irreplaceable role as an important component of land transportation infrastructure (Mei and Li 2015). During several decades or over a hundred years of operation time, they inevitably suffer from various damages such as material corrosion, structural cracks, performance degradation, and functional failure due to the coupling effects of the external environment, frequent traffic and aging (Zhao *et al.* 2022, Zhang *et al.* 2021). The impact of a strong earthquake can also cause severe damage to structures (Zhu *et al.* 2021). The accumulation of these damages may result in partial or complete collapses of bridges, causing human casualties and economic loss. Therefore, it is essential to timely detect structural damage and evaluate

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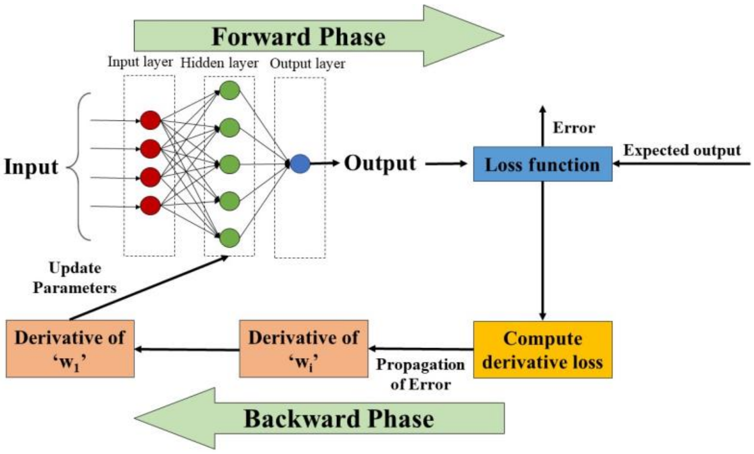

Fig. 1 Basic process of an ANN (Nayak *et al*. 2020)

structural safety of bridges through effective methods (Estes and Frangopol 2001). In the past, the periodic inspection is usually employed to assess bridge performance (Gattulli and Chiaramonte 2005). However, this method is problematic due to its subjectivity, inefficiency, high cost, low reliability, and inability to track quantitatively in real time (Jiao *et al*. 2017).

Bridge health monitoring (BHM), which employs sensor arrays to monitor the external environment, operating loading and critical responses of a bridge, is capable of reliably evaluating the operational status, detecting the damage and tracking the performance deterioration and has been considered as a potential technology to replace the periodic inspection (He *et al*. 2022, Jiao *et al*. 2021, Ju *et al*. 2023). Successful applications of BHM systems on real bridges have been widely reported throughout the world (Sun *et al*. 2020b, Azhar *et al*. 2023). In a sophisticated BHM system, reasonably interpreting monitoring data to accurately indicate the structural serviceability, safety, reliability and sustainability is the core task (Jiao *et al*. 2020, Zhang *et al*. 2022).

In the early stages, researchers typically used monitored vibration data to detect changes in structural properties such as frequencies, stiffness and damping and indicate structural damages (Lifshitz and Rotem 1969). To identify these changes and damages, researchers have developed physical model-based methods as well as statistical methods. The physical model-based method indicates the presence of damages by comparing the difference between the predictions obtained by the finite element (FE) model of the undamaged bridge and the monitored data (Zhu *et al*. 2015, Xiao *et al*. 2015). This approach is difficult to accurately detect anomalies in bridges because the simplification and the uncertainties of the geometrical and material properties in the FE model cannot be completely eliminated (Sun *et al*. 2020b). The statistical method treats the task as a statistical pattern recognition problem. However, as the amount and complexity of data increases, the computational difficulty increases significantly (Farrar and Worden 2012).

Artificial neural networks (ANNs), which are novel data-driven models, offer viable solutions to interpret monitoring data and identify structural abnormality and have been widely applied in bridge engineering (Cha *et al*. 2024). Nguyen *et al.* (2019) applied ANN to bridge damage detection and the results showed that ANN can accurately identify the location and degree of damage. Fig. 1 shows the scheme of a typical ANN (Nayak *et al*. 2020). The ANN is featured by fast feed-forward fitting, powerful function approximation, and high accuracy as the depth of the network increases (Huang and Wang 2022). Therefore, ANN can effectively model complex nonlinear relationships in monitoring data to predict the nonlinear response of bridges (An and Lee 2022). In an ANN model, each layer extracts a distinct feature from the data, forming an end-to-end system that eliminates the need for human involvement. This capability enables it to be broadly applicable in BHM with minimal professional knowledge of specific bridges (Azimi *et al*. 2020a). Furthermore, the ANN excels at solving complex physical problems and handling complex multidimensional data (Wang *et al*. 2020). Therefore, neural networks can detect abnormal data by capturing local features and overall trends between multiple types of data (Lee *et al*. 2024, Shajihan *et al*. 2022, Liu *et al*. 2022, Du *et al*. 2022). Some classical ANNs such as the fully connected neural network, the long short-term memory network (LSTM), the convolutional neural network (CNN), the autoencoder, the deep belief network, the deep Boltzmann machine, and the generative adversarial network have shown their superiority in processing bridge monitoring data. With the continuous development of neural networks, new neural networks with stronger capabilities also play a better role in dealing with various special problems (Pham *et al*. 2023).

Although the ANN has many advantages over traditional methods, it still encounters many problems and challenges when dealing with complicated bridge monitoring data (Karpatne *et al*. 2017). Firstly, the ANN training requires a large amount of extremely high-quality monitoring data and errors cannot be tolerated (Wedel and Marx 2022). The acquisition of high-quality monitoring data remains difficult because environmental noise and sensor damages are difficult to avoid, although BHM systems have been well designed and installed. Secondly, the ANN has black-box properties that make it difficult for researchers to thoroughly understand and interpret the theories contained in the training process (Liang *et al*. 2021). The black-box property also makes some experts and researchers question the reliability as well as the accuracy of the results calculated by the ANN. Finally, the ANN may not be able to perform specific physical analyses without providing them with domain-specific prior knowledge, which notably decreases its generalization ability (Zhang *et al*. 2020c).

To address these issues mentioned above, researchers in different fields make efforts to introduce physical knowledge into data-driven ANNs aiming at improving their accuracy and interpretability of the established models (Jiao *et al*. 2020, Huang and Wang 2022, Li *et al.* 2024, Pang *et al.* 2023, Diao *et al.* 2023, Zhang *et al.* 2023, Dai *et al.* 2023, Ogliari *et al.* 2021). The most recent developed one embodying this learning paradigm is the physics-guided neural network (PGNN). PGNNs have better generalization performance, interpretability and scalability compared to traditional ANNs and rely significantly less on monitored data. PGNN is also a bridge between physical and data-driven methods. On the one hand, physical knowledge can be adopted to solve forward modelling problems (Brunton *et al*. 2016). On the other hand, data-driven methods can be used to explain underlying physical patterns and correlations among variables (Schmidt and Lipson 2009). For data collected by BHM systems, inherent physical relationships exist among temperatures (Zhu *et al.* 2024, Zhou *et al.* 2024, Li *et al.* 2023), displacements (Sun *et al*. 2023, Guan *et al*. 2018), vibrations (Zhang *et al*. 2024, Liu *et al*. 2023b, Mazzeo *et al*. 2023, Lv *et al*. 2023), wind and vehicles (Han *et al*. 2020, Zhang and Zhou 2023, Hester *et al*. 2017). These physical relationships can be employed to train PGNNs and enhance the ability of PGNNs to identify structural damages. Therefore, the utilization of PGNNs to evaluate structural performance has become a research frontier in the community of BHM and the improvements of PGNNs have been performed by many researchers.

