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The effects of thermal relaxation times in thermo-viscoelastic tissues during hyperthermia treatment

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Abstract. The paper is a study on the biothermoelastic analysis in viscoelastic biological tissues in the presence of thermal relaxation times. Using Laplace transforms and related methodologies, we explore how living tissue responds to an exponentially decaying pulse of heat flux at the boundary. The Laplace transformations are reversed using the numerical method. The Tzuo technique was used to measure the reversal. Temperature, displacement, and stress distributions are affected by single-phase and delay relaxation coefficients as well as volume rheological factors, are provided with numerical findings and graphically depicted. In addition, we carry out a parametric analysis to provide assistance in choosing the design variables that are the most successful, which finally results in an improvement in the accuracy of hyperthermia treatments.

Keywords: bio-thermo-viscoelastic model; biological tissue; Laplace transform; thermal relaxation times; thermo-mechanical interactions

1. Introduction

In the modern world, one of the main causes of death is cancer. It is a class of disorders in which an aberrant population of cells grows out of control against the rules of normal cell division. Signals that control whether cells will divide, die, or differentiate into various cells are frequently used to regulate natural cells. Since cancer cells can reproduce themselves, they grow and proliferate uncontrollably. A subsequent mutation (changed DNA sequence) causes an additional aberrant phase, which is followed by the unchecked proliferation of abnormal cells. Tumors must develop to be recognized and treated early because it takes a long time to form and spread. Thermal ablation, one of the least invasive cancer treatments, kills malignant tissue or restores normal function through a change in temperature without causing damage to other vital tissues. Extremely high temperatures expedite the formation of the desired tissue, whereas low temperatures necessitate extended treatment durations. Thermal ablation produces significant tissue ablation and a successful outcome at temperatures ranging from 46 to 56 °C for 2 to 6 minutes.

Analysis of thermal transmission and skin tissue damage is crucial and can aid in clinical treatments. Materials with viscoelasticity display both elastic and viscous characteristics when deformed. Ilioushin and Pobedria's book (Ilioushin and Pobedria 1970) includes an explanation of the thermal viscoelasticity theory. Rheological materials called linear viscoelastics show load-

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dependent time-temperature dependence. All biological tissues have mechanical viscoelastic qualities, which are crucial to their distinctive activities. This is due to the presence of viscoelastic components in structural proteins, extracellular matrices, and tissue cells. Viscoelasticity has been demonstrated in even hard tissue. The bio-heat transfer equation developed by Pennes (1948) explains the heated conduct that is based on the original Fourier's law. Fourier's law, as is widely known, predicts infinitely accelerated thermal signal transmission, which is obviously at odds with the way things work. Furthermore, due to the substantial internal inhomogeneity of living biological tissues, heat nevertheless spreads through them at a finite rate. However, Luikov *et al.* (1968) and Kaminski (1990) have demonstrated in many published investigations that warm conduct in non-homogeneous media demands a longer period of relaxation to build up enough warm energy to go to the adjacent section. The first is attributed to Cattaneo (1958), who established a wave-type heat condition by suggesting another law of heat conduction to replace the classic Fourier law. For the bloodstream's moderating influence, Pennes added a medium reaction term to the fundamental heat condition. Heat movement via processes other than diffusion is referred to as convective heat transport. Among a few papers devoted to the application of Cattaneo theory to Pennes' bioheat move condition. By creating a novel understanding of multi-mode energy coupling, Liu and Bilston (2000) looked into the mechanism underlying wave-like heat transport behaviors in biological tissues. Shih *et al.* (2007) investigated the influence of sinusoidal heat flux at the skin on the temperature response of a semi-infinite biological tissue.

