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Time harmonic interactions in a nonlocal isotropic thermoelastic thick circular plate without energy dissipation

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Abstract. The research paper is devoted to study of the thermomechanical deformations occurring in a nonlocal homogeneous isotropic thick circular plate with frequency domain and without energy dissipation. The upper and lower surfaces of the thick circular plate are traction free subjected to axisymmetric heat supply. Hankel transform has been used to find the analytical solutions. The expressions for physical quantities such as displacement components, stress components and conductive temperature have been obtained in the transformed domain. The resulting quantities in the physical domain have been obtained by using the numerical inversion technique. The numerical simulated results have been depicted graphically to study the effect of nonlocality and two temperature on the components of displacement, stress components and conductive temperature.

Keywords: axisymmetric; displacement components; frequency domain; Hankel transform; nonlocality; stress components; thermoelastic; thick circular plate; two temperature

1. Introduction

Some well-known research work has been done to study the concept of nonlocality in various thermoelastic bodies by different researchers. As we are already aware that the effect of nonlocality is concerned with the dependence of the stress components of a material not just due to the strain at that single point only but due to strain at all the points of the body. Two temperature thermoelasticity theory is concerned about the dependence of heat conduction upon two distinct temperature known as conductive temperature and thermodynamical temperature.

The theory of nonlocal elasticity was established due to Edelen *et al.* (1971), Edelen and Laws (1971) and Eringen and Edelen (1972). The theory of nonlocal elasticity was given and the constitutive equations were derived due to their combined efforts. GN theories of type I, II and III were given by Green and Naghdi (1992, 1993). GN-I theory of thermoelasticity corresponds to the classical thermoelastic model. While, GN-II admits undamped oscillations in thermoelastic waves and thus is famous as theory of thermoelasticity without energy dissipation. GN-III considers both theories as a special case and allows dissipation of energy in general.

Chen and Gurtin (1968) and Chen *et al.* (1969) formulated a theory of heat conduction in

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deformable bodies which depends upon two distinct temperatures i.e., conductive temperature φ and thermodynamical temperature T. Youssef (2006) and Youssef and Al-Lehaibi (2007) gave generalized two temperature theory of thermoelasticity.

Marin (1996) gave the generalized solutions in elasticity of micropolar bodies with voids. Marin (1997) discussed the effects on the domain of influence in thermoelastic bodies with voids. Mohamed *et al.* (2009) analysed the hydromagnetic flow and heat transfer of a heat generation fluid over a surface embedded in a non-Darcian porous medium.

Abo-Dahab and Abbas (2011) gave LS model on thermal shock problem of generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity. Fabrizio *et al.* (2011) derived the expressions of internal powers from the power balance laws for some non-local materials. Zenkour and Abbas (2013) studied the magneto-thermoelastic response of an infinite functionally graded cylinder using the finite element method. Hu *et al.* (2013) introduced a new theoretical framework named generalized multi-symplectic integrator for a class of nonlinear wave PDEs based on the multi-symplectic idea.

Abbas (2014a) derived a GN model based upon two-temperature generalized thermoelastic theory in an unbounded medium with a spherical cavity while Abbas (2014b) gave fractional order GN model on thermoelastic interaction in an infinite fibre-reinforced anisotropic plate containing a circular hole. Kumar *et al.* (2015) studied the axisymmetric distributions of a thick thermoelastic circular plate in the context of the theories of thermodiffusion elastic solid with one and two relaxation time.

Abbas *et al.* (2015) investigated the wave propagation of generalized thermoelastic diffusion on a traction free half space with energy dissipation. Abbas *et al.* (2016) studied the propagation of wave in a generalized thermoelastic plate using eigenvalue approach. Abouelregal and Zenkour (2017) used the theory of two-temperature generalized thermoelasticity with phase lags to solve the problem of micropolar generalized thermoelasticity for a traction-free half-space.