Currently, a large number of papers related to PGNNs in BHM mainly focus on solving specific problems, and there are fewer review papers on this method. To fill this gap, this paper attempts to summarize the relevant papers about PGNNs within the last few years, hoping to provide new impetus for the optimization and application of this method in BHM. Meanwhile, potential research directions for PGNNs in BHM is proposed, considering the development of related theories and the advancement of data-driven technologies.

2. Concept of PGNN

Physical information generally guides the establishment of ANN models by three approaches, the physics-guided loss function, the physical data enhancement and the digital twin. The physics-guided loss function embeds physical information into loss functions to improve the accuracy of neural networks. The physical data enhancement integrates physical information into inputs to jointly train neural networks. And the digital twin interacts the neural network with the physical model thus enabling the incorporation of physical information. The specific ideas of the three approaches are summarized in the following paragraphs.

2.1 Physics-guided loss functions

The loss function is the criterion for measuring the difference between the outputs of the neural network and the real values. The neural network carries out inverse optimization according to the loss function, so as to accurately map the relationship between inputs and outputs (Bai *et al*. 2023). The physics-guided loss function incorporates the equations describing physical laws into the loss function of the neural network. Researchers have argued that this can help neural networks capture generic patterns consistent with established physical laws (Willard *et al*. 2020).

The neural network embeds physical information in the loss function is also named as the physically informed neural network (PINN). Its concept was first proposed by Raissi *et al.* (2019). The general physics-guided loss function is defined as follows

$$L = L_{data}(\hat{y} - y) + \lambda R(W, b) + \gamma R_{phy}(X, \hat{y}) \tag{1}$$

Where L is the physics-guided loss function; $L_{data}(\cdot)$ is the loss function of the conventional neural network, which is usually used to measure the distance between the neural network output $\hat{y}$ and the target result y; $R(\cdot)$ is the regularization term in the loss function, which is used to constrain the weight W and bias b of the neural network (Wang *et al*. 2023b). $R_{phy}(\cdot)$ is the physical regularization term based on the input X, output $\hat{y}$ and the physical prior knowledge (Willard *et al*. 2020, Zhang 2020, Cuomo *et al*. 2022); λ and γ are regularized hyperparameters.

The commonly used loss functions of conventional neural networks are the mean absolute error (Zhang 2020), mean square error (Bommidi *et al*. 2023), cross-entropy loss (Mao *et al*. 2023), and combined loss (Fu *et al*. 2023). The regularization term in the loss function is usually regularized by the L1 and L2 norm regularizations (Yang *et al*. 2023, Wang *et al*. 2023a). The form of the physical regularization term $R_{phy}(\cdot)$ varies according to the adopted physical knowledge.

Liu *et al.* (2023a) and Zhang *et al.* (2020a) added physical regularization terms to the loss function of the LSTM to study structural dynamic responses excited by ground motions. Structural displacements, accelerations and restoring forces are employed to formulate the loss function of the LSTM; while structural velocity, structural acceleration, structural restoring forces and the

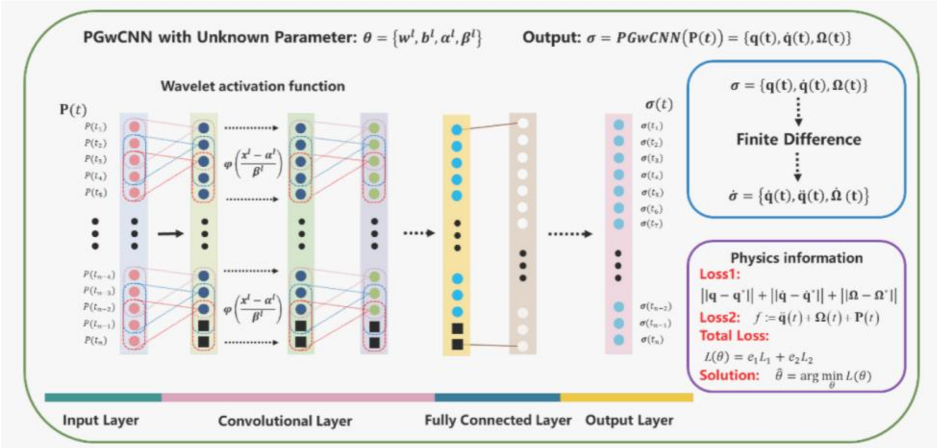

Fig. 2 Structure of a physical information-guided CNN (Xu *et al*. 2023)

ground acceleration are adopted to describe the physical regularization term. Structural displacements, velocity, accelerations and restoring forces obey the basic dynamics equation. The specific formulas are given by Eqs. (2)-(6). The results show that adding a physical regularization term to the loss function can effectively improve the accuracy and narrow the prediction range.

$$L = \alpha L_{data} + \beta R_{phy} \tag{2}$$

$$m\ddot{x}^p + c\dot{x}^p + kx^p = -m\ddot{x} \tag{3}$$

$$m\ddot{x}^m + c\dot{x}^m + kx^m = -m\ddot{x}_g \tag{4}$$

$$L_{data} = \frac{1}{N}\left(\|x^p - x^m\|_2^2 + \|\dot{x}^p - \dot{x}^m\|_2^2 + \left\|F_{rs}^p - F_{rs}^m\right\|_2^2\right) \tag{5}$$

$$R_{phy} = \frac{1}{N}\left(\left\|\dot{x}^p - \dot{x}_t^p\right\|_2^2 + \left\|\ddot{x}_t^p + F_{rs}^p + \ddot{x}_g\right\|_2^2\right) \tag{6}$$

where α and β are scale factors; m, c and k are the mass, damping coefficient and stiffness of the structure, respectively; $\ddot{x}^p$, $\dot{x}^p$ and x^p are the predicted displacement, velocity, and acceleration, respectively; $\ddot{x}_g$ is the ground acceleration; $\ddot{x}^m$, $\dot{x}^m$ and x^m are the measured displacement, velocity, and acceleration, respectively; F_{rs}^p and F_{rs}^m are the predicted and measured mass-normalized restoring forces, respectively; the subscript t denotes the derivative of the prediction; N is the total number of samples ; $\|\cdot\|_2$ is the L2 norm regularization.

Xu *et al.* (2023) also constructed a physical neural network by embedding the structural equation of motion into the loss function of a wavelet CNN, as shown in Fig. 2.

