

Optimizing the size and geometry of the 3D frame using an enhanced meta-heuristic algorithm

Behzad Rahimi^a, Seyed Arash Mousavi Ghasemi^{*},
Reza Goli Ejlali^b and Adel Ferdousi^c

Department of Civil Engineering, Tabriz Branch, Islamic Azad University, Tabriz, Iran

(Received February 11, 2025, Revised April 17, 2025, Accepted April 21, 2025)

Abstract. This paper presents a study on optimizing the size and geometry of 3D frames using an improved meta-heuristic algorithm. 3D frames are integral parts of various engineering designs and require efficient optimization techniques to improve their performance and minimize material consumption. The proposed meta-heuristic algorithm builds on existing methods and incorporates novel improvements to increase search efficiency and solution quality. Through rigorous testing on benchmark problems, the algorithm demonstrates superior performance in achieving optimal design solutions that ensure structural integrity while reducing overall weight and cost. The results exhibit the potential of the improved algorithm to advance the field of structural optimization.

Keywords: genetic algorithm; meta-heuristic algorithm; optimization; particle perturbation algorithm; special bending frame

1. Introduction

Due to the increasing cost of construction materials, the optimal design of structures is of particular importance. Although several factors affect the cost of building a structure, the material cost is considered one of the most important factors in building a structure, and its reduction by minimizing the weight or volume of the system is considered. The selection and allocation of sections to the components of a structure that simultaneously meet the design specifications and achieve the minimum weight of the structure is an objective that can be achieved using optimization techniques. To date, numerous optimization techniques have been used by various researchers to solve optimal design problems. Structural optimizations have been proposed, but in recent decades, meta-innovative methods have received more attention due to their advantages over other methods. Therefore, in this study, three of the most well-known meta-innovative methods, including genetic algorithm, ant colony optimization, and particle perturbation optimization, were considered to get familiar with the application method and compare the

*Corresponding author, Assistant Professor, E-mail: Amousavi2000@iau.ac.ir

^a Ph.D., E-mail: bezadrahimiIAU@gmail.com

^b Assistant Professor, E-mail: r.ejlali@iau.ir

^c Assistant Professor, E-mail: Adel.ferdousi@gmail.com

performance of these algorithms in solving the structural optimization problem.

Many researchers deal with optimization problems of structures Zakaulla (2025), Eskandar (2024), Mashayekhi (2024), Heydari (2024, 2011, 2013, 2015, 2019, 2020a, b), Bijari (2023), Heydari and Gholizade (2022), Heydari and Jalali (2017), Heydari *et al.* (2017), Heydari and Kazemi (2009), Heydari and Li (2021), Heydary and Shariati (2018), Ismail *et al.* (2018), Toghroli *et al.* (2018), Rostami *et al.* (2024). In general, the optimization problem is defined as achieving the best result under a given input and subject to certain constraints Jakiela *et al.* (2000). The optimization of the input is usually a problem of cross-sectional area or one of the characteristics of the cross-sections of the structural elements, called the design variable, and the output is often the weight or cost of the structure; Although in some optimization problems, called multi-objective optimization problems, other parameters besides the weight or cost of the structure can be considered as the objective and outcome of the problem. Constraints and limitations are also the problem of optimizing instruments according to the design criteria in the regulations chosen for the design, as well as some of the constraints raised by the client or the designer as an example of the considerations related to the design are applied to the optimization problem. The history of structural optimization can be traced back to 1638, when Galileo “proposed the rule of uniform resistance for a bending beam. Perhaps the first analytical work on structural optimization was by Maxwell (1890). This was followed by the better known work of Mitchell (1904) on the optimal design of trusses under only one loading condition and only under stress constraints. Although these papers were highly idealized, they provided considerable insight into the optimization problem and the design process Minimum weight design was first developed during World War II to find the optimal design of aircraft components under drag constraints by Cox and Smith (1943), Zahersky and Shanley (1944), Glover and Kochenberger (2003).

Since the 1950s, many researchers have worked in this field and introduced various optimization methods Gong (2003). In general, the optimization techniques used in the design of structures can be divided into three different approaches, including mathematical programming (MP) methods of optimality criteria (OC) and meta-heuristic methods Camp *et al.* (1998) from many applications of the above methods in the discussion of device optimization published in publications, however, meta-heuristic search methods have some advantages over the other two methods They have received more attention in recent decades. In the following, these three optimization approaches are briefly described.