To calculate the thermal damage and temperature in live tissue brought on by laser irradiation, Alzahrani and Abbas (2019) provided an analytical method using Laplace transformation, experimental temperature data, and a sequential concept across time. One of the earliest innovations in Fox's (Fox 1969) second thermodynamic sound theory may be seen in the way the generalized theory was constructed to reflect the finite speed in thermal signal propagation known as the second sound effect. Furthermore, The two well-known and extensively researched generalized thermocouple theories were developed by Lord and Shulman (1967) and Green and Lindsay (1972). Ezzat and El-Karamany (2002) demonstrate the uniqueness of theorems for generalized thermo-viscoelasticity under various situations. Ezzat *et al.* (2001) developed a model of the equation of generalized thermo-viscoelasticity with one relaxation time and used a state-space approach to solve a one-dimensional half-space thermal shock issue with or without heat sources Ezzat (2008). The generalized thermoelasticity theory was expanded by Othman *et al.* (2002) to include the Green and Lindsay theory, a two-dimensional generalized thermo-viscoelasticity model with two relaxation durations. The precise equations for temperature distribution, thermal stresses, and displacement components are computed using the normal mode analysis. Additionally, a viscoelastic half-space problem in one and two dimensions is investigated. Some difficulties in this subject were handled by Abbas and Youssef (2012), Marin (2010), Marin *et al.* (2017), Zenkour *et al.* (2015), Ezzat and Youssef (2010), Abbas and Marin (2017), Abbas and Marin (2018), Hobiny and Abbas (2017, 2018), Hobiny *et al.* (2020), El-Bary *et al.* (2019), Youssef and El-Bary (2016). Several researchers have solved the various causes of linear and nonlinear thermoelasticity and their solutions (Xu *et al.* 2009, Zenkour and Abbas 2014, Abbas 2015, Abbas and Kumar 2016, Ezzat *et al.* 2016, Lotfy *et al.* 2019, Yasein *et al.* 2019, Ezzat 2020, Ezzat 2020, Ezzat 2021, Ezzat 2021, Ezzat 2021, Hobiny and Abbas 2021, Ezzat and Lewis 2022, Lata 2022, Zhang *et al.* 2022, Ahmed *et al.* 2023, Ezzat 2023, Ezzat and Alabdulhadi 2023, Fahmy and Almehmadi 2023, Lata and Kaur 2023, Singh and Lata 2023).

The aim of this work is to quantitatively analyze the impact of temperature on biological tissue using the SPL model. Research investigates the dynamic bio-thermo-viscoelastic reactions of live skin tissue to various heat-loading scenarios. The thermo-viscoelasticity hypothesis involves considering the relaxing effects of volume. The reversal is determined using the Tzuo (2014) procedure, and the Laplace transformation approach is employed. For various theories, the effects of changing the thermal material characteristics and the relaxation period on temperature, displacement, and thermal stress are discussed and graphically represented.

2. Basic equations

According to Lord-Shulman (Lord and Shulman 1967), The thermo-viscoelasticity theory's governing equations are based on the heat conduction model of Pennes' (Pennes 1948) 1- Motion's equation

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} \tag{1}$$

2-Stress-displacement-temperature relations

$$\sigma_{ij} = \mu\left(u_{i,j} + u_{j,i}\right) + (\lambda u_{k,k} - \gamma(T - T_o))\delta_{ij}. \tag{2}$$

The strain-displacement relation

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{3}$$

3-The Lord-Shulman equation for heat conduction (Lord and Shulman 1967)

$$\left(1 + \tau_o \frac{\partial}{\partial t}\right) q_i = -k\nabla^2 T. \tag{4}$$

4- Equation for energy conservation according to Pennes (1948)

$$k\nabla^2 T = (1 + \tau_0 \frac{\partial}{\partial t})(\rho c \frac{\partial T}{\partial t} + \gamma T_0 \frac{\partial^2 u}{\partial x \partial t} + \omega_b \rho_b c_b (T - T_b) - Q_m) \tag{5}$$

where *i*, *j*=1, 2, 3 refer to general coordinates. The previous equations, q_i is heat flux, T is the temperature of the tissue, T_b is blood temperature, *t* is time, ρ_b is blood mass density, C_b is the specific heat of blood and σ_{ij} the stresses tensor, ε_{ij} is strain tensor, e_{ij} is strain deviator tensor, μ, λ are Lame's constants and, α_T is the coefficient of linear thermal expansion, $\gamma = (3\lambda + 2\mu)\alpha_T$, ω_b is the blood perfusion rate, c is the specific heat of tissue, τ_o is the thermal relaxation time, and Q_m is the heat produced by metabolic procedures as a result of various physiological processes in the remainder of the body. The production of metabolic heat is expressed by Mitchell *et al.* (1970). Because some materials have a viscoelastic reaction. Viscoelasticity research is important in many fields of materials science, manufacturing processes, and biological systems. The Kelvin-Voigt viscoelastic model is used to describe the viscoelastic characteristics of isotropic materials in this manner. When the viscoelastic effect is considered, the parameters μ, λ, and γ are as follows