Intensive research work has been performed on improving neural networks incorporating physics-guided loss functions. The improvements include the activation function (Jagtap *et al*. 2020b, c), the sampling method (Nabian *et al*. 2021, Wight and Zhao 2020, Wu *et al*. 2023, Guo *et al*. 2023), the fusion of loss functions (Wang *et al*. 2021a, Son *et al*. 2021), the updating pattern of weights (Wang *et al*. 2022, Xiang *et al*. 2022, McClenny and Braga-Neto 2020), the network architectures (Zhang *et al*. 2020b, Rodriguez-Torrado *et al*. 2021, Ren *et al*. 2022), and the analysis of generalization error (Shin *et al*. 2020, Mishra and Molinaro 2022). Neural networks have a wide range of applications in solving problems related to partial differential equations using physics-

Table 1 Neural networks combining physics-guided loss functions

Ref.	Network name	Applicable issues
(Yang *et al*. 2021)	B-PINN	Low- and high-dimensional linear and nonlinear partial differential equations with noisy data
(Jagtap *et al*.2020d)	cPINN	High-dimensional nonlinear higher-order differential equations
(Jagtap *et al*.2020a)	XPINN	Any differential equation
(Kharazmi *et al*. 2021)	hp-PINN	Partial differential equations with non-smooth solutions
(Yu *et al*. 2022a)	gPINN	Positive and Inverse Partial Differential Equation Problems
(Dwivedi *et al*. 2019)	DPINN	Nonlinear partial differential equation
(Pang *et al*. 2019)	fPINN	Fractional partial differential equation
(Guo *et al*. 2022)	MC-PINN	High-dimensional fractional partial differential equations
(Yuan *et al*. 2022)	A-PINN	Integral differential equation
(Pang *et al*. 2020)	nPINN	Parametric and functional reasoning for integral equations
(Chiu *et al*. 2022)	can-PINN	Fluid dynamics problems
(Peng *et al*. 2022)	RPINN	Stationary partial differential equation
(Lu *et al*. 2021)	DeepONet	Learning a variety of continuous nonlinear operators
(Wang *et al*. 2021b)	Physics-informed DeepONets	Learning solution operators for arbitrary partial differential equations
(Xing *et al*. 2024)	GRK4-PINN	The second order differential equation for the equivalent single DOF system

guided loss functions. Table 1 summarizes the results of research on solving partial differential equation problems.

It was demonstrated that there are at least three advantages when physical regularization terms are added. Firstly, non-monitored data are allowed to be included in training since real data is not required to compute the physically-based loss term. Secondly, the constraints of the physical formulas can reduce the search space of network parameters, which may significantly improve the network accuracy and relaxes the requirement for monitored data. Finally, neural networks are more likely to generalize to scenarios outside of the training environment (Li *et al*. 2024).

2.2 Physical data enhancement

The amount and error of data govern the training results of data-driven neural networks. In real application, obtaining high-quality measured data is very difficult especially in BHM systems for the reason that noise contamination and electromagnetic interference are inevitable. Furthermore, the amount of data is sometimes limited, for examples, in the first several months of BHM system operation and the case that sensors are very sparse. As a result, employing the simulated data obtained from the physical model to help the neural network training, which not only increases the amount of data and reduce the dependence on the measured data but also improves the accuracy and convergence speed of the neural network, becomes an advisable choice. This strategy is called physical data enhancement. Common approaches for data enhancement include the physical information initialization and the input data enhancement.

The physical information initialization, which is inspired by the idea of transfer learning, utilizes physical information to assist in determining the initial weights of neural networks,

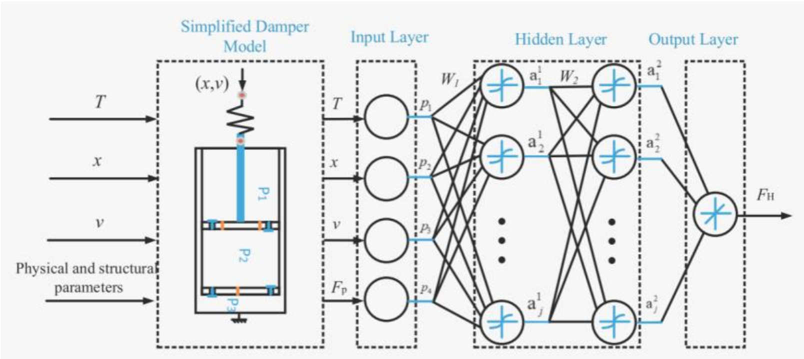

Fig. 3 Hybrid neural network model for damping force prediction (Dai *et al.* 2023)

considering the fact that the initial weights can have a significant impact on the convergence speed and accuracy of the model. The data simulated by physical models is firstly fed into the initial neural network to pre-train the parameters. These pre-trained parameters are then migrated to the desired model as initial parameters and the training is continued using real monitoring data. Kohtz *et al.* (2022) adopted this concept to predict the battery health condition from partial charging data and proved that the model initialized by physical information can predict the battery health condition more quickly and accurately. Jia *et al.* (2021) pre-trained the neural network model using simulated data and continued to train the model using real data. The obtained prediction model can achieve better results of the lake temperatures. Therefore, the physical information initialization can effectively mitigate problems that the amount of real data is small and the error of real data is significant.

The input data enhancement combines the simulation data generated by the physical model with the real monitoring data to form a new augmented dataset and uses the new dataset to train the neural network (Von *et al.* 2021). This method is also effective in increasing the amount of training data and improving the training accuracy. Moreover, the physical model can generate data from different scenarios to increase the diversity of the training dataset, which strengthens the generalization ability of the neural network. Dai *et al.* (2023) proposed a hybrid neural network model integrating a physical parameter model with a neural network model to investigate the influence of boundary conditions on the dynamic performance of hydraulic dampers, as shown in Fig. 3. The approximate damping force was first calculated using a simplified physical model. Subsequently, the calculated damping force together with the real measured relative displacement, velocity and temperature are used as inputs to train the neural network and predict the exact damping force. The results demonstrated that input data enhanced by physical information is an effective method to improve the reliability of the neural network.

In addition to the two main methods mentioned above, there are a number of other ways of using physical information to initialize models and enhance datasets, such as physics-informed design of architectures, physics-informed data preprocessing and physics-informed data generation (Li *et al.* 2024). The physics-informed design of architectures employs physical information to guide the network architecture and assign some physical meaning to some of the neuron's outputs or weights of the hidden layers (Daw *et al.* 2020, Sun *et al.* 2020a). Xiong *et al.* (2023) investigated a generative adversarial network constrained by multiple physical factors. However, these ideas have been less studied in the BHM community.