El-Nabulsi (2018) studied non-equilibrium thermodynamics and heat diffusion processes based on Suykens's nonlocal-in-time kinetic energy approach. Sellitto and Di Domenico (2019) discussed and studied the nonlocal and nonlinear effects in the propagation of thermal and elastic high-frequency waves in nanosystems in case of both non-rigid and rigid bodies.

Othman *et al.* (2019) used the three-phase-lag model along with GN-II and GN-III to study the influence of the gravity field on a thermoelastic medium with two temperature. Lata and Singh (2019) studied the effect of nonlocality and two temperature on a nonlocal thermoelastic solid subjected to an inclined load.

Lata and Singh (2020) investigated the deformations in a nonlocal thermoelastic solid with two temperatures due to time harmonic sources. Saeed *et al.* (2020) studied the GL model on thermo-elastic interaction in a poroelastic material using finite element method. Belmahi *et al.* (2019), Benahmed *et al.* (2019), Gao *et al.* (2019), Hosseini (2020), Hussain *et al.* (2020) and Liani *et al.* (2022) studied and discussed the concept of nonlocality in different thermoelastic mediums and proved various results to prove the nonlocal effects.

Zine *et al.* (2020) analysed the bending of functionally graded porous plates via a refined shear deformation theory. Hu *et al.* (2020) obtained the internal resonance conditions of the spatial flexible beam suspended by two springs in a spatial on-orbit tethered system based on the method of multiple scales. Hu *et al.* (2020) studied the vibration and wave propagation characteristics in the spatial flexible damping panel attached to four special springs using the complex structure-preserving approach.

Hu *et al.* (2021a) proved that the coupling effects between the deformation of the beam, the active stretching of the beam and the rotation of the hub are reflected in the stretching stage and the transverse vibration of the beam is enhanced by the stretching effect of the beam. Hu *et al.* (2021b) established a mechanoelectrical flexible hub-beam model of ionic-type solvent-free nanofluids on the basis of microscopic mechanisms.

Lata and Singh (2021a) investigated the propagation of Stoneley waves at the interface of two nonlocal magneto-thermoelastic media and derived the corresponding secular equations. Lata and Singh (2021b) studied the thermomechanical interactions in a homogeneous nonlocal magneto-thermoelastic rotating medium under the effect of hall current and two temperature with memory dependent derivatives. Lata and Singh (2022) studied the axisymmetric deformations in a nonlocal isotropic thermoelastic solid with two temperature. Hu *et al.* (2022) investigated and studied the dynamic response of the SMS modeled by a flexible hub-beam with a step-variable cross-section excited by an external torque by employing a new complex structure-preserving method.

Hu *et al.* (2023a) studied the FORQ equation from the viewpoint of multi-symplectic structures. Hu *et al.* (2023b) studied the effects of the crack's parameters, the axially moving speed and the circular frequency of the transverse harmonic load on the steady vibration amplitude of the cracked beam by the structure-preserving method. Huai *et al.* (2023) derived the dynamic equations of the flexible magnetic hub-beam model subjected to the external magnetic field force and the viscous resistance in the Lagrangian framework.

Abouelregal *et al.* (2023) studied memory-dependent dynamic response for a thermo-piezoelectric functionally graded rotating rod. Singh (2023) discussed the effects of two temperature and nonlocality in an isotropic thermoelastic thick circular plate without energy dissipation. Yadav *et al.* (2024) studied the reflection of hygrothermal waves in nonlocal theory of coupled thermo-elasticity.

In this paper, a two-dimensional axisymmetric problem in a thick circular plate made up of a nonlocal homogeneous isotropic thermoelastic solid with two temperature and without energy dissipation under time harmonic conditions has been studied. The effects of nonlocal parameter and frequency have been depicted graphically to study the variations in various physical quantities such as the components of stresses and displacements subjected to different values of two temperature parameter. The results obtained will be helpful in the various research fields and might be useful for extension works.