2. Mathematical programming methods

The mathematical programming method itself can be divided into linear and non-linear programming. The main feature of linear programming is that in this method the objective function and the dependent constraints are expressed as a linear combination of design variables. To use linear programming techniques for the optimization of structures, the relationship between the objective function and the constraints should be linearized with design variables, taking into account the possibility of nonlinear behavior of structures during severe earthquakes. A linear relationship for modeling a nonlinear response leads to errors. Nonlinear mathematical programming was developed for unconstrained optimization problems. In this way, the famous Kar-Tucker condition provides the necessary conditions to achieve optimal solutions. The direct application of these conditions is very difficult for most problems. The calculation of gradients and the solution of nonlinear equations depends on the direct application of the Kar-Tucker conditions for most engineering problems Camp *et al.* (1998).

2.1 Methods of optimality criteria

The method of optimality criteria was developed based on the collaboration of many researchers including Barnett (1961), Preiger (1968) and Venkaiah *et al.* (1968). It was found that the Kahn-Tucker condition provides the necessary conditions for the optimal solution, and the Lagrange coefficients are used to consider the dependent constraints Camp *et al.* (1998).

2.2 Innovative methods

Meta-innovative methods are other optimization techniques that have been developed in the last two decades Kaveh and Talatahari (2010). These techniques are often random and iterative methods that deal with discrete design variables. However, these methods have also been used to optimize problems with continuous variables. Meta-heuristic methods can be used for unconstrained or constrained optimization problems, such as instrument design problems. One of the main advantages of these methods is their ability to find the optimal solution for the growth of the problem dimensions, as well as their suitability and relative simplicity in solving problems where the implementation of the optimization process becomes complicated due to the complexity and recognizability of the design domain; Cited. The basis of these algorithms is often their similarity to natural and social processes Hasańçebi and Çarbaş (2011).

These methods include evolutionary algorithms (EAS) such as genetic algorithms (GA) and evolutionary strategies (ES), particle perturbation optimization (PSO), Ant Colony Optimization (ACO), Simulation Annealing (SA), Harmony Search (HS), Charged Particle Search (CSS), Colonial Competition Algorithm (ICA) and Big Bang-Big Crunch Algorithm (BBC). In order to improve the performance of some of the above methods, some of them have been used simultaneously in structural optimization in recent years, and new algorithms such as Discrete Ant Colony Particle Perturbation Optimization (DPSACO) have been proposed.

3. Optimization using genetic algorithm

The genetic algorithm was first introduced by Holland (1975). This algorithm simulates the process of natural evolution to create a superior species or generation for survival in the best survival environment. The title of a design (answer) contains a series of values that are assigned to the design variables and represented by a sequence of numbers. The algorithm can initially create a population of initial designs at random. The composition of the individuals of the designs is done to create a population for the next generation of reproduction. In this process, which mimics the natural evolution of living organisms, the composition of individuals is based on a selection process. Each individual is first evaluated and assigned a performance value that corresponds to the value of the target function. The individuals with a high performance value are then used for reproduction. An individual with high fitness can have several chances to mate during the reproduction phase. Therefore, each field in the population is assigned a certain probability of being selected as a parent field based on its merit. The next generation is generated by selecting suitable pairs from the population and using the heuristic operators mutation (1975) and crossover (1975) Erbatur *et al.* (2000). The different steps of a conventional genetic algorithm are shown in Fig. 1 Standard genetic algorithm. The most main steps of this algorithm are described below Cao and Wu (1999).