$$\mu = \left(1 + \tau_1 \frac{\partial}{\partial t}\right)\mu_e, \lambda = \left(1 + \tau_2 \frac{\partial}{\partial t}\right)\lambda_e, \gamma = (3(1 + \tau_2 \frac{\partial}{\partial t})\lambda_e + 2\,(1 + \tau_1 \frac{\partial}{\partial t})\mu_e)\alpha_T \tag{6}$$

where τ_1 and τ_2 denote the viscoelastic relaxation times.

3. Formulation of the problem

We suppose that the cancer layer's surface exhibits linear, thermo-viscoelastic, homogenous, and isotropic features. The *y*-and *z*-axes are chosen to be perpendicular to the tumor surface, which is thought to be infinitely long in *y*- and *z*-axes, and the *x*-axis to the surface of the tumor. Since just position *x* and time *t* are employed in all functions, the issue can be thought of as a one-dimensional case. The displacement component's form is as follows

$$u_x = u(x,t), u_y = 0, u_z = 0. \tag{7}$$

The strain-displacement relation

$$e = \frac{\partial u}{\partial x}. \tag{8}$$

The stress tensor's *x*-component is

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \gamma(T - T_o). \tag{9}$$

The motion equation has the following form

$$\rho\frac{\partial^2 u_i}{\partial t^2} = (\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial T}{\partial x}. \tag{10}$$

It is possible to formulate the heat equation as

$$k\frac{\partial^2 T}{\partial x^2} = \left(1 + \tau_0\frac{\partial}{\partial t}\right)\left(\rho c\frac{\partial T}{\partial t} + \gamma T_0\frac{\partial^2 u}{\partial x\partial t} + \omega_b\rho_b c_b(T - T_b) - Q_m\right). \tag{11}$$

4. The initial and boundary conditions

In order to derive solutions for the equations, two sets of initial and boundary conditions are required, aligning with the physical description of the model.

$$T(x,0) = T_b, \left.\frac{\partial T(x,t)}{\partial t}\right|_{t=0} = 0,\ u(x,0) = 0, \left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = 0, \tag{12}$$

$$u(0,t) = 0, -k\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=0} = q_o\frac{t^2 e^{-\frac{t}{t_p}}}{16t_p^2},\ u(L,t) = 0, T(L,t) = 0. \tag{13}$$

where t_p indicates the duration of the heat flow pulse and q_0 denotes the heat flow's intensity.

For simplicity, we use the following non-dimensional variables

$$\begin{gathered}(x`, u`) = ʒc(x,u), (t`, \tau_0',, \tau_1') = ʒc^2(t, \tau_o, \tau_1), T` = \frac{T - T_o}{T_o},\\ \omega`_b = \frac{\omega_b}{ʒc^2}, \sigma`_{xx} = \frac{\sigma}{\lambda+2\mu}, Q_m' = \frac{Q_m}{kT_o c_e^2 ʒ^2}, ʒ = \frac{\rho c_e}{k}, c_e = \sqrt{\frac{\lambda+2\mu}{\rho}},\end{gathered} \tag{14}$$

The governing Eqs. (9)-(11) can be stated in the following way by using the non-dimensional components mentioned above and omitting the reference bullets for convenience

$$\sigma_{xx} = \left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x} - a_1\left(1 + \tau_1\frac{\partial}{\partial t}\right)T, \tag{15}$$

$$\frac{\partial^2 u}{\partial t^2} = \left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial x^2} - a_1\left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}, \tag{16}$$

$$\frac{\partial^2 T}{\partial x^2}=\left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\frac{\partial T}{\partial t}+a_2T+a_3\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial t\partial x}-Q_m\right), \tag{17}$$

where $a_1=\frac{T_o\gamma_e}{(\lambda_e+2\mu_e)}$, $a_2=\frac{\rho_b c_b\omega_b}{Kʒ^2c^2}$, $a_3=\frac{\gamma_e T_0}{\rho C_e}, \tau_2=\tau_1$.