2. Basic equations

Following Eringen (2002), Green and Naghdi (1993) and Youssef (2006) the equations of motion, heat conduction equation and the constitutive relations in a nonlocal homogeneous isotropic thermoelastic solid with two temperatures and without energy dissipation are given by

$$(\lambda + 2\mu)\nabla(\nabla.\boldsymbol{u}) - \mu(\nabla \times \nabla \times \boldsymbol{u}) - \beta\nabla T = (1 - \epsilon^2\nabla^2)\rho\frac{\partial^2\boldsymbol{u}}{\partial t^2}, \tag{1}$$

$$K^*\nabla^2\varphi = \rho\, C^*\frac{\partial^2\boldsymbol{T}}{\partial t^2} + \beta T_0\frac{\partial^2}{\partial t^2}(\nabla.\boldsymbol{u}), \tag{2}$$

where

$$T = (1 - a\nabla^2)\varphi. \tag{3}$$

$$(1-\epsilon^2\nabla^2)t_{ij}=\lambda \boldsymbol{u}_{k,k}\delta_{ij}+\mu\left(\boldsymbol{u}_{i,j}+\boldsymbol{u}_{j,i}\right)-\beta T\delta_{ij}, \tag{4}$$

where λ, μ are material constants, ϵ is the nonlocal parameter, ρ is the mass density, $\boldsymbol{u}$ is the displacement vector, φ is the conductive temperature, a is two temperature parameter, T is absolute temperature and T_0 is reference temperature, K^* is a materialistic constant, C^* the specific heat at constant strain, $\beta=(3\lambda+2\mu)\alpha$ where α is coefficient of linear thermal expansion, δ_{ij} is the Kronecker delta, t_{ij} are the components of stress tensor.

3. Formulation and solution of the problem

We consider a nonlocal homogeneous isotropic thermoelastic thick plate of thickeness $2d$ occupying the region defined by $0\leq r\leq\infty, -d\leq z\leq d$, which is initially at uniform temperature T_0. We take a cylindrical polar co-ordinate system (r,θ,z) with symmetry about z-axis with origin on the surface $z=0$, the heat flux $g_0F(r,z)$ is prescribed on the upper and lower boundary surfaces of the plate. For two-dimensional problem, we consider

$$\boldsymbol{u}=(u,0,w). \tag{5}$$

Using Eq. (5) in Eq. (1) and Eq. (2), yields

$$(\lambda+\mu)\frac{\partial e}{\partial r}+\mu\left(\nabla^2-\frac{1}{r^2}\right)u-\beta\frac{\partial T}{\partial r}=\rho(1-\epsilon^2\nabla^2)\frac{\partial^2 u}{\partial t^2}, \tag{6}$$

$$(\lambda+\mu)\frac{\partial e}{\partial z}+\mu\nabla^2 w-\beta\frac{\partial T}{\partial z}=\rho(1-\epsilon^2\nabla^2)\frac{\partial^2 w}{\partial t^2}, \tag{7}$$

$$K^*\nabla^2\varphi=\rho C^*\frac{\partial^2 T}{\partial t^2}+\beta T_0\frac{\partial^2 e}{\partial t^2}. \tag{8}$$

where

$$T=\left[1-a\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}\right)\right]\varphi. \tag{9}$$

Constitutive relations are

$$\begin{aligned}(1-\epsilon^2\nabla^2)t_{rr}&=\lambda e+2\mu e_{rr}-\beta T,\\ (1-\epsilon^2\nabla^2)t_{zr}&=2\mu e_{zr},\\ (1-\epsilon^2\nabla^2)t_{zz}&=\lambda e+2\mu e_{zz}-\beta T.\end{aligned} \tag{10}$$

where

$$e=e_{rr}+e_{\theta\theta}+e_{zz},\ e_{rr}=\frac{\partial u}{\partial r},\ e_{\theta\theta}=\frac{u}{r},\ e_{zz}=\frac{\partial w}{\partial z},\ e_{rz}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right),\ \nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}.$$