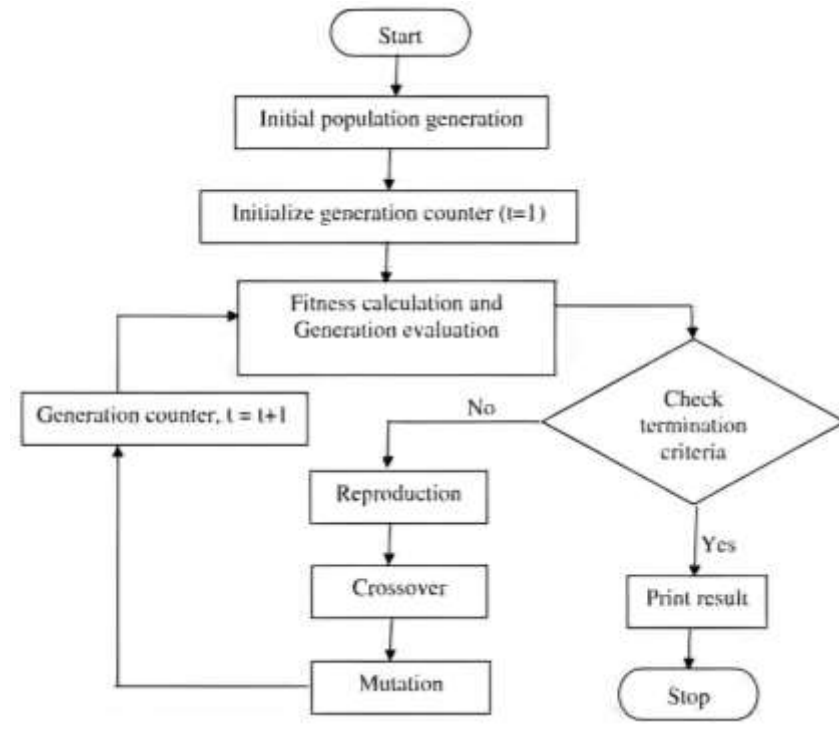


Fig. 1 Standard genetic algorithm

3.1 Initialization

To initialize the genetic algorithm, the answers must be coded. An answer, or in GA terms, a chromosome, usually takes the form of a sequence of zeros and a one or a sequence of integers. Each of these binary or integer components in the chromosomes is classified as a gene. The number of bits that should be used to describe the parameters depends on the problem. In this way, each solution to the problem is defined as a sequence of digits of a certain length. Normally, the initial population consists of m answers, which are chosen completely randomly. The selection operator of the genetic algorithm selects the population of parents to generate the next generation. However, the most main method for implementing this operator is the roulette wheel method. This method uses a probability distribution for selection, where the probability of selecting each string as a parent is proportional to its value.

Fig. 2 shows a simple example of the roulette wheel method. In this example, it is assumed that the population consists of five fields with the values 25 17 17 9 32. As shown. The probability of each person (string) being selected is proportional to the height of the part of the roulette wheel to which it is assigned. The random numbers are normalized to a normalized and are used in sequence to select parent strings, for example the random numbers 013 and 068. They select the first and fourth courses.

Crossover is an important random operator in the genetic algorithm that generates new chromosomes (children) from the chromosomes of two parents by combining the information it receives from the parents. There are several different ways to implement this operator. The

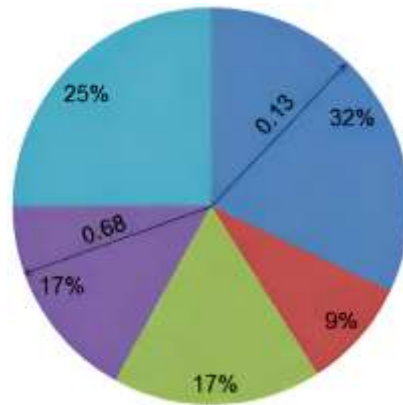


Fig. 2 The method of selection by means of the roulette wheel

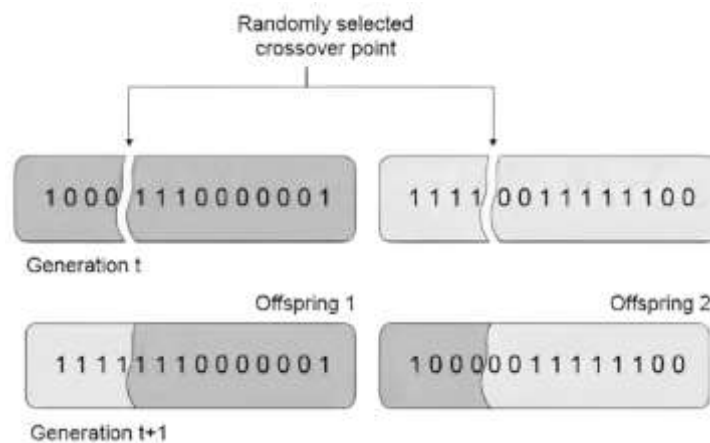


Fig. 3 Intersection operator

intersection method used in the conventional genetic algorithm is the one-point intersection. In this method, as shown in Fig. 3, for a chromosome with a length of l , a random number (C) between 1 and l is generated and the genes of the two parents are merged after and before this cut point. Normally, the intersection probability is between 0.6 and 0.95 Cao and Wu (1999). In addition to the one-point intersection method, other intersection methods such as the two-point intersection method, the distributed method, the intermediate method, the heuristic method and the arithmetic method are also used in the genetic algorithm.