5. Methods of solution

Applying the Laplace transform with parameter s defined by the formulas

$$\bar{F}(\chi,s)=L\{f(\chi,t)\}=\int_0^\infty f(\chi,t)e^{-st}\,dt, s>0. \tag{18}$$

Hence, we obtain the following system of differential equations.

$$\bar{\sigma}_{xx}=\zeta_1\frac{d\bar{u}}{dx}-\zeta_2\bar{T}, \tag{19}$$

$$\frac{d^2\bar{u}}{dx^2}-\zeta_3\bar{u}=\zeta_4\frac{d\bar{T}}{dx}, \tag{20}$$

$$\frac{d^2\bar{T}}{dx^2}-\zeta_5\bar{T}=\zeta_6\frac{d\bar{u}}{dx}-\frac{\bar{Q}_m}{s}, \tag{21}$$

with the boundary conditions

$$\frac{d\bar{T}(0,t)}{dx}=-\frac{q_0t_b}{8(st_b+1)^3}, \frac{d\bar{T}(L,t)}{dx}=0, \bar{u}(0,t)=0, \bar{u}(L,t)=0, \tag{22}$$

where $\zeta_1=(1+\tau_1 s)$, $\zeta_2=a_1(1+\tau_1 s)$, $\zeta_3=\frac{s^2}{(1+\tau_1 s)}$, $\zeta_4=a_2$, $\zeta_5=(1+\tau_o s)(s+a_1)$, $\zeta_6=sa_3(1+\tau_o s)(1+\tau_1 s)$.

6. Exact solution

Eliminating *T* from the Eqs. (20) and (21) we get

$$\frac{d^4\bar{u}}{dx^4}-ƺ_1\frac{d^2\bar{u}}{dx^2}+ƺ_2\bar{u}=0, \tag{23}$$

where $ƺ_1=\zeta_3+\zeta_4\zeta_6+\zeta_5$, $ƺ_2=s^2\zeta_5$.

The solutions of Eq. (23) can be written in the form

$$\bar{u}=Ʀ_1e^{ͱ_1x}+Ʀ_2e^{-ͱ_1x}+Ʀ_3e^{ͱ_2x}+Ʀ_4e^{-ͱ_2x}, \tag{24}$$

where $Ʀ_1$, $Ʀ_2$, $Ʀ_3$ and $Ʀ_4$ are parameters to be determined from the boundary conditions, $ͱ_1$, $-ͱ_1$, $ͱ_2$ and $-ͱ_2$ are the roots of the following characteristic equation

$$ͱ^4-ƺ_1ͱ^2+ƺ_2=0, \tag{25}$$

where $ͱ_1$ and $ͱ_2$ are given by $ͱ_1=\pm\sqrt{\frac{ƺ_1+\sqrt{ƺ_1^2-4ƺ_2}}{2}}$, $ͱ_2=\pm\sqrt{\frac{ƺ_1-\sqrt{ƺ_1^2-4ƺ_2}}{2}}$.

Using Eq. (24) in Eqs. (20) and (21), the expression for temperature can be written in the form

$$\bar{T} = \text{ƙ}_1\text{Ꞧ}_1e^{\text{ᴍ}_1x} - \text{ƙ}_1\text{Ꞧ}_2e^{-\text{ᴍ}_1x} + \text{ƙ}_2\text{Ꞧ}_3e^{\text{ᴍ}_2x} - \text{ƙ}_2\text{Ꞧ}_4e^{-\text{ᴍ}_2x} + \frac{\bar{Q}_m}{s\zeta_5}, \tag{26}$$

where $\text{ƙ}_1 = \frac{\text{ᴍ}_1^3 - \text{ᴍ}_1s^2 - \text{ᴍ}_1\zeta_4\zeta_6}{\zeta_4\zeta_5}$ and $\text{ƙ}_2 = \frac{\text{ᴍ}_2^3 - \text{ᴍ}_2s^2 - \text{ᴍ}_2\zeta_4\zeta_6}{\zeta_4\zeta_5}$.