We introduce the following dimensionless quantities

$$(r',z',u',w')=\frac{\omega_1}{c_1}(r,z,u,w),\ t'_{ij}=\frac{t_{ij}}{\beta T_0},\ t'=\omega_1 t,\ a'=\frac{\omega_1}{c_1}a,\ T'=\frac{\beta}{\rho c_1^2}T. \tag{11}$$

where, $c_1^2=\frac{\lambda+2\mu}{\rho}$ and $\omega_1=\frac{\rho C^* c_1^2}{K^*}$.

Upon introducing Eq. (11) in Eqs. (6)-(8) and after suppressing the primes, we obtain

$$\left(\frac{\lambda+\mu}{\rho c_1^2}\right)\frac{\partial e}{\partial r}+\frac{\mu}{\rho c_1^2}\left(\nabla^2-\frac{1}{r^2}\right)u-\frac{\partial T}{\partial r}=(1-\epsilon^2\nabla^2)\frac{\partial^2 u}{\partial t^2}, \tag{12}$$

$$\left(\frac{\lambda+\mu}{\rho c_1^2}\right)\frac{\partial e}{\partial z}+\frac{\mu}{\rho c_1^2}\nabla^2 w-\frac{\partial T}{\partial z}=(1-\epsilon^2\nabla^2)\frac{\partial^2 w}{\partial t^2}, \tag{13}$$

$$\nabla^2\varphi=\rho C^*\frac{\partial^2 T}{\partial t^2}+\beta T_0\frac{\partial^2 e}{\partial t^2}. \tag{14}$$

The potential functions ψ_1 and ψ_2 in dimensionless form are defined as follows

$$u=\frac{\partial\psi_1}{\partial r}+\frac{\partial\psi_2}{\partial r\partial z},\ w=\frac{\partial\psi_1}{\partial z}-\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\psi_2. \tag{15}$$

Assuming the harmonic behavior as

$$(\psi_1,\psi_3,\varphi,e,T)(r,z,t)=(\psi_1,\psi_3,\varphi,e,T)(r,z)e^{i\omega t}, \tag{16}$$

where ω is the angular frequency.

Hankel Transform is defined as

$$\hat{f}(\xi,z,t)=\int_{-\infty}^{\infty}\bar{f}(r,z,t)r\,\mathrm{J}_n(r\xi)dr. \tag{17}$$

Upon introducing the quantities defined by Eq. (15) in Eqs. (12)-(14) and applying Eqs. (16)-(17) in the resulting equations, after some simplifications, we obtain

$$[AD^4+\mathrm{B}D^2+C](\widehat{\psi_1},\hat{\varphi})=0, \tag{18}$$

$$[D^2+E]\widehat{\psi_2}=0, \tag{19}$$

where

$$A=\zeta_2\zeta_5+a\zeta_6,\ B=\zeta_3\zeta_5+\zeta_2\zeta_7-\zeta_4\zeta_6-a\xi^2\zeta_6,\ C=\zeta_3\zeta_7+\zeta_4\zeta_6\xi^2, E=\frac{\zeta_8}{\zeta_9},\ D=\frac{d}{dz}.$$

$$\zeta_1=\omega^2(1+\epsilon^2\xi^2)+\xi^2,\ \zeta_2=\omega^2\epsilon^2\ \ \zeta_3=\xi^2-\zeta_1,\ \ \zeta_4=1+a\xi^2,\ \zeta_5=1-a\omega^2,\ \zeta_6=a_1\omega^2,\ \zeta_7=\zeta_4\omega^2-\xi^2,\ \zeta_8=\rho{c_1}^2\zeta_1-\mu\xi^2,\ \zeta_9=\mu-\rho{c_1}^2\zeta_2,\ a_1=\frac{\beta^2T_0}{K^*\rho\omega_1}.$$

The solutions of Eq. (18) can be written as

$$\widehat{\psi_1}=\sum_{i=1}^{2}A_i\cosh(m_iz), \tag{20}$$

$$\hat{\varphi}=\sum_{i=1}^{2}R_iA_i\cosh(m_iz). \tag{21}$$

where m_1 and m_2 are roots of the Eq. (18).