3.2 Mutation

Mutation is another important operator that randomly changes the genes of each chromosome independently. A common method for mutation used in the conventional genetic algorithm is to generate a random number (v) between one and l and produce a random change in (v) of the t th member of the desired string. This process is illustrated in Fig. 4. In general, the mutation probability for a bit is between 0.001 and 0.01 Cao and Wu (1999).

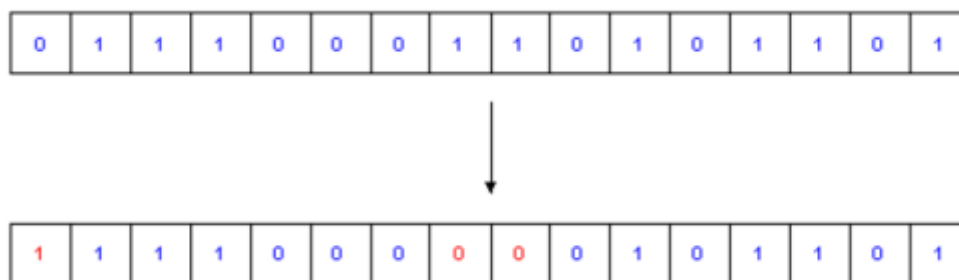


Fig. 4 Mutation operator

This operator makes it possible to create properties that are not present in the parent strings for their children. Without this feature, some areas of the search space may never be discovered Camp *et al.* (1998).

Besides the mentioned method, there are other methods in the genetic algorithm, such as mutation depending on the uniform and consistent Gaussian constraint. In this way, the three basic genetic operators of selection, crossover and mutation are used to generate the next generation of the population, which are likely to be better individuals than the current generation. The basic philosophy of a GA is that at any point in the process, by combining individuals with higher merit, the average merit of the population should be increased and the algorithm should converge to the optimal point Erbatur *et al.* (2000).

So far, the standard genetic algorithm (GA) and its improved versions have been used by various researchers to optimize different engineering problems. Some of these applications can be found in 63 65.

4. Optimizing the ant colony

Ant Colony Optimization (ACO) is an example of a meta-heuristic algorithm originally proposed by Cleroni *et al.* for TSP. This method is derived from the behavior of real ants, which are able to find the shortest path between their nest and the food source by using the effect of a substance called pheromone. To illustrate the ants' behavior in finding the shortest path, an experimental setup as shown in Figure 5 is used. There may be a path along the movement of the ants, e.g. from the food source (A) to the nest (F) and vice versa, when suddenly an obstacle appears on the path and the path is cut off. Therefore, ants going from A to E in position B or in the opposite direction in position D must choose one of the two paths on the right or left. Their choice depends on the intensity of the pheromone left by the previous ants. A higher pheromone value on the right-hand path gives an ant an incentive and a higher probability of taking this path. The first ant to reach point B (or) (D) has the same probability of moving on, as there are no pheromones from the previous ant on either path. Since BCD is shorter, the first ant following BCD will reach point D B HD earlier than the slowest ant. Therefore, an ant returning from E to D will find a stronger pheromone effect on the DCB path than on half of all ants that happened to choose the BCD path. As a result, they prefer the DCB route over the DHB route due to probability. This causes the amount of pheromone to grow faster on the shorter route than on the longer route, increasing the probability that each ant will choose the shorter route. Eventually, all ants will quickly choose the shorter route Dorigo *et al.* (1996).