Substituting from Eqs. (24) and (26) into Eq. (19), we obtain

$$\bar{\sigma}_{xx} = \text{Ꞇ}_1\text{Ꞧ}_1e^{\text{ᴍ}_1x} - \text{Ꞇ}_1\text{Ꞧ}_2e^{-\text{ᴍ}_1x} + \text{Ꞇ}_2\text{Ꞧ}_3e^{\text{ᴍ}_2x} - \text{Ꞇ}_2\text{Ꞧ}_4e^{-\text{ᴍ}_2x} - \frac{\zeta_2\bar{Q}_m}{s\zeta_5}, \tag{27}$$

where $\text{Ꞇ}_1 = (1 + s\tau_1)\text{ᴍ}_1 - \zeta_2\text{ƙ}_1$ and $\text{Ꞇ}_2 = (1 + s\tau_1)\text{ᴍ}_2 - \zeta_2\text{ƙ}_2$.

7. Numerical results and discussion

Depending on the Riemann-sum approximation approach used to study the numerical outcomes, a numerical reversal procedure is utilized for the final answer. This approach allows any function in the Laplace domain to be translated into the domain of time, such as

$$f(x,t) = \frac{e^{\text{ʂ}t}}{t}\left\{\frac{1}{2}R_e\left(\bar{F}(x,\text{ʂ})\right) + R_e\left[\sum_{k=0}^{N}(-1)^k\bar{F}\left(x,\text{ʂ} + \frac{ik\pi}{t}\right)\right]\right\}, \tag{28}$$

whereas R_e is the real part and i is the imaginary number unit. For quicker assemblage, numerical methods were decided. that $\text{ʂ} = \frac{3.7}{t}$ that satisfies the above equation. The computations are performed with MATLAB software, and the results are displayed graphically. For numerical computations, exemplary thermal property values for living tissue have been selected (Li *et al.* 2018)

$$\rho_b = 1060(kg)(m^{-3}), \omega_b = 1.87 \times 10^{-3}(s^{-1}), C_b = 3860(J)(kg^{-1})(k^{-1}), T_b = 37\ ℃,$$
$$C = 3600(J)(kg^{-1})(k^{-1}), \tau_o = 0.2(s), \tau_1 = 0.002(s), \lambda_e = 8.27 \times 10^8(kg)(m^{-1})(s^{-2}),$$
$$\mu_e = 3.446 \times 10^7(kg)(m^{-1})(s^{-2}), \rho = 1190(kg)(m^{-3}), k = 0.235(W)(m^{-1})(k^{-1}),$$
$$Q_m = 1.19 \times 10^3(W)(m^{-3}), T_o = 37\ ℃, \alpha_t = 1 \times 10^{-4}(k^{-1}), L = 0.02(m),$$
$$Q_o = 1 \times 10^3(W)(m^{-3})$$

Through the bio-thermoelastic model, we will display the physical quantities' numerical values that were calculated taking into account the times of thermal relaxation and using the aforementioned parameters in Figs. 1-12. We carried out the numerical calculations at time t=0.4 to determine the changes in stress, displacement, and temperature during the distance x through different values of the physical parameters that were studied. It is clear from Figs. 1-12 that each of the stress, displacement, and temperature changes along the x-axis, as a result of the change of physical parameters, which are the relaxation time τ_o, the relaxation time τ_1, the change in the characteristic time of q_o and the pulsating heat flow t_b. The following four groupings can be made up of them: In the first group, Figs. 1-3 show the change of stress, displacement, and temperature under different values of thermal relaxation time $(\tau_0 = 0{,}0.01{,}0.02)$ when $(\tau_1 = \tau_2 = 0.002,\ t_b = 0.2,\ q_o = 10)$.

In the next group, which is the second, Figs. 4-6 show the change of stress, displacement, and temperature under different values of thermal relaxation time $(\tau_1 = \tau_1 = 0, 0.001, 0.002)$ when $(\tau_o = 0.02,\ t_b = 0.2,\ q_o = 10)$.