The solutions of Eq. (19) can be written as

$$\widehat{\psi_2}=A_3\sinh(m_3z), \tag{22}$$

where

$$R_i=\frac{\zeta_7+\zeta_5{m_i}^2}{\zeta_3+\zeta_2{m_i}^2},\ i=1,2. \tag{23}$$

4. Boundary conditions and applications

The appropriate boundary conditions for the problem are:

(1) We assume that the upper and lower boundary of the planes of the plate are subjected to axismmetric temperature distribution, mathematically these can be written as

$$\frac{\partial\varphi}{\partial z} = \pm g_0 F(r,z) \text{ at } z = \pm d. \tag{24}$$

(2) Both the upper and lower boundary of the planes of the plate are traction free, thus the mechanical boundary conditions are

$$t_{zz} = t_{zr} = 0 \text{ at } z = \pm d. \tag{25}$$

where

$$F(r,z) = z^2 e^{-\omega r}. \tag{26}$$

Using non-dimensional quantities as defined by Eq. (11) and applying Eqs. (16) and (17) on Eqs. (24)-(26), we get

$$F(\xi,z) = \frac{z^2\omega}{(\omega^2+\xi^2)^{\frac{3}{2}}}. \tag{27}$$

Using non-dimensional quantities as defined by Eq. (11) and applying Eqs. (16) and (17) on Eq. (10) and solving, we get the components of displacement, stress and conductive temperature as

$$\hat{u} = \frac{\xi}{\Delta}\left\{\pm g_0 \hat{F}(\xi,z)(\Delta_{11}\nu_1 + \Delta_{12}\nu_2 + m_3\Delta_{13}\nu_3)\right\}, \tag{28}$$

$$\widehat{w} = \frac{1}{\Delta}\left\{\pm g_0 \hat{F}(\xi,z)(m_1\Delta_{11}\theta_1 + m_2\Delta_{12}\theta_2 + \xi^2\Delta_{13}\theta_3)\right\}, \tag{29}$$

$$\hat{\varphi} = \frac{1}{\Delta}\left\{\pm g_0 \hat{F}(\xi,z)(R_1\Delta_{11}\nu_1 + R_2\Delta_{12}\nu_2)\right\}, \tag{30}$$

$$\widehat{t_{zz}} = \frac{1}{\Delta}\left\{\pm g_0 \hat{F}(\xi,z)(k_1\Delta_{11}\nu_1 + k_2\Delta_{12}\nu_2 + k_3\Delta_{13}\nu_3)\right\}, \tag{31}$$

$$\widehat{t_{zr}} = \frac{\mu}{\Delta}\left\{\pm g_0 \hat{F}(\xi,z)(2m_1\Delta_{11}\theta_1 + 2m_2\Delta_{12}\theta_2 + ({m_3}^2 - 1)\Delta_{13}\theta_3)\right\}, \tag{32}$$

$$\widehat{t_{rr}} = \frac{1}{\Delta}\left\{\pm g_0 \hat{F}(\xi,z)(l_1\Delta_{11}\nu_1 + l_2\Delta_{12}\nu_2 + l_3\Delta_{13}\nu_3)\right\}, \tag{33}$$

$$\Delta = R_1\Delta_{11}m_1 + R_2\Delta_{12}m_2. \tag{34}$$

where

$$\mathrm{k}_1 = -\lambda\xi^2 + \rho {c_1}^2[({m_1}^2 - 1) + a({m_1}^2 - \xi^2)],\ \mathrm{k}_2 = -\lambda\xi^2 + \rho {c_1}^2[({m_2}^2 - 1) + a({m_2}^2 - \xi^2)],\ \mathrm{k}_3 = 2\mu\xi^2 m_3,\ \nu_i = \cosh(m_i z),\ \theta_i = \sinh(m_i z),\ i = 1,2,3.$$