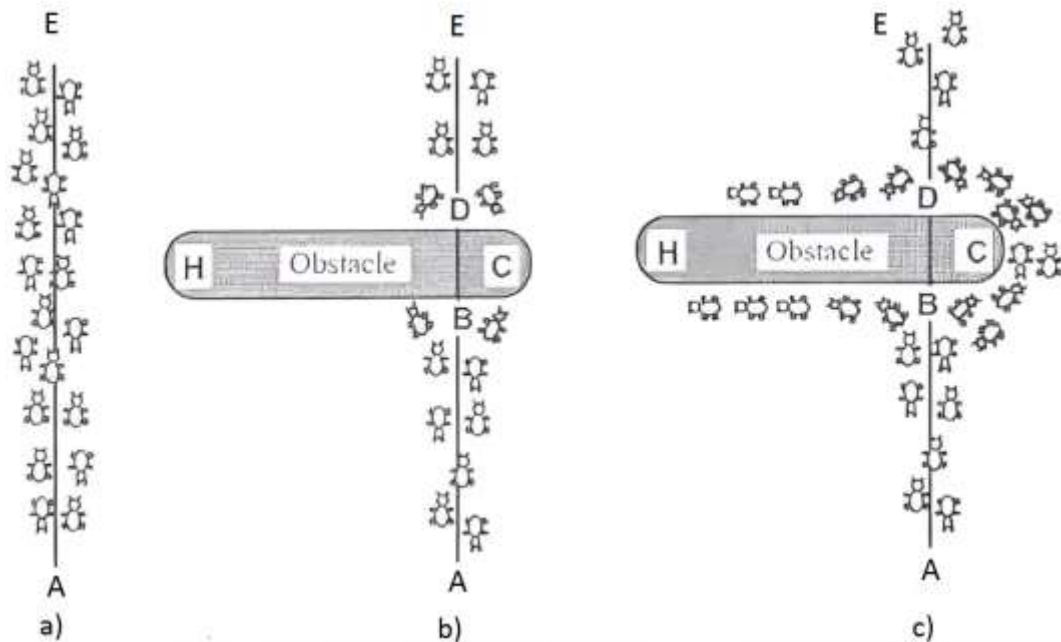


Fig. 5 An example with a real ant

4.1 Application of the ant algorithm for optimization of devices

By simulating the behavior of real ants, various optimization problems can be solved, including the problem of optimal design of structures, just as real ants are able to find the shortest path between the nest and the food source. Artificial ants will also be able to find the lowest weight for a structure. The paths to be taken by the artificial ant colony are actually the allowable sections to be considered for each link of the structure. Thus, between the two nodes of the structure, there is the number of allowable sections defined for the member between these two circular paths in the direction of movement of the ants. This situation is illustrated in Figure 6 for a two-story frame with one span and six members Camp Charles *et al.* (2005).

Each ant of the colony starts its migration randomly from a link of the structure and leaves a pheromone trail on this path (section) with a specific section to the corresponding link and then moves to the next link. Therefore, at the end of its journey, each ant creates a blueprint by assigning a series of sections to the members of the structure. At the end of each cycle of the ant colony, including m , ant m creates a blueprint for the structure, for example in Figure 6 of the journey Each ant is complete when the ant has given each of these nine frames a specific section. At the beginning of the optimization process, when there is no effect of the pheromone on the routers or the pheromone is the same on all paths, the selection of paths for the members is based on a parameter called visibility. Visibility is an additional defined parameter that allows the selection of shorter paths. It provides light for artificial ants. Through the accumulation of pheromone effects on the paths chosen by previous ants, the selection of ants gradually occurs based on a combination of pheromone effects and visibility parameters, ending the journey of all ants in a colony. The ACO cycle is completed and the design m for the structure is created. In this step, the colony is evaluated and the corresponding merit values are determined for each ant.

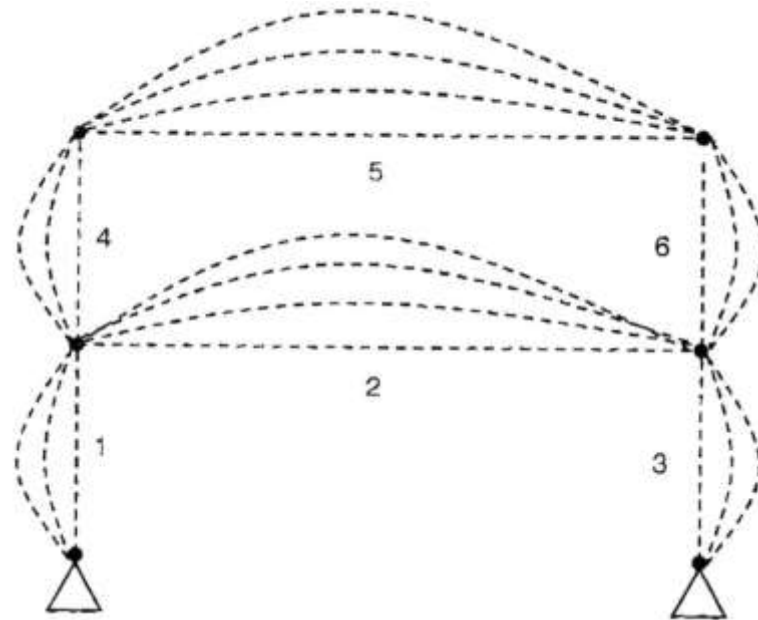


Fig. 6 Possible virtual paths for a frame with six members Camp Charles *et al.* (2005)