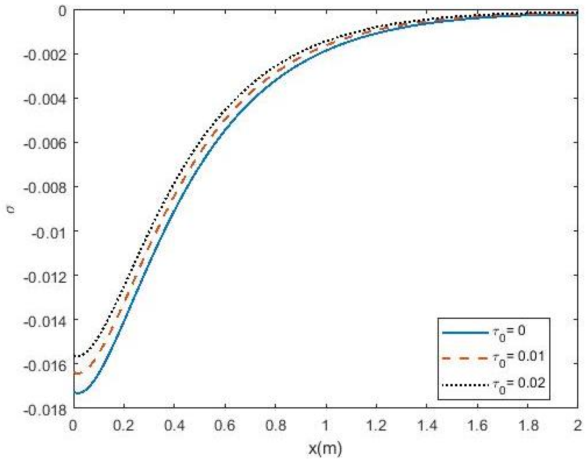

Fig. 1 Variation of stress

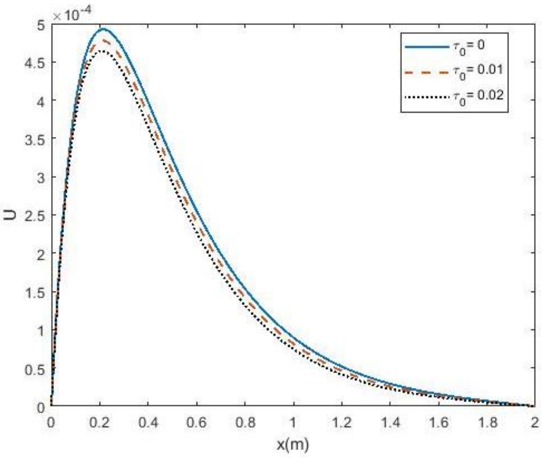

Fig. 2 Variation of displacement

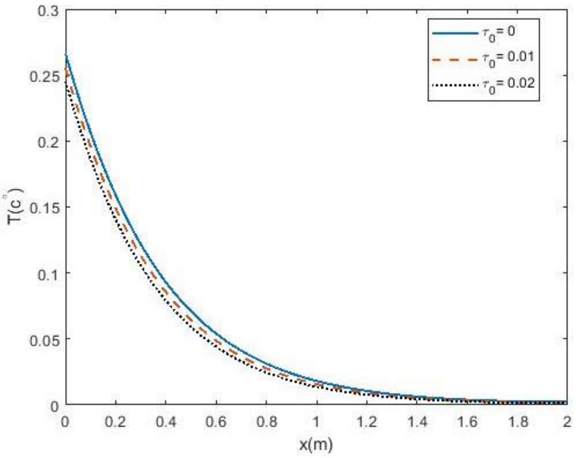

Fig. 3 Variation of temperature

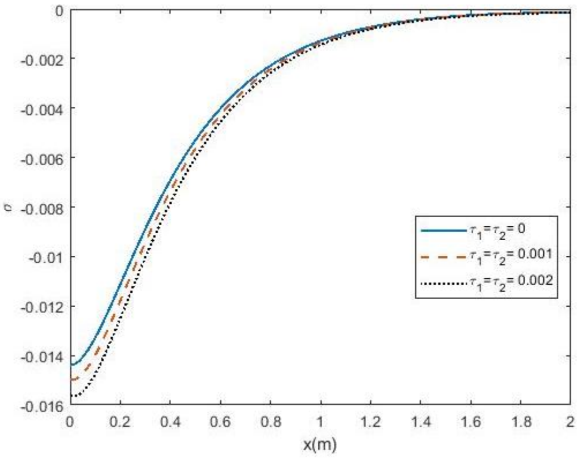

Fig. 4 Variation of stress

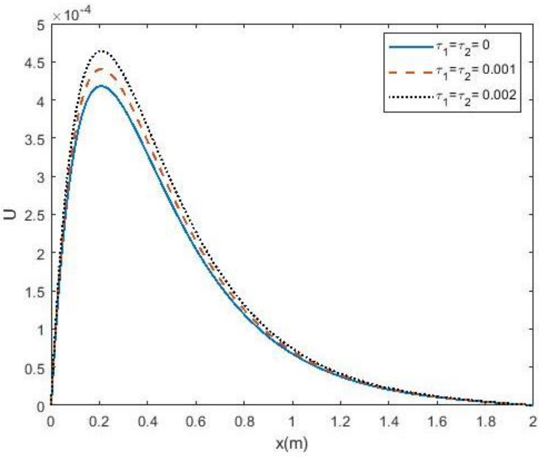

Fig. 5 Variation of displacement

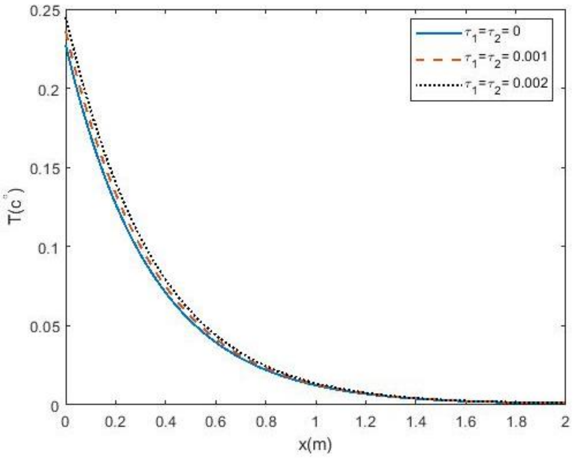

Fig. 6 Variation of temperature

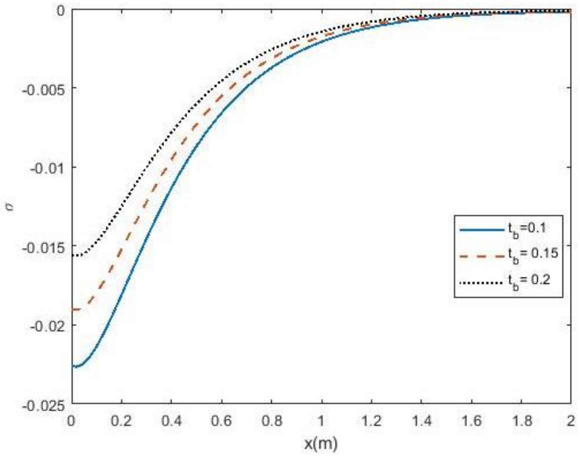

Fig. 7 Variation of stress

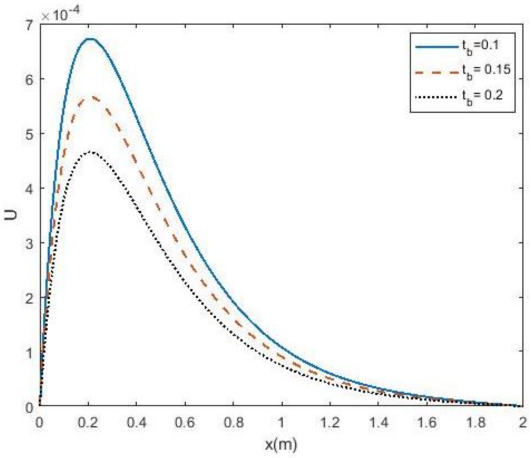

Fig. 8 Variation of displacement

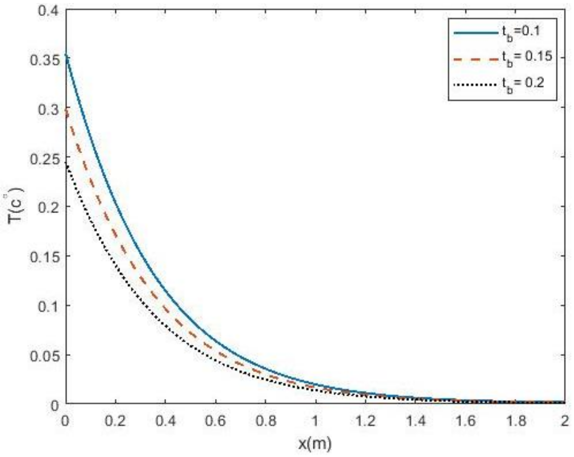

Fig. 9 Variation of temperature

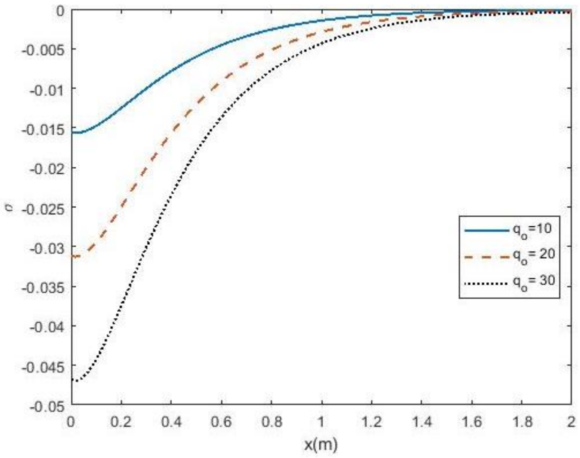

Fig. 10 Variation of stress

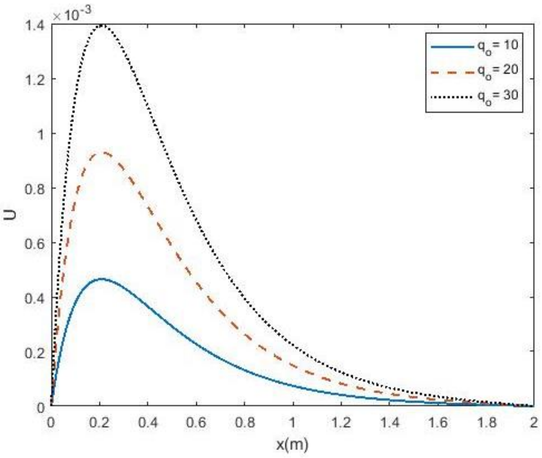

Fig. 11 Variation of displacement

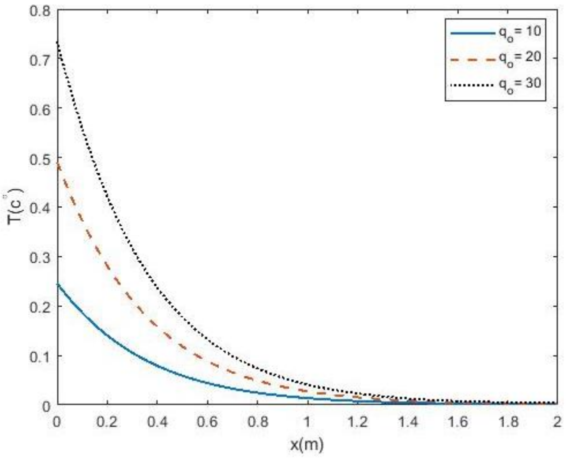

Fig. 12 Variation of temperature

In the next group, which is the third, Figs. 7-9 show the change of stress, displacement, and temperature under different values of the indicates the duration of the heat flow pulse (t_b=0.1, 0.15, 0.2) when ($\tau_o = 0.02,\ \tau_1 = \tau_2 = 0.002,\ q_o = 10$).