$$\Delta_{11} = k_2\nu_2({m_3}^2 - 1)\theta_3 - 2k_3\nu_3 m_1\theta_1,\ \Delta_{12} = 2k_3\nu_3 m_1\theta_1 - k_1\nu_1({m_3}^2 - 1)\theta_3, \Delta_{13} = 2k_1\nu_1 m_2\theta_2 - 2k_2\nu_2 m_1\theta_1.$$

$$\Delta_{21} = m_2R_2({m_3}^2 - 1)\theta_3,\ \Delta_{22} = m_1R_1({m_3}^2 - 1)\theta_3, \Delta_{23} = 2m_1m_2(\theta_2R_1 - R_2\theta_1).$$

$$l_1 = \rho {c_1}^2\xi^2 + \lambda {m_1}^2 - \rho {c_1}^2(1 + a\xi^2 - a{m_1}^2)R_1,\ l_2 = \rho {c_1}^2\xi^2 + \lambda {m_2}^2 - \rho {c_1}^2(1 + a\xi^2 - a{m_2}^2)R_2,\ l_3 = m_3\xi^2(\rho {c_1}^2 - \lambda).$$

5. Particular cases

- If $a = 0$, then from Eqs. (28)-(34), we obtain the corresponding expressions for displacements, stresses and conductive temperature for nonlocal isotropic solid without energy dissipation.
- If $\epsilon = 0$, then from Eqs. (28)-(34), the corresponding expressions for displacements, stresses, and conductive temperature for local isotropic solid with two temperature are obtained.
- If $\epsilon = a = 0$, then from Eqs. (28)-(34), the corresponding expressions for displacements, stresses, and conductive temperature for local isotropic solid without two temperature are obtained.

6. Inversion of the transforms

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (28)-(34). Here the displacement components, stress components and conductive temperature are functions of z and the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $\bar{f}(\xi, z, s)$. To obtain the function $f(x, z, t)$ in the physical domain, we invert the Hankel transform using

$$\bar{f}(r, z, s) = \int_0^\infty \xi\, \hat{f}(\xi, z, s) J_n(\xi r) d\xi. \tag{35}$$

Now for fixed values of ξ, z and r, $\hat{f}(r, z, s)$ in above expression can be considered as the Laplace transform $\hat{g}(s)$ of $g(t)$. Following (Honig and Hirdes 1984), the Laplace transform function can be inverted.

The Last step is to calculate the integral in Eq. (35). The method for evaluating this integral is described in Press *et al.* (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7. Numerical results and discussion

Following Dhaliwal and Singh (1980), Magnesium material is chosen for the purpose of numerical calculation, the numerical data for which is given as

$$\lambda = 9.4 \times 10^{10} Nm^{-2},\ \mu = 3.278 \times 10^{10} Nm^{-2}, K^* = 1.7 \times 10^2 Wm^{-1}K^{-1},\ \rho = 1.74 \times 10^3 Kgm^{-3}, \theta_0 = 298\ K, C^* = 10.4 \times 10^2 JKg^{-1}deg^{-1}, \omega_1 = 3.58,\ a = 0.05.$$

A comparison of values of displacement components u and w, stress components $t_{zz},\ t_{rr},\ t_{zr}$ and conductive temperature φ for an isotropic thermoelasic solid with distance r has been made for the nonlocal parameter ($\epsilon = 0$ and $\epsilon = 0.1$) and non-dimensional frequencies ($\omega = 0.1$ and $\omega = 0.2$).