To increase the pheromone level on the better routes, the pheromone information is updated and the next cycle begins. By increasing the number of cycles and accumulating pheromone effects on specific paths specific to each member, most ants in the colony travel the same paths on their journey and the algorithm converges to the optimal point.

5. Optimization by particle perturbation method

Particle Perturbation Optimization (PSO) was first proposed by Kennedy and Eberhart Sammut and Webb (2011) as a method for optimizing nonlinear functions based on simulating the collective behavior of animals such as flocks of birds or fish. A group of birds or fish search for food or protect themselves from hunting in a very simple way. When a member of the swarm discovers a desired route. The other members of the group will quickly follow it. Each member searches for the best position based on their own best experience and the experience of the others. The PSO method is based on the simulation of the behavior described above Plevris *et al.* (2011).

The PSO method consists of a number of particles that have a defined position and velocity and are initially placed randomly in a multidimensional search space of an target function. Each particle represents an actual solution to the problem and the degree of its goodness is The title of a problem solution depends on the value of the corresponding objective function. The set of particles is generally considered as a cluster. These particles fly in the multidimensional search space and have two important reasoning capabilities, namely each particle's memory of its own best position and its knowledge of the overall best position or neighborhood. is a particle. In an optimization problem in the form of minimizing a function, the best position objective refers to a position of the particle (x) whose value corresponds to the following relation. Numerically, the position of a particle is updated as follows:

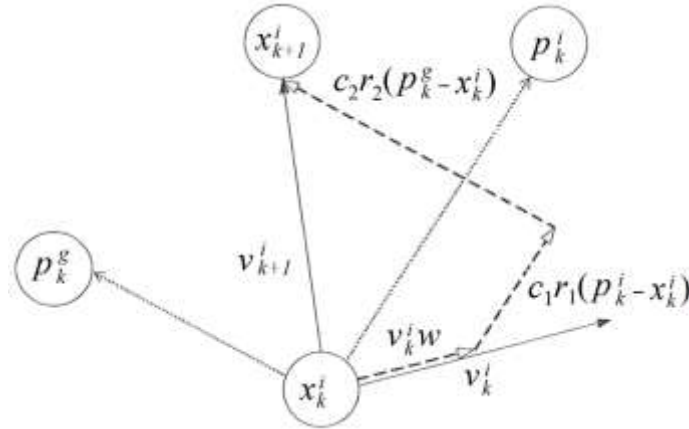


Fig. 7 Position and speed update in PSO

$$x_{k+1}^i = x_k^i + v_{k+1}^i \Delta t \quad (1)$$

The corresponding objective function is the smallest value, $\min f(x)$. The members of a group of good positions transfer to each other and adjust their positions and velocities based on the information about good positions, so that for each particle there is a set of design variables (x) and corresponding velocities (v) that represent an actual solution to an optimization problem Gomes (2011).

5.1 Mathematical formulation of PSO

The optimization process by the particle perturbation method is inherently a random process that uses a velocity vector to update the current position of each particle in the cluster. Its velocity vector is based on the memory obtained by each particle and the information obtained by the whole. The cluster is updated. Therefore, the position of each particle in the cluster is determined based on the social behavior of the cluster, so that the social behavior of the cluster is adjusted at any moment by returning to the desired areas of the space that have already been discovered and also searching for better positions with its environment. The velocity vector of each particle in each step is calculated as follows.

$$v_{k+1}^i = \omega v_k^i + c_1 r_1 \frac{(p_k^i - x_k^i)}{\Delta t} + c_2 r_2 \frac{(p_k^g - x_k^i)}{\Delta t} \quad (2)$$

In relation 2, the velocity vector in the repetition P and P are the best position of the particle and the best position among all particles in the batch up to the repetition, respectively, and ω and ω are two random numbers between zero and one. The remaining terms are the configuration parameters that play an important role in the convergence behavior of PSO, so the coefficients C_1 and C_2 are the cognitive and social parameters, respectively, indicating the degree of confidence in the best solution found by each particle in the batch and the best answer found by the overall category. High inertia weights cause high speed updates. In contrast to the low values of inertia, it causes the updates of the velocity vector to concentrate on the regions adjacent to the explored