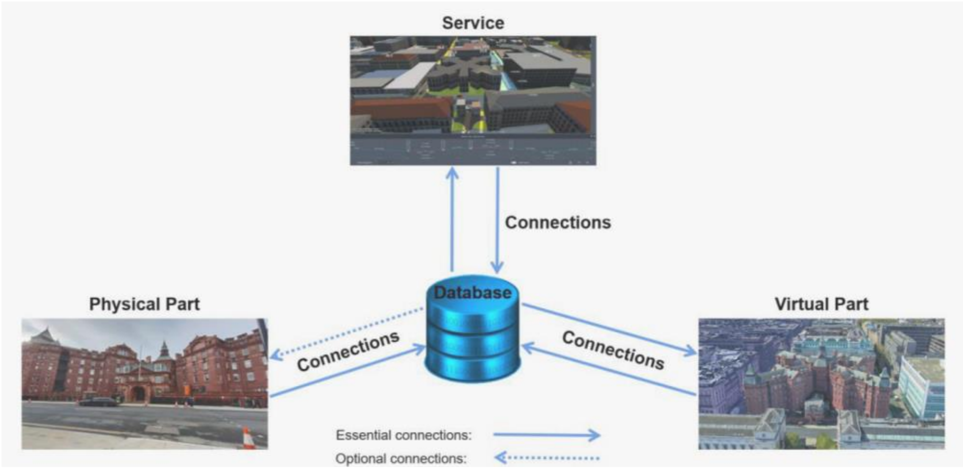

Fig. 4 Framework of the digital twin (Jiang *et al*. 2021c)

2.3 Digital twins

Being different from the physics-guided loss function and the physical data enhancement, which only use physical information to control the data-driven neural networks, the digital twin provides a bi-directional data flow. Thus, the physical-data co-driving and the virtual-real interaction between the real model and the data model are realized. Specifically, the digital twin is a technical tool to digitally create a virtual entity of a physical entity to simulate, validate, predict, and control the whole life cycle process of the physical entity with the help of historical data, real-time data, and algorithmic models (Jiang *et al*. 2021c). The framework of the digital twin is shown in Fig. 4.

The concept of the digital twin was first appeared in studies related to the product life cycle (Grieves 2005). In 2011, NASA first introduced the terminology of "twin" and defined it in detail during the Apollo program (Tuegel 2012, Glaessgen and Stargel 2012). Researchers then have conducted exploratory studies on the digital twin in product design (Liu *et al*. 2021, Lo *et al*. 2021, Tao *et al*. 2019, Tao *et al*. 2020), manufacturing (Tao *et al*. 2020, Tao *et al*. 2018a, Zhou *et al*. 2020), and operational status monitoring (Balta *et al*. 2019, Tao *et al*. 2018a, Qamsane *et al*. 2022, Fera *et al*. 2019). In bridge engineering, the digital twin utilizes continuous, high-frequency monitoring of physical structures as well as accurate virtual models (Tao *et al*. 2022).

The study of the digital twin in bridge engineering has started in the last few years (Yang *et al*. 2024). Based on the original concept of the digital twin, the whole system is divided into a physical bridge subsystem, a virtual bridge subsystem and the information that connects the two subsystems (Jiang *et al*. 2021a). The digital twin can be used throughout the life cycle of a bridge, including the design phase, the construction phase, and the operation and maintenance phase. This paper focuses on the operation and maintenance phase, in which a large amount of monitoring data is available.

For bridges equipped with BHM systems, the large number of sensors enables data to be collected in real time (He *et al*. 2022). At the same time, FEM or other numerical methods provide virtual models (Kim *et al*. 2023). The connection between virtual models and real data can be achieved through data-driven methods such as statistical modelling and ANN (Qi *et al*. 2021). The digital twin technology can construct virtual models to describe physical entities, which allows for physical information enhancement to improve their accuracy (Tao *et al*. 2018b). Digital twin technology can also modify physical models to minimize modeling uncertainty and significantly improve the predictive ability of the model (Rojas *et al*. 2022).

3. Applications of PGNN in BHM

The incorporation of physical information could improve the accuracy of the neural networks describing the relationship between inputs and outputs of bridges. The response anomalies, structural damages, and performance degradation of bridges could be identified more reliably. Therefore, the application of PGNN has attracted widespread attention in the BHM community and is summarized here.

3.1 Application of physics-guided loss functions

3.1.1 Bridge damage identification

Bridge damage identification aims at detecting and locating structural damages so as to prevent possible structural failures, extend the service life, and ensure the safety and reliability. Yamaguchi and Mizutani (2024) proposed a physically driven neural networks for the nonlinear damage identification of a reinforced concrete (RC) bridge pier, as shown in Fig. 5. A bilinear rotational-spring-damping model is developed as a physical model of the RC piers to obtain simulation data. A physical loss function that combines neural network predictions, physical model predictions, and measured data from full-size pier shaking table experiments was used to guide the learning of the network. The results show that the neural network containing physical information is robust to the effects of noise and ground motion. The method is proved to effectively reveal the distribution of elastic stiffness and ductility coefficients and their deterioration along the height of abutment under different working conditions by foot-size simulation experiments. Guo and Liu (2024) proposed an efficient cable force identification method using a physically driven auto-encoder network through frequency response functions (FRFs) obtained from monitored cable vibration data. The framework of the method is displayed in Fig. 6. The cross-signature assurance criterion is introduced into the loss function as a constraint in order to make the network fully take into account the properties of the FRF and to improve the interpretability. It was demonstrated that the proposed method achieves not only high prediction accuracy, but also a good fit between the predicted and actual developmental trends of cable forces, and is well-suited for the different types of bridges. Chen and Liu (2021) developed a PGNN to estimate the probabilistic fatigue S-N curve. The loss function of the network takes into account the type of input data and reflects the probabilistic properties. The distribution of the loss function also has an impact on the type and amount of output data. In addition, the structure and parameters of the network are also constrained by physical knowledge. The results show that this method can effectively deal with failure and beating data and ensure that the output fatigue parameters conform to physical laws. Furthermore, PGNN can be used as a flexible and robust model for general fitting and uncertainty quantification of fatigue data and does not impose constraints on the type of function, average stress or other factors at different stress levels.

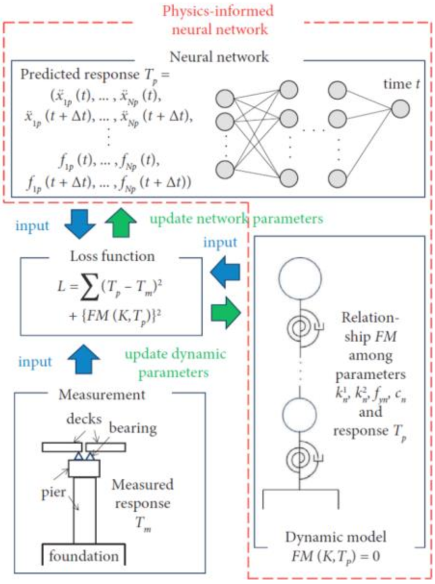

Fig. 5 Concept of the PGNN for dynamic system identification of the bridge pier (Yamaguchi and Mizutani 2024)