1) The solid purple colored line with center symbol square corresponds to local parameter $\epsilon = 0$

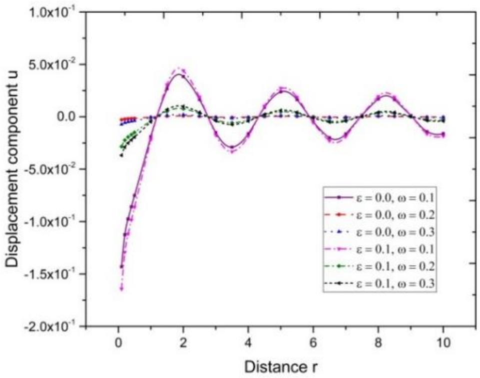

Fig. 1 Variation of displacement component u with distance r

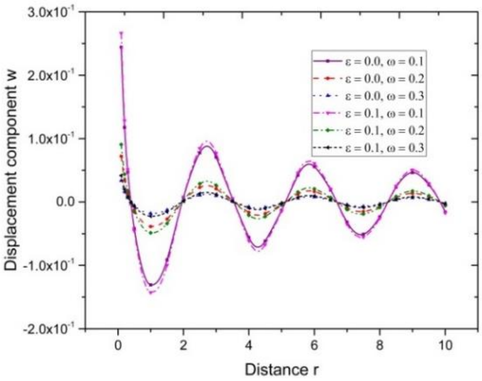

Fig. 2 Variation of displacement component w with distance r

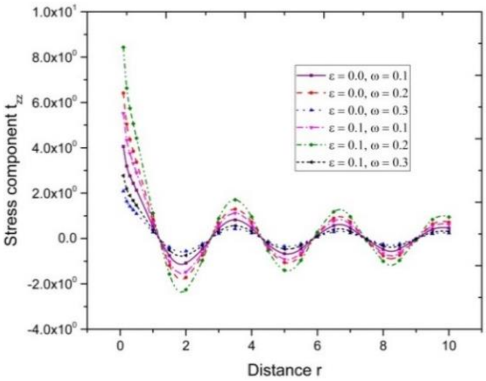

Fig. 3 Variation of stress component t_{zz} with distance r

and frequency $\omega = 0.1$.

2) The dashed red blue colored line with center symbol circle corresponds to local parameter $\epsilon = 0$ and frequency $\omega = 0.2$.

3) The dotted blue colored line with center symbol upward triangle corresponds to nonlocal parameter $\epsilon = 0$ and frequency $\omega = 0.3$.