Table 1 Sections used for beams and columns

Sections for columns																			
W5X19	W6X12	W6X25	W8X15	W8X28	W8X40	W8X67	W10X100	W12X45	W14X53	W14X82	W14X311	W14X455	W14X730	W18X119	W18X143	W18X175	W18X211	W24X250	W40X372
Sections for beams																			
W5X16	W6X16	W8X15	W8X21	W8X40	W8X58	W10X112	W12X50	W12X136	W18X106	W21X132	W24X162	W24X250	W24X370	W27X539					

region of each particle in the design space. Fig. 7 shows the updating of particle positions and velocities in a two-dimensional vector space. As can be seen in the figure, the movement of the particle in the design space is influenced by its velocity in the previous iteration and a random measurement, which is a combination of the previous best position and the best overall position using the parameters C and $C2$. The coefficient C tilts the position of the particle towards the best solution found by that particle, and the coefficient and $C2$ tilts it towards the best overall solution found by the whole group.

6. Case study

To compare the efficiency and accuracy of the investigated meta-heuristic algorithms, these algorithms were applied in this part to a metal bending frame with 6 floors and 3 openings and their performance was compared. In the mentioned frame, the height of the second floor is 4 meters and the other floors are 3 meters high. The length of the openings is also 4 meters. The uniform dead load of all floors is 2 kN/m, the uniform live load is 219,687 m² and the uniform live load of the roof is 1.5 kN/m. They are applied to the beams. The span of each beam is 5 meters and the specifications of the steel used are as follows for beams and columns. The cross-sections of all W-beams are taken into account and 15 or 20 cross-sections can be selected for beams and columns.

$$\begin{aligned}
 E &= 1.99e11 & \text{N/m}^2 \\
 F_y &= 3.44e8 & \text{N/m}^2 \\
 F_{ye} &= 3.79e8 & \text{N/m}^2 \\
 F_u &= 4.48e8 & \text{N/m}^2 \\
 F_{ue} &= 4.92e8 & \text{N/m}^2 \\
 G &= 99.2e9 & \text{N/m}^2
 \end{aligned} \tag{3}$$

Nonlinear dynamic analysis was used to investigate the effects of earthquakes on the structure. To model the nonlinear behavior of the beams, a combination of elastic elements and nonlinear rotational springs was used in both beams. To model the hysteresis behavior of the springs, the nonlinear damping model of Ibarra Krawinkler was used. The model by Gupta and Krawinkler was used to model the connecting spring.

6.1 Loading combinations

According to AISC/SEI 10-7 and to the values of dead and live loads, the following load combinations predominate among the other load combinations and therefore the following load combinations are considered for the design and the structure must meet the desired limits under all load combinations. It must satisfy the following:

$$\begin{aligned}
 &1.2D + 1.6L + 0.5L_r \\
 &1.2D + 1.0L + 1.6L_r \\
 &1.2D + 1.0L \pm 1.0E \\
 &0.9D \pm 1.0E
 \end{aligned} \tag{4}$$

Constraints considered for the optimization problem:

- Limitations of AISC Code 100-360

When checking the strength of members according to the AISC360-10 rules for all members, the mutual effect of axial and bending forces in symmetrical sections of a plane frame is controlled by the following relationship:

$$1.0 - \left[\frac{P_r}{P_c} + \frac{8}{9} \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \right] > 0 \tag{5}$$

and

$$1 - \left[\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \right] \geq 0 \tag{6}$$

- Limitations of AISC Code 10-341

Weak Axis Width to Thickness Ratio Limit: According to the AISC 10-341 provisions, which are considered ductility requirements for the width to thickness ratio of compression members, a section must meet the following limits to be considered a compression section:

$$\begin{aligned}
 &b/t \leq 0.30 \sqrt{E/F_y} \\
 &\text{If } C_a \leq 0.125; \\
 &h/t_w \leq 2.45 \sqrt{E/F_y} (1 - 0.93C_a) \\
 &\text{If } C_a > 0.125; \\
 &h/t_w \leq 0.77 \sqrt{E/F_y} (2.93 - C_a) \\
 &\geq 1.49 \sqrt{E/F_y}
 \end{aligned} \tag{7}$$

where

$$C_a = \frac{P_u}{\phi_c P_y} \tag{8}$$

Table 2 Comparison results of the accuracy of meta-heuristic algorithms

	Genetic algorithm	Ant colony algorithm	Particle Confusion Algorithm
Optimal weight of the best thigh	11367	11784	12104
Average optimal weight	13549	15971	16743
standard deviation	2624	6279	6143
Number of failed runs	12	29	35