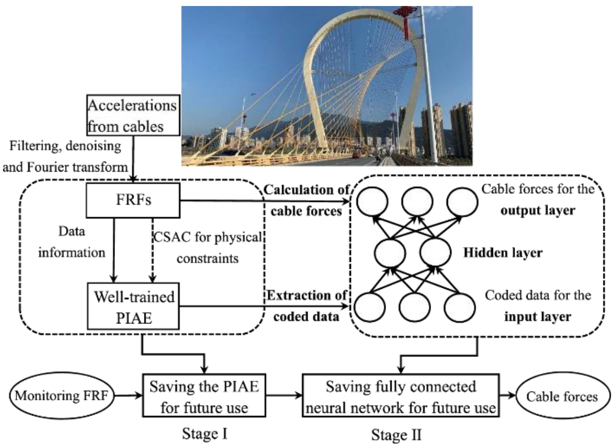

Fig. 6 Framework for cable force identification (Guo and Liu 2024)

3.1.2 Bridge response prediction

The comparison of bridge responses predicted by models of intact structures and monitored results is a direct way to discover structural anomalies. Li and Zhu (2024) presented a new method combining physical information and neural networks to simplify the identification process of bridge influence lines and vehicle loads. The framework of this method is shown in Fig. 7. Physical information was introduced into the neural network by transforming the equations representing the relationship between vehicle loads and bridge responses into the loss function. The method avoids the complex computational steps in traditional methods, such as matrix

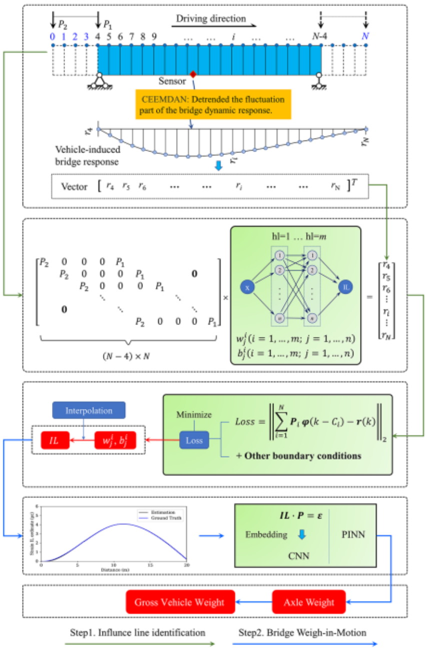

Fig. 7 Framework of the proposed method (Li and Zhu 2024)

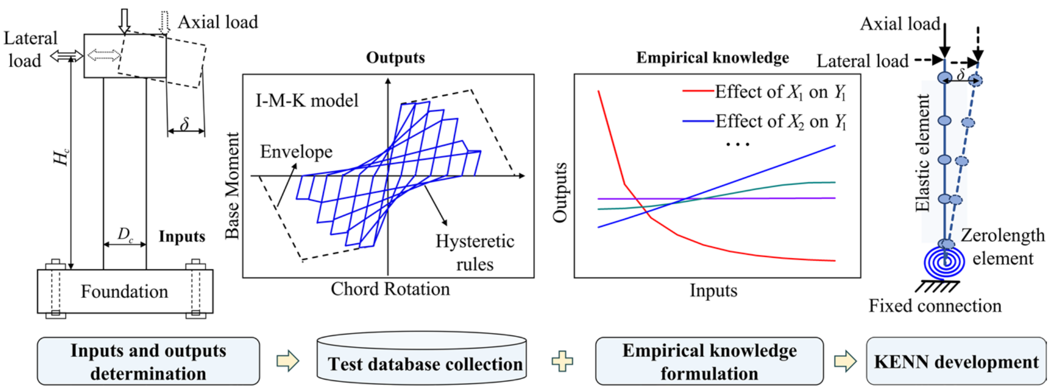

Fig. 8 Framework of the KENN-calibrated model (Liu *et al*. 2024b)

decomposition and regularization coefficient optimization, while improving the accuracy of the identification. The results of theoretical derivation and simulation show that the method is able to identify the vehicle load response accurately and has good numerical stability. Moreover, the network requires only a small amount of data to obtain accurate results due to the constraints of the physical equations. He *et al*. (2024) extended the PINN for dynamic response analysis of bridges under moving loads. The Fourier embedding layer and causal weight have been added in the deep neural network and loss function, respectively. The results show that, even if the bridge parameters are unknown, the bridge responses can still be predicted by PINN driven by both data and physic with small amount of monitored response.

Besides, using physical information to set or strengthen certain parameters in neural networks is also an effective approach to improve the performance of the model. Liu *et al.* (2024b) developed a parametric FEM model of the bridge pier containing simplified component units and used a knowledge-enhanced neural network (KENN) to calibrate model parameters, as shown in Fig. 8. In conjunction with a database of historical experimental results, the effects of key characteristics describing reinforced concrete columns on the model parameters were investigated and formulated as physical laws to supervise the training of the KENN. The methodology is successfully applied to rapid seismic response prediction of typical bridges.

3.1.3 Bridge performance prediction

By predicting bridge performance, potential structural problems and safety risks can be identified in advance. Yan *et al.* (2023) used the PINN to assess bridge wind loads from flow visualization data, thereby reducing the high cost of traditional wind tunnel experiments and high-precision numerical simulations. This PINN obtained hydrodynamic information by introducing the Navier-Stokes equations and momentum conservation equations into the loss function. The PINN was used to analyze a two-dimensional viscous incompressible fluid through a generic bridge deck cross-section and two cases were tested with different Reynolds numbers. The results indicate that the PINN is able to accurately extract the velocity and pressure fields in the concentration field and reliably predict the drag and lift coefficients. Xing *et al.* (2024) proposed a physics-driven neural network to predict seismic responses of bridges. The method not only embeds the physical information into the loss function, but also integrates the gradient-enhanced fourth-order Runge-Kutta method into the neural network. The prediction performance is improved by directly embedding additional gradient equations in the loss function. The results of numerical experiments conducted on a large-span continuous girder high-speed railroad bridge show that the method can accurately identify the unknown parameters in the equivalent single-degree-of-freedom system and predict the ground vibration response of the bridge.

Ruan *et al.* (2024) combined FE analysis and physically guided LSTM (PG-LSTM) networks to predict the failure process of prefabricated bridge deck joints. The governing equations for displacement and bending are introduced as physical constraint terms in the loss function. An extensive database of failure processes was established through bending tests and parametric analyses, and a PG-LSTM neural network was constructed for accurate sequence prediction. The complete framework of the method is shown in Fig. 9. The prediction results were validated by supplementary tests and show good agreement. It was demonstrated that the method is capable of accurately predicting structural key parameters such as initial stiffness, ultimate deflection, failure mode and ultimate bearing capacity.