In the next group, which is the fourth, Figs. 10-12 show the change of stress, displacement, and temperature under different values of the intensity of the heat flow ($q_o = 10$, 20, 30) when ($\tau_0 = 0.02,\ t_b = 0.2,\ \tau_1 = \tau_2 = 0{,}002$).

We note from the results and through the change of physical parameters that the temperature changes with the change of physical parameters and at its beginning it is on the fabric's surface, where it peaks (*x*=0) due to the heat flow, then it decreases and continues to decrease with the increase of the distance x, and we notice a steadily decreasing temperature. Also, through the change of physical parameters, we notice the change of stress σ_{xx} along the x-axis, and it can be seen that the stress σ_{xx} starts from its highest negative values and ends at zero to comply with the boundary conditions. We also note the change of displacement along the x-axis with the change of physical parameters. We can see that the displacement starts from the lowest values on the surface of the tissue (*x*=0). Before returning to zero, it progressively rises to maximum values near the surface. From the above, we conclude that the maximum amplitude of stress, displacement, and temperature decreases with the increase in thermal relaxation times, which means that thermal relaxation times are suitable for weakening the effect of mechanical thermal diffusion. An increase in the time characteristic of the pulsating heat flow also weakens the effect of thermomechanical diffusion, which is indicated by a decrease in the maximum stress, displacement, and temperature.

8. Conclusions

In this work, the objective was to study the response of living tissue to a sudden pulsing heat flux load using the generalized thermoviscoelasticity model under thermal relaxation times. The biothermoelastic analysis presented in this paper offers valuable insights into the thermal behavior of viscoelastic biological tissues. The findings can aid in optimizing hyperthermia treatments by selecting appropriate design variables. Overall, this study contributes to improving the effectiveness and accuracy of thermal treatments in medical applications.

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