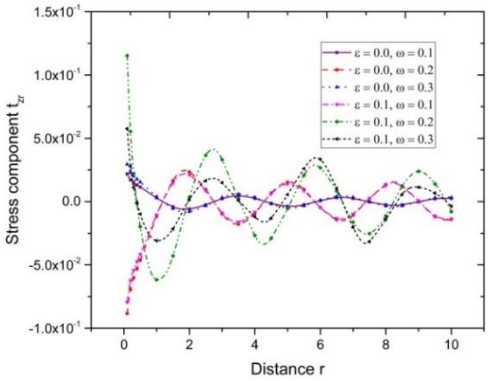

Fig. 4 Variation of stress component t_{zr} with distance *r*

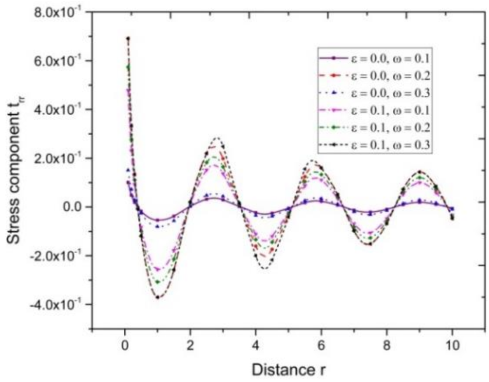

Fig. 5 Variation of stress component t_{rr} with distance *r*

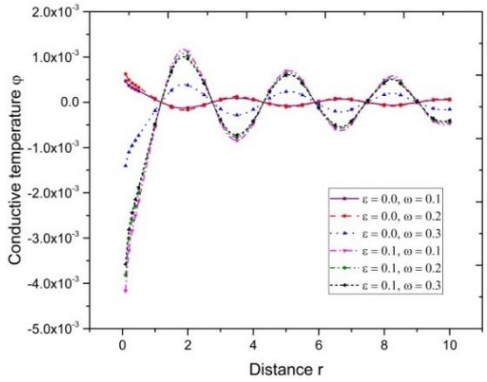

Fig. 6 Variation of conductive temperature φ with distance *r*

4) The dash-dotted magenta colored line with center symbol downward triangle corresponds to nonlocal parameter $\epsilon = 0.1$ and frequency $\omega = 0.1$.
5) The dash-dot-dot olive colored line with center symbol tilted square corresponds to nonlocal parameter $\epsilon = 0.1$ and frequency $\omega = 0.2$.
6) The short dash black colored line with center symbol left triangle corresponds to nonlocal

parameter $\epsilon = 0.1$ and frequency $\omega = 0.3$.

In Fig. 1, the variations of displacement component u have been studied with the effects of nonlocal parameter and frequency with respect to displacement r. It is clear that the local and nonlocal effects are there with changes due to frequency. While for both local and nonlocal parameters the magnitude of variations decrease with increase in frequency but the varaiations are higher for local parameter.

In Fig. 2, the variations of displacement component w have been studied. The change in magnitude is highly oscillatory in nature with difference in variations due to local and nonlocal parameter. The magnitude of variations decrease with increase in frequency and the difference is clear for local and nonlocal parameters.

From Fig. 3, the variations of stress component t_{zz} with respect to displacement r have ben studied. The variations due to local and nonlocal effects are there with the effect of frequency domain. The path followed is oscillatory in nature. Also, the variation of oscillations is comparatively more in magnitude with respect to nonlocal parameter. The magnitude of oscillations decrease with increase in frequency for nonlocal parameter while trend is reversed for local parameter.

The variations of stress component t_{zr} with respect to displacement r have been studied in Fig. 4. The variations in magnitude are more for nonlocal parameter. The magnitude of oscillations follows an oscillatory path. In case of frequency, the variations first increase and then decreases for both parameters but the magnitude is less for local parameter comparatively.

Fig. 5 shows the variations of stress component t_{rr} with respect to displacement r. As per the trend, the variations in magnitude follow an oscillatory path. The variations increase with increase in frequency in case of nonlocal parameter while for local parameter the variations increase and then decrease.

Fig. 6 shows the variations of conductive temperature with respect to distance r. The effect of local and nonlocal parameters is visibly clear. The path followed is oscillatory with a sharp increase during initial values of r for local parameter. The variations increase with the introduction of nonlocal parameter. In case of nonlocal parameter, the variations decrease slightly in magnitude with the increase in frequency but in case of local parameter, the difference is less and pattern is different for initial frequencies and becomes similar to nonlocal parameter for frequency $\omega = 0.3$ with comparatively lesser magnitude values.

8. Conclusions

From above investigation, it is found that there is a considerable difference in the deformation of nonlocal isotropic thermoelastic thick circular plate while considering the effect of presence and absence of nonlocal parameter with variations frequency. The variation of r from point of application causes the components of displacements, stresses and conductive temperature to follow an oscillatory pattern. The nonlocal parameter affects the magnitude of variation more as compared to local parameter in most of the cases. Also, in case of nonlocal parameter, the trend followed can be generalised but in case of local parameter in some graphs the pattern followed is not conclusive as it varies a lot with sudden changes. The results of this problem can be very useful for the researchers working in the field of nonlocal theory of thermoelasticity. The results can be further extended beyond the without energy dissipation concept. There might be some useful applications in the fields of geophysics, seismology and various other industries too.

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