3.2 Applications of physical data enhancement

Physical data enhancement could reduce the quality and quantity requirement of monitored data and shorten training time. This approach performs particularly well when data is scarce and computational resources are limited. Azimi and Pekcan (2020) developed a new damage identification method based on CNNs. Compared with the traditional CNN, this method first pre-trained the CNN model by the acceleration response history obtained from the numerical baseline model and the compression response data obtained from the discrete histogram. Then the pre-trained CNN is fine-tuned based on the field monitoring data. A wide range of numerical simulations provide pre-training data corresponding to almost all possible damage scenarios. As a

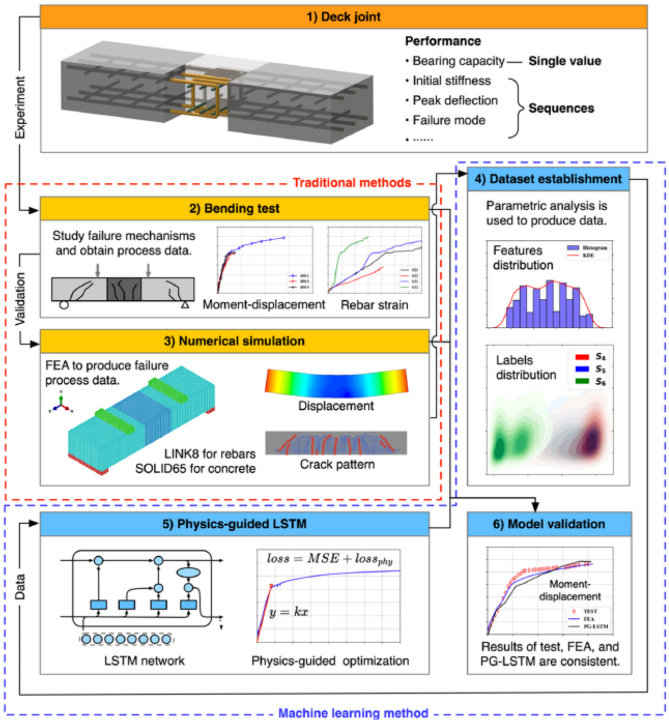

Fig. 9 Flowchart for predicting the failure process using PG-LSTM (Ruan *et al.* 2024)

result, this method is able to accurately detect potential damage scenarios of structures and use unsupervised clustering for damage pattern identification. Considering that BHM systems usually only have access to a limited range of data and it is difficult to obtain data under damage conditions, Figueiredo *et al.* (2019) also used a FE model to generate data with structural damages. The combination of these simulated data and monitoring data were fed into a machine learning algorithm to promote damage classification. It was found that the damage detection performance is improved when data from the FE model are added in the training process. Yin *et al.* (2023) constructed a neural network for recognizing bridge damage under moving vehicle loads. The physical enhancement dataset contains simulation data from refined FE models, simulation data from simplified FE models, and monitoring data from real structures. The physically enhanced dataset was utilized for network training and the training process is supervised by physical information. The framework of the damage recognition model is shown in Fig. 10. The results indicate that physical data can significantly enhance the training samples thereby improving the performance of the trained neural network and increasing the accuracy of damage identification. Eventekidis *et al.* (2020) employed an updated FE model to generate labelled BHM vibration data and combined them with random excitation to train a deep learning CNN classifier, as shown in Fig. 11. The investigation shows that CNN can effectively classify and predict the experimental results by training with the data generated from the updated FE model. It is also verified that the accuracy of the FE model has a significant impact on the recognition accuracy of the CNN and the combination of an accurate FE model and a neural network is a more reliable method for structural damage identification when the measured data is limited.

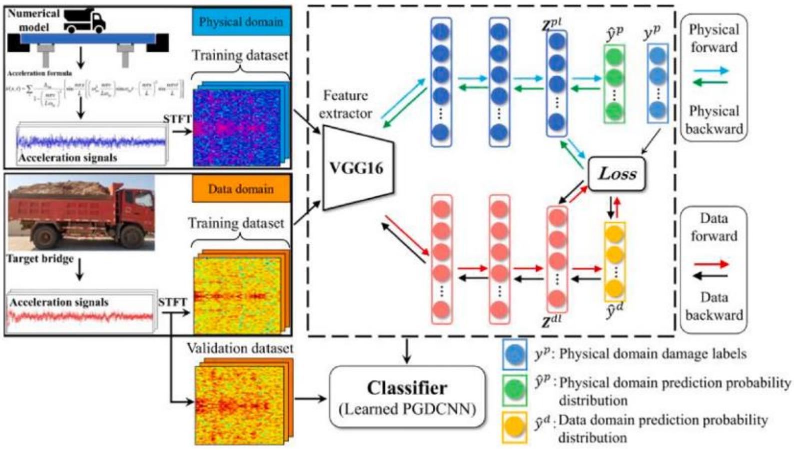

Fig. 10 Framework of the damage identification model (Yin *et al*. 2023)

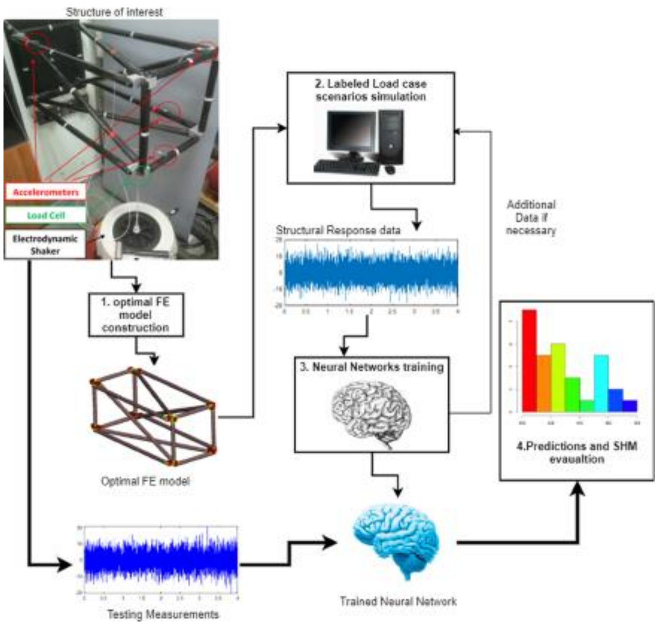

Fig. 11 Workflow of the proposed method with core stages numbered (Eventekidis *et al*. 2020)

In addition, some researchers have constructed databases depending on expertise from specific physical problems (Su and Ye 2005). Rai and Mitra (2021) created a parameter database with damages from complex response data generated by FE simulations in order to detect damages from Lamb waves. And a hybrid physically-assisted multilayer feedforward neural network was proposed, which was trained using the simulated data as input and supervised by the robust Levenberg-Marquardt algorithm. Preliminary tests show that the model is able to predict the damage location with 99.94% success rate.