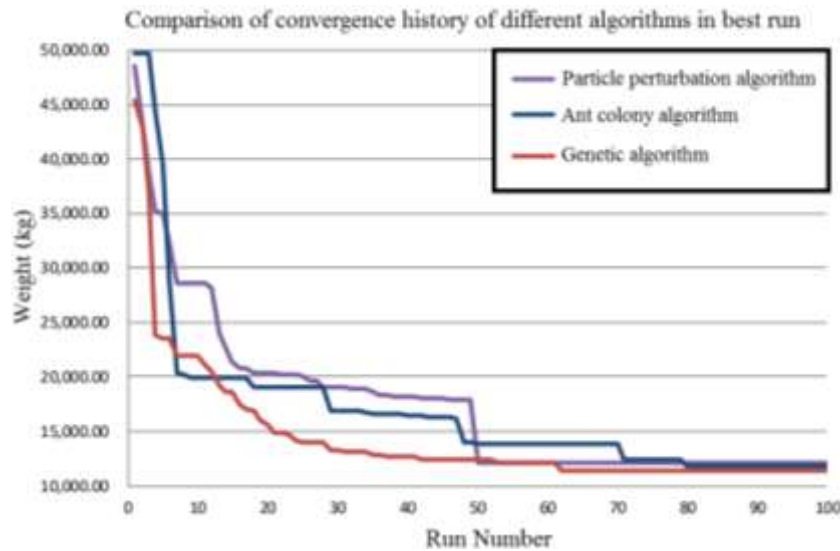


Fig. 8 Comparison of convergence speed of different algorithms

Criterion of strong column - weak beam: If a number of columns of one floor fail at the same time, the floor mechanism may be created and the structure will have a soft floor and lead to a lot of damage. ; Therefore, the sections of the column and related beams should be selected in such a way that plastic joints are created in the beams or connecting springs and the columns remain as elastic as possible. For the occurrence of this situation, AISC 10- 341 expresses the adverb of strong column weak beam in the form of a relation

$$\Delta_a - \delta_M > 0 \quad (9)$$

Restrictions on the change of the lateral position of the floor: the relative lateral position change of the floors should not exceed a certain value. This relative displacement is determined from the relationships of Sections 128 and 12, 12 of AISC/SEI Regulation 7-10. The maximum relative inelastic drift must satisfy the condition of Eq. 10:

$$\Delta_a - \delta_M > 0 \quad (10)$$

7. Results and discussion

To compare the performance of the studied algorithms according to the random nature of the

meta-heuristic algorithms, the optimization of the target structure was performed 100 times with each of the algorithms and the algorithms were repeated 100 times in each optimization. The table shows the optimal weight value extracted for the best case, the average of all runs, the standard deviation of the results of each algorithm and the number of cases in which the algorithm failed to converge.

As can be seen in Table 2, the genetic algorithm stands better in terms of accuracy and performance of the algorithm and seems to be a more reliable algorithm for solving the structural optimization problem, although this algorithm failed to produce convergence in 12% of the runs, but the ant colony ratio with 29% and particle disturbance with 35% is in a much better condition. In addition, Fig. 8 compares the convergence curve of all three algorithms for the best run of each algorithm. As can be seen in the figure, the convergence trajectory of the genetic algorithm for the best pass approaches the final optimal solution with a better speed and in a relatively uniform way, while the two algorithms, despite the reasonable speed in the initial iterations, in the final iterations and in fact they have some problems in approaching the final optimal solution.

8. Conclusions

In this study, three algorithms including genetic, particle perturbation and ant colony are briefly introduced. These algorithms were used to optimize metal bending frames and compared with each other in terms of their performance, accuracy, efficiency and convergence rate. The results show the better performance of the genetic algorithm in solving the studied optimization problem, such that in the average condition it was up to 19 times better than the particle perturbation algorithm and up to 15 times better than the ant colony algorithm. It was observed that the genetic algorithm gradually reached the optimal solution at a better rate compared to the other two algorithms.

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