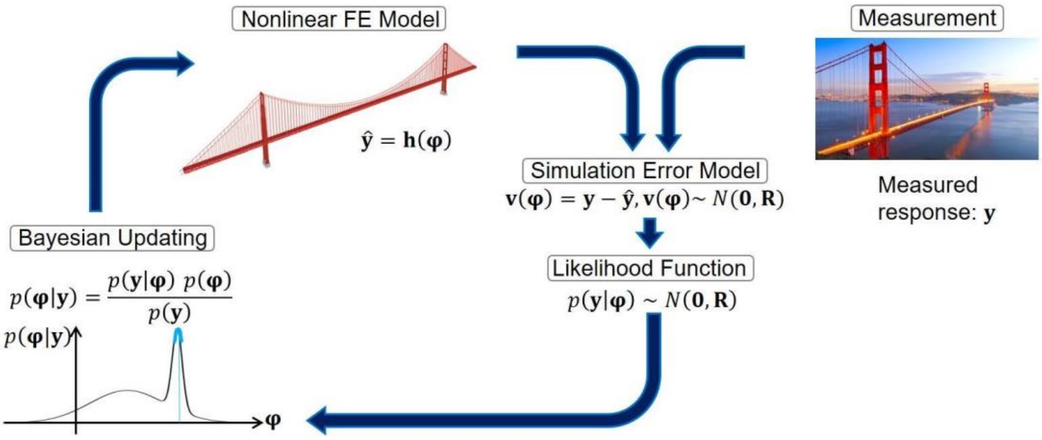

Fig. 12 Schematic representation of the sequential Bayesian inference method for model updating (Ghahari *et al.* 2022)

3.3 Applications of digital twin

Compared with the pure data model and physical model, the interaction and joint action of the two can further improve the performance of the model. Digital twins, which combine physical and data models, are now widely used in bridge monitoring.

3.3.1 Bridge damage identification

The digital twin technology can be used to efficiently and accurately analyze and judge the bridge safety, so as to clarify the location and extent of damages and provide guidance for maintenance. Ye *et al.* (2020) concluded that a data-driven approach based on a large amount of experimental data can improve the accuracy of the digital twin model in identifying various types of performance defects in bridges. They obtained a large number of structural responses of an old bridge by ambient vibration, forced vibration, controlled loading experiments and forced excitation tests. The experimental data were then used to update and calibrate the FE model using a data-driven approach. The results of the damage detection showed that the updated model was able to help the researchers accurately determine the damage and the root cause of the bridge defects. Yu *et al.* (2022b) proposed a methodology combining monitoring data with FE results for fatigue assessment of orthotropic anisotropic steel bridge panels. Firstly, the sub-model method in the digital twin technique combined with the multi-scale simulation technique is used for modelling. Subsequently, the monitoring data were adopted to update the FE model to obtain the full-field stress distribution of the substructure. The fatigue experiment results show that the updated model can determine the distribution characteristics of welded residual stresses in the nodes of U-shaped ribs and top plates of orthotropic anisotropic steel bridge panels as well as their coupling effects with vehicle stresses and temperature stresses.

However, it is impossible to carry out an exhaustive field tests or to install a large number of sensors to monitor all bridges. The low quantity and poor quality of monitoring data is a real problem that must be coped. Since seismic damage detection of bridges using FE models is susceptible to many uncertainties in the environment, Ghahari *et al.* (2022) proposed a

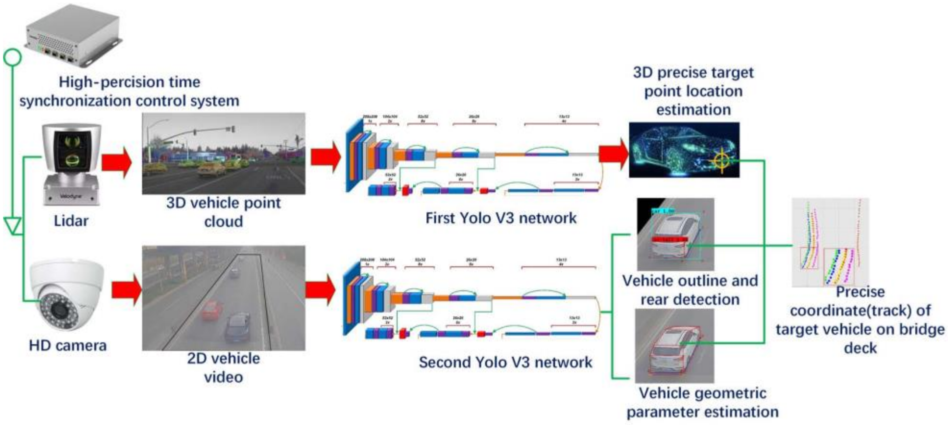

Fig. 13 Multivariate information fusion model (Dan *et al*. 2021)

correction technique based on the sequential Bayesian model. The specific update process is shown in Fig. 12. In this method, the pure seismic response is combined with a mechanics-based FE model of the bridge through a Bayesian inference approach in order to jointly estimate the foundation input motion and unknown model parameters. The performance of the model before and after the update was evaluated by numerical simulation experiments as well as real seismic data. The results show that the updated model, as a digital twin, can accurately identify the local response of bridges under seismic actions. Considering that neural networks can efficiently estimate parameters and the accuracy of a FE model relies on boundary conditions, Park *et al*. (2017) proposed a new technique for evaluating bridge boundary conditions using neural networks. They obtained the rotational spring constant of the bridge by feeding the relationship between the bridge response and the rotational spring constant into the neural network. The rotational spring constant was then used as an aging and constraint effect of the boundary conditions to update the model. Results from laboratory tests and field tests on steel girder bridges show that the method can effectively reduce the uncertainty of boundary conditions in FE model updating.

Due to the development of various types of sensors, research on updating digital twin models using multiple types of data is possible. Dan *et al*. (2021) proposed a digital twin system for bridge clusters in a regional transport infrastructure network, interconnected by measured traffic loads. In the physical space, a traffic load monitoring system based on dynamic weighing and image recognition using neural networks is set up on the target bridges, while lightweight sensors are installed on the bridge clusters to obtain structural response information. The multi-information fusion model for recognizing vehicle information is shown in Fig. 13. By building a mechanical analysis model in the corresponding digital space and using the measured traffic loads as links, the working condition and safety warning of all bridges in the regional traffic network is achieved. Some researchers have also attempted to use advanced image acquisition tools (i.e. 3D point cloud, UAV inspection and computer vision) to obtain changes in structural and material properties to update FE models and assess structural damages (Shu *et al*. 2019, Yoon *et al*. 2022, Lai *et al*. 2022).

3.3.2 Bridge performance prediction

Since the digital twin model is a mapping of the real bridge in the virtual world, the model can be used to accurately predict the various properties of the real bridge. FE models are often inconsistent with the properties of the actual structure, Febrianto *et al.* (2022) proposed a statistical FE model as a numerical twin for strain prediction. The output of a physical model and statistical analysis of real sensor data were performed on the statistical FE model. Experimental results on real bridges show that the model is able to generate reasonable strain distribution predictions at locations where no measurement data are available. The model can also be used to optimize sensor network settings and to make accurate predictions in the event of sensor damage. Jiang *et al.* (2021b) proposed a probabilistic multiscale model as a digital twin to describe the fatigue evolution of steel bridges over their life cycles. The microstructure uses the modified Fine and Bhat model as a baseline model, which is corrected and updated by crystal plasticity FE simulations and historical fatigue data. Cracking at the macro level is done using the Paris law with random expansion parameters. The macroscopic model is also calibrated in real time using Bayesian inference and Markov chain Monte Carlo simulation. The fatigue test results of segmental rigid bridge deck panels show that the model can accurately predict the fatigue onset life and the remaining fatigue life.

Liu *et al.* (2024a) proposed a dual-drive method based on the concept of the digital twin that combines machine learning and FE models with the Global Navigation Satellite System and Earth observation techniques to bridge the gap between the application of these methods for small, medium-sized and long-span bridges. The displacement and dynamic responses of a structure under strong winds are predicted using a pair of Bayesian neural networks and FE models based on genetic algorithms and multi-objective optimization of the response surface methodology. The prediction results show that the Bayesian neural network has advantages in predicting speed while the FE model has better interpretability. The fusion of the two can improve the prediction ability of the digital twin model. Lin *et al.* (2021) proposed a digital twin framework for evaluating the seismic performance of a long-span cable-stayed bridge. The framework uses the design literature to construct physical FE models and performs linear and non-linear model updates. The FE model is validated by shaking table tests on a scaled down model of a large span bridge. By comparing the FE model data with the experimental data, it was found that the model updated nonlinearly provides close predictions of the bridge collapse. Updating FE models using a novel data-driven approach is a viable method for constructing accurate digital twin models in the future.

3.3.3 Hybrid modelling

The joint use of physical and advanced data models is also a viable idea to take full advantage of their respective strengths. Gallup *et al.* (2023) connect purely physical models to neural network models, where these models pass information to each other and work in parallel, or replace some of the physical models with neural networks. The three ways of mixing are named as the substitution, series conjunction, parallel conjunction. The specific structure is shown in Fig. 14. The results show that the performance and generalization ability of these mixed models are considerably better than typical neural networks. However, these models are prone to overfitting and parameter estimation errors due to the lack of regularization.

Bridge data can be obtained not only through real-time transmission from sensors but also through various information databases or information models. The joint use of information models and FE simulations can also improve the accuracy of digital twin models. Marzouk and Hisham (2012) developed a Bridge Information Modeling (BrIM) framework for structural assessment, incorporating a comprehensive database of bridge components, elements, and inspection logs. The framework evaluates bridge conditions by simulating incremental loads to determine the load factor. Corrosion in steel members is modeled within the 3D BrIM environment and subsequently

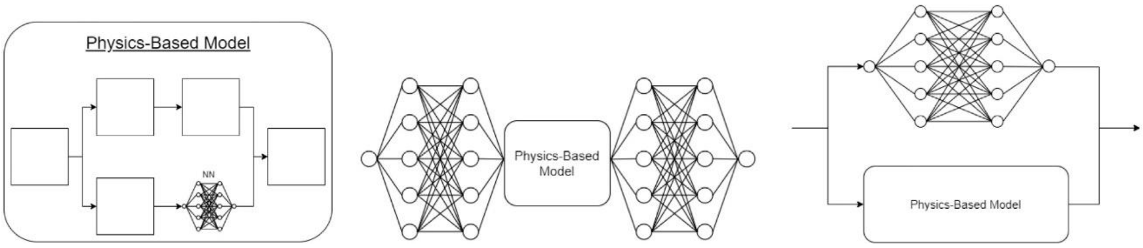

Fig. 14 Structures of the hybrid model (Gallup *et al.* 2023)

exported to the FE model for further analysis and simulation. The results demonstrate that the information model can provide more accurate geometric parameters and material properties for the physical model. Shim *et al.* (2019) introduced a framework for bridge maintenance utilizing a 3D digital twin. This method involves creating a federated model that integrates historical data and the as-built geometry of the bridge. The development process includes bridge alignment, parametric modeling, 3D scanning, and reverse engineering. The federated digital twin model is then connected to an interoperable FE model to simulate and understand the structural behavior under current conditions. The model provides a rich information resource for bridge maintenance and is continuously upgraded by real-time scanning during the bridge life cycle. The data for damage recognition in this framework comes from traditional image recognition methods therefore there may be inaccuracies in the recognition. The use of neural networks for recognition in future applications can effectively improve the accuracy of the model. Sofia *et al.* (2020) proposed a digital twin framework for monitoring and maintaining bridge structures. The framework utilizes dynamic laser scanning to generate point clouds, which are then used to create a information model from geographic information systems data. Internet-of-things technology and sensor data are integrated to feed machine learning algorithms for detecting bridge damages. The digital twin is subsequently modeled and analyzed using this data, facilitating informed decision-making processes.

4. Discussions

The successful applications of PGNN methods in BHM prove its potential. However, the research of the PGNN methods is far from fruitful. This section discusses the challenges and prospects of PGNN methods in the field of BHM.

When physical information is integrated into the loss function, the loss function becomes more complicated. This would dramatically increase the time, computational complexity, and computational resources during neural network training when solving multi-scale and multi-physics field problems. The physics-guided loss function still follows the architecture of conventional neural networks, which makes the neural network unable to be interpretable and restricts the application in large-scale or complex BHM problems. To improve computational efficiency, reduce hardware requirements, and enhance interpretability, it becomes necessary to develop novel training strategies, new network architectures, and employ other excellent methods such as quantum machine learning (Farhi and Neven 2018), symbolic regression (Udrescu and Tegmark 2020) and Kolmogorov-Arnold Networks (Liu *et al.* 2024c).

Although the use of physical models allows for a large amount of simulation data, the reliance on high quality and large amounts of real monitoring data during neural network training is not fully relieved. In real application, data problems such as noise, loss and insufficiency can still seriously affect the accuracy of trained neural networks even if the physical data enhancement is adopted. At the same time, if the physical model differs greatly from the real bridge and the simulated data cannot be consistent with monitoring data, the difficulty of subsequent training may increase. Developing effective methods and criteria to calibrate physical models and select monitored data are an effective approach to improve data quality. Designing high-performance sensors or adding sensors to obtain more bridge monitoring data is also an essential step.

The interaction between physical and data models using the digital twin technology could promote the deep fusion of physical information and artificial neural networks. However, accurately establishing a physical model and a data model for a real civil structure remains a challenge due to its non-uniformity, complexity, load uncertainty, and instability of constraints. Real time data transfer between physical models and data models has not yet been achieved. The development of more efficient, fidelity and intelligent physical and data models or new data frameworks with digital twin characteristics can promote the use of the digital twin in BHM. New algorithms and computer hardware could be adopted to speed up data transmission between physical and data models to achieve real-time analysis, calculation, evaluation, and visualization of operation bridges.

5. Conclusions

This study aims to provide a comprehensive review of the application of PGNNs for BHM. The concept of PGNNs is clarified and classifies into three categories: physics-guided loss functions, physical data enhancement, and digital twins. The idea and characteristics of each category are described in detail. The applications of the three types of PGNNs in bridge damage identification, bridge response prediction and bridge performance prediction are summarized. The limitations, challenges and potential research directions for PGNN are discussed.

It should be noted that omissions and bias could not be avoided for such a daunting task because of preference and limitations of authors